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Inequalities on coefficients for certain classes of m-fold symmetric and bi-univalent functions equipped with Faber polynomial

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Abstract: In this work, considering a new subclass of bi-univalent functions which are m-fold symmetric and analytic functions in the open unit disk, we determine estimates for the general Taylor–Maclaurin coefficient of the functions in this class. Furthermore, initial upper bounds of coefficients for m-fold symmetric, analytic and bi-univalent functions were found in this study. For this purpose, we used the Faber polynomial expansions. In certain cases, the coefficient bounds presented in this paper would generalize and improve some recent works in the literature. We hope that this paper will inspire future researchers in applying our approach to other related problems.

Key words: Analytic functions, univalent functions, bi-univalent functions, coefficient estimate

1. Introduction, preliminaries, known results

Let A denote the class of functions $f(z)$ which are analytic in the open unit disk $\mathcal{U} = \{z \in C :| z |< 1\}$ and normalized by the conditions $f(0) = f^{'}(0) - 1 = 0$ showing in the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1.1}$$

We denote by the subclass of all functions in $\mathcal{S}$ that are univalent in $\mathcal{U}$. It is well known that every function f has an inverse f^{-1}, which is defined by

$$f^{-1}(f(z)) = z \qquad z \in \mathcal{U},$$

$$f(f^{-1}(w)) = w \qquad \left(|w| < r_0(f), \quad r_0(f) \geq \frac{1}{4}\right),$$

where

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots . \tag{1.2}$$

A function $f \in A$ is said to be bi-univalent in $\mathcal{U}$ if both f and f^{-1} are univalent in $\mathcal{U}$. Let Σ denote the class of all functions $f \in A$ which are bi-univalent in $\mathcal{U}$ and are given by the equation (1.1).

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In 1967, Lewin [6] investigated the class Σ and showed that $|a_2| < 1.51$. Brannan and Clunie improved Lewin's result to $|a_2| < \sqrt{2}$. On the other hand, Netanyahu [8] showed that $\max|a_2| < \frac{4}{3}$. Brannan and Taha [3] introduced certain subclasses of the bi-univalent function class Σ similar to the familiar subclasses. On the other hand, the Faber polynomials introduced by Faber [4] play an important role in various areas of mathematical sciences, especially in geometric function theory. Let $m \in \mathbb{N} = \{1, 2, 3, ...\}$. A domain E is known as m-fold symmetric if a rotation of E around origin with an angle $\frac{2\pi}{m}$ maps E on itself. A function $f(z)$ analytic in $\mathcal{U}$ is said to be m-fold symmetric $(m \in \mathbb{N})$

$$f(e^{i\frac{2\pi}{m}}z) = e^{i\frac{2\pi}{m}}f(z).$$

Especially, every f is 1-fold symetric every odd f is 2-fold symmetric. We denote by $\mathcal{S}_m$ the class of m-fold symmetric univalent functions in $\mathcal{U}$. A simple argument shows that $f \in \mathcal{S}_m$ is characterized by having a power series of the form

$$f(z) = z + \sum_{k=1}^{\infty} a_{mk+1}z^{mk+1} \qquad z \in \mathcal{U}, m \in \mathbb{N}. \tag{1.3}$$

Each bi-univalent function generates an m-fold symmetric bi-univalent function each integer $m \in \mathbb{N}$.

In [9] Srivastava et al. defined m-fold symmetric bi-univalent function analogues to the concept of m-fold symmetric univalent functions. They gave some important results, such as each function $f \in \Sigma$ generates an m-fold symmetric bi-univalent function for each $f \in \mathbb{N}$. In their work, for normalized form given by (1.3), they obtained the series expansion for f^{-1} as follows:

$$\begin{aligned} g(w) &= w - a_{m+1}w^{m+1} + [(m+1)a_{m+1}^2 - a_{2m+1}]w^{2m+1} \\ &\quad -[\frac{1}{2}(m+1)(3m+2)a_{m+1}^3 - (3m+2)a_{m+1}a_{2m+1} + a_{3m+1}]w^{3m+1} + ... \\ &= w + \sum_{k=1}^{\infty} A_{mk+1}w^{mk+1} \end{aligned} \tag{1.4}$$

where $f^{-1} = g$. We denote by Σ_m the class of m-fold symmetric bi-univalent functions in $\mathcal{U}$. The functions in the class Σ are said to be one-fold symmetric.

For $m = 1$, the equality (1.4) coincides with the function (1.2) of the class Σ_m. Here are some remarkable examples on m-fold symmetric bi-univalent functions (see, e.g., [7] and [9]):

$$\left(\frac{z^m}{1-z^m}\right)^{\frac{1}{m}}, \quad [-\log(1-z^m)]^{\frac{1}{m}}, \quad \left[\frac{1}{2}\log\left(\frac{1+z^m}{1-z^m}\right)^{\frac{1}{m}}\right],$$

with the corresponding inverse functions

$$\left(\frac{w^m}{1-w^m}\right)^{\frac{1}{m}}, \left(\frac{e^{w^n}-1}{e^{w^n}}\right)^{\frac{1}{m}}, \left(\frac{e^{2w^m}-1}{e^{2w^m}+1}^{\frac{1}{m}}\right).$$

The coefficient problem for m-fold symmetric analytic bi-univalent functions is one of the favorite subjects of geometric function theory. Bounds for the initial coefficients of different classes of m-fold symmetric bi-univalent functions were also investigated by other authors (see, e.g., [5], [9], and [10]). In this paper, we use the Faber polynomial expansions for a subclass of m-fold symmetric analytic bi-univalent functions to determine estimates for the general coefficient bounds $|a_{mk+1}|$. Firstly, we consider a comprehensive class of m-fold symmetric analytic bi-univalent functions defined by Kant and Vyas [11].

2. The class $\mathcal{S}_{\Sigma_m}(\lambda,\tau,\beta,\xi)$

Definition 1 For $0\leq\lambda\leq 1, 0\leq\beta\leq 1$, and $m\in\mathcal{N}$, a function $f\in\Sigma_m$ given by (1.1) is said to be in class $\mathcal{S}_{\Sigma_m}(\lambda,\tau,\beta,\xi)$ if the following two conditions are satisfied:

$$1+\frac{1}{\tau}\left[\frac{zf'(z)+\lambda z^2f''(z)}{\lambda zf'(z)+(1-\lambda)f(z)}-1\right]\in P_\xi(\beta) \qquad and \tag{2.1}$$

$$1+\frac{1}{\tau}\left[\frac{wg'(w)+\lambda w^2g''(w)}{\lambda wg'(w)+(1-\lambda)g(w)}-1\right]\in P_\xi(\beta), \tag{2.2}$$

where $\tau\in\mathcal{C}-\{0\}$, the function $g=f^{-1}$ given by (1.4), and $z,w\in\mathcal{U}$. In order to derive our main results, we shall need the following Lemma A.

Lemma A Let the function $\Phi(z)=1+h_1z+h_2z^2+....z\in\mathcal{U}$ such that $\Phi\in P_\xi(\beta)$ then

$$|h_n|\leq\xi(1-\beta), n\geq 1.$$

3. Coefficient estimates

In general, for any $n\geq 2$ and for any $p\in\mathbb{R}$ an expansion is [1],

$$K_n^p=pa_n+\frac{p(p-1)}{2}D_n^2+\frac{p!}{(p-3)!3!}D_n^3+...+\frac{p!}{(p-n)!n!},$$

where $D_n^l=D_n^l(a_2,a_3,...a_n)$ and by [2]

$$D_n^l=D_n^l(a_2,a_3,...a_n)=\sum_{n=2}^{\infty}\frac{l!}{i_1!...i_{n-1}!}a_2^{i_1}...a_n^{i_{n-1}},$$

and the sum is taken over all non-negative integers $i_1,....,i_{n-1}$ satisfying

$$i_1+i_2+...+i_{n-1}=li_1+2i_2+...+(n-1)i_{n-1}=n-1.$$

It is clear that $D_n^n(a_2,a_3,...a_n)=a_2^n$.

Similarly, using the Faber polynomial expansion of functions $f\in\mathcal{A}$ of the form (1.3), that is

$$f(z)=z+\sum_{k=1}^{\infty}a_{mk+1}z^{mk+1}=z+\sum_{k=1}^{\infty}K_k^{\frac{1}{m}}(a_2,a_3,...a_{k+1})z^{mk+1}$$

the coefficients of its map $g = f^{-1}$ may be expressed as

$$g(w) = f^{-1}(w) = w + \sum_{k=1}^{\infty} \frac{1}{mk+1} K_k^{-(m+1)}(a_{m+1}, a_{2m+1}, ...a_{mk+1}) w^{mk+1}. \tag{3.1}$$

Consequently, for functions $f \in S_\Sigma(\lambda, \tau, \beta, \xi)$ of the form (1.3), we can write

$$1 + \frac{1}{\tau}\left[\frac{zf^{'}(z) + \lambda z^2 f^{''}(z)}{\lambda z f^{'}(z) + (1-\lambda)f(z)} - 1\right] = 1 + \sum_{k=1}^{\infty} F_k(a_{m+1}, a_{2m+1}, ..., a_{mk+1}) z^{mk}. \tag{3.2}$$

In particular, the first three terms of $F_k(a_{m+1}, a_{2m+1}, ..., a_{mk+1})$ are

$$F_1 = \frac{1}{\tau} m(\lambda m + 1) a_{m+1},$$

$$F_2 = \frac{1}{\tau} m[2(2\lambda m + 1) a_{2m+1} - (\lambda m + 1)^2 a_{m+1}^2],$$

$$F_3 = \frac{1}{\tau} m[3(3\lambda m + 1) a_{3m+1} - 3(\lambda m + 1)(2\lambda m + 1) a_{m+1} a_{2m+1} + (\lambda m + 1)^3 a_{m+1}^3].$$

In our first theorem, we introduced an upper bound for the coefficients $|a_{mk+1}|$ of m-fold symmetric analytic bi-univalent functions in the class $\mathcal{S}_{\Sigma_m}(\lambda, \tau, \beta, \xi)$.

Theorem 3.1 *For $0 \le \lambda \le 1$ and $0 \le \beta \le 1$, let the function $f \in \mathcal{S}_{\Sigma_m}(\lambda, \tau, \beta, \xi)$ be given by (1.3). If $a_{mj+1} = 0$, $0 \le j \le k-1$, then*

$$|a_{mk+1}| \le \frac{\xi(1-\beta)|\tau|}{mk(\lambda mk + 1)}, \qquad (k \ge 2).$$

Proof For the function $f \in \mathcal{S}_{\Sigma_m}(\lambda, \tau, \beta, \xi)$ of the form (1.3); we have the expansion (3.2) and for the inverse map $g = f^{-1}$, considering (1.4) and (3.1), we obtain

$$1 + \frac{1}{\tau}\left[\frac{wg^{'}(w) + \lambda w^2 g^{''}(w)}{\lambda w g^{'}(w) + (1-\lambda)g(w)} - 1\right] = 1 + \sum_{k=1}^{\infty} F_k(A_{m+1}, A_{2m+1}, ..., A_{mk+1}) w^{mk}, \tag{3.3}$$

with

$$A_{mk+1} = \frac{1}{mk+1} K_k^{-(mk+1)}(a_{m+1}, a_{2m+1}, ..., a_{mk+1}) \qquad k \ge 1.$$

On the other hand, since $f \in \mathcal{S}_\Sigma(\lambda, \tau, \beta, \xi)$ and $g = f^{-1} \in \mathcal{S}_{\Sigma_m}(\lambda, \tau, \beta, \xi)$, by definition, there exist two positive real part functions:

$$p(z) = 1 + \sum_{k=1}^{\infty} p_{mk} z^{mk} \quad and \quad q(w) = 1 + \sum_{k=1}^{\infty} q_{mk} w^{mk}, \tag{3.4}$$

where $\Re(p(z)) > 0$ and $\Re(q(z)) > 0$ in $\mathcal{U}$. So that

$$1 + \frac{1}{\tau}\left[\frac{zf^{'}(z) + \lambda z^2 f^{''}(z)}{\lambda z f'(z) + (1-\lambda)f(z)} - 1\right] = p(z),$$

$$\frac{1}{\tau}\left[\frac{zf^{'}(z) + \lambda z^2 f^{''}(z)}{\lambda z f'(z) + (1-\lambda)f(z)} - 1\right] = \sum_{k=1}^{\infty} K_k^1(p_m, p_{2m}, ..., p_{km})z^{km}, \tag{3.5}$$

and

$$1 + \frac{1}{\tau}\left[\frac{wg^{'}(w) + \lambda w^2 g^{''}(w)}{\lambda w g'(w) + (1-\lambda)g(w)} - 1\right] = q(w),$$

$$\frac{1}{\tau}\left[\frac{wg^{'}(w) + \lambda g^2 g^{''}(w)}{\lambda w g'(w) + (1-\lambda)g(w)} - 1\right] = \sum_{k=1}^{\infty} K_k^1(q_m, q_{2m}, ..., q_{km})w^{km}. \tag{3.6}$$

From Lemma A, $|p_k| \leq \xi(1-\beta)$ and $|q_k| \leq \xi(1-\beta)$ for all $k \geq 1$.

Comparing the corresponding coefficients of (3.2) and (3.4), for any $k \geq 1$

$$F_k(A_{m+1}, A_{2m+1}, ..., A_{mk+1}) = K_k^1(q_m, q_{2m}, ..., q_{km}).$$

Note that for $a_{mj+1} = 0, \quad (1 \leq j \leq k-1)$ we have

$$A_{mk+1} = -a_{mk+1},$$

and so

$$\frac{1}{\tau} mk(\lambda mk + 1)a_{mk+1} = p_{mk},$$

$$\frac{1}{\tau} mk(\lambda mk + 1)A_{mk+1} = q_{mk}.$$

From here we can write

$$-\frac{1}{\tau} mk(\lambda mk + 1)a_{mk+1} = q_{mk}.$$

Taking the absolute values of the above equalities, we obtain

$$|a_{mk+1}| = \frac{|p_{mk}||\tau|}{|mk(\lambda mk + 1)|} = \frac{|q_{mk}||\tau|}{|-mk(\lambda mk + 1)|}.$$

By using Lemma A

$$|a_{mk+1}| \leq \frac{\xi(1-\beta)|\tau|}{mk(\lambda mk + 1)}, \qquad (k \geq 2).$$

This completes the proof. $\square$

By setting $\tau = 1, \xi = 2, \lambda = 0, \beta = \alpha$ in the Theorem 3.1, we get following consequence.

Corollary 1. [5] For $0 \leq \lambda \leq 1$ and $m \in \mathcal{N}$ let the function $f \in \mathcal{S}_{\Sigma_m}(\lambda, \tau, \beta, \xi) = S^*_{\Sigma_m}$ be given by (1.3) if $a_{mj+1} = 0,\ (1 \leq j \leq k-1)$, then

$$|a_{mk+1}| \leq \frac{2(1-\alpha)}{mk}, (k \geq 2).$$

Remark 1. For one-fold case, we get the class $\mathcal{S}_\Sigma(0,1,\alpha,2) = S^*_\Sigma(\alpha)$ and the coefficient estimate as follows:

$$|a_{k+1}| \leq \frac{2(1-\alpha)}{k}, (k \geq 2).$$

Theorem 3.2 *For* $0 \leq \lambda \leq 1$ *and* $0 \leq \beta \leq 1$, *let the function* $f \in \mathcal{S}_{\Sigma_m}(\lambda, \tau, \beta, \xi)$ *be given by (1.3) then one has the following*

$$|a_{m+1}| \leq \begin{cases} \sqrt{\frac{\xi(1-\beta)|\tau|}{m[(m+1)(1+2m\lambda)-(1+m\lambda)^2]}}, & 0 \leq \beta < \frac{\xi[(m+1)(1+2m\lambda)-(1+m\lambda)^2]|\tau|-m(1+m\lambda)^2}{\xi[(m+1)(1+2m\lambda)-(1+m\lambda)^2]|\tau|} \\ \frac{\xi(1-\beta)|\tau|}{m(1+m\lambda)}, & \frac{\xi[(m+1)(1+2m\lambda)-(1+m\lambda)^2]|\tau|-m(1+m\lambda)^2}{\xi[(m+1)(1+2m\lambda)-(1+m\lambda)^2]|\tau|} < \beta < 1, \end{cases} \tag{3.7}$$

$$|a_{2m+1}| \leq min\left\{\frac{\xi(1-\beta)\left\{\left|2(m+1)(1+2m\lambda)-(1+m\lambda)^2\right| + (1+m\lambda)^2\right\}|\tau|}{4m(1+2m\lambda)\left|(m+1)(1+2m\lambda)-(1+m\lambda)^2\right|}, \frac{\xi(1-\beta)|\tau|}{2m(1+m\lambda)} + \frac{\xi^2(1-\beta)^2(m+1)|\tau|^2}{2m^2(1+m\lambda)^2}\right\}, \tag{3.8}$$

$$\left|\frac{[2(1+2m\lambda)(m+1)-(1+m\lambda)^2]}{2(1+2m\lambda)}a_{m+1}^2 - a_{2m+1}\right| \leq \frac{\xi(1-\beta)|\tau|}{2m(1+2m\lambda)}.$$

Proof We set $k=1$ and $k=2$ in equalities (3.5) and (3.6), respectively, and we get the following equations:

$$\frac{1}{\tau}m(m\lambda+1)a_{m+1} = p_1, \tag{3.9}$$

$$\frac{1}{\tau}[2m(2m\lambda+1)a_{2m+1} - m(m\lambda+1)^2 a_{m+1}^2] = p_2, \tag{3.10}$$

$$-\frac{1}{\tau}m(m\lambda+1)a_{m+1} = q_1, \tag{3.11}$$

$$\frac{1}{\tau}\left\{[2m(2m\lambda+1)(m+1) - m(m\lambda+1)^2]a_{m+1}^2 - 2m(2m\lambda+1)a_{2m+1}\right\} = q_2. \tag{3.12}$$

From the Lemma A, we find

$$|a_{m+1}| \leq \frac{\xi(1-\beta)|\tau|}{m(m\lambda+1)}. \tag{3.13}$$

From (3.10) and (3.12), we obtain

$$\frac{2m}{\tau}[(m+1)(1+2m\lambda) - (1+m\lambda)^2]a_{m+1}^2 = p_2 + q_2. \tag{3.14}$$

Using Lemma A, we get

$$|a_{m+1}| \leq \sqrt{\frac{\xi(1-\beta)|\tau|}{m[(m+1)(1+2\lambda m)-(1+\lambda m)^2]}},$$

and combining this with the inequality (3.13), we obtain the desired estimate on the coefficient $|a_{m+1}|$ as asserted in (3.7). Next, in order to find the bound on the coefficient, we subtract (3.12) from (3.10). We thus get

$$\frac{1}{\tau}\left\{4m(1+2\lambda m)a_{2m+1}-2m(1+2\lambda m)(1+m)a_{m+1}^2\right\}=p_2-q_2$$

or

$$a_{2m+1}=\frac{m+1}{2}a_{m+1}^2+\frac{(p_2-q_2)\tau}{4m(1+2m\lambda)}. \tag{3.15}$$

Upon substituting the value of a_{m+1}^2 from (3.9) into (3.15), it follows that

$$a_{2m+1}=\frac{(m+1)p_1^2\tau^2}{2m^2(1+m\lambda)^2}+\frac{\tau(p_2-q_2)}{4m(1+2m\lambda)}. \tag{3.16}$$

We thus easily find (by using Lemma A) that

$$|a_{2m+1}| \leq \frac{\xi(1-\beta)|\tau|}{2m(1+2m\lambda)}+\frac{\xi^2(1-\beta)^2(m+1)|\tau^2|}{2m^2(1+m\lambda)^2}. \tag{3.17}$$

On the other hand, upon substituting the value of a_{m+1}^2 from (3.14) into (3.15), it follows that

$$a_{2m+1}=\frac{(m+1)(p_2+q_2)\tau}{4m[(m+1)(1+2m\lambda)-(1+m\lambda)^2]}+\frac{(p_2-q_2)\tau}{4m(1+2m\lambda)}. \tag{3.18}$$

Appliying Lemma A and making some arrangements, we can easily obtain the inequality as follows:

$$|a_{2m+1}| \leq \frac{\xi(1-\beta)\left\{\left|2(m+1)(1+2m\lambda)-(1+m\lambda)^2\right|+(1+m\lambda)^2\right\}|\tau|}{4m(1+2m\lambda)\left|(m+1)(1+2m\lambda)-(1+m\lambda)^2\right|}. \tag{3.19}$$

Finally from (3.12) by using the Lemma A, we find that,

$$\left|\frac{2(1+2m\lambda)(m+1)-(1+m\lambda)^2}{2(1+2m\lambda)}a_{m+1}^2-a_{2m+1}\right| \leq \frac{\xi(1-\beta)|\tau|}{2m(1+2m\lambda)}. \tag{3.20}$$

This completes the proof of Theorem 3.2. $\square$

By setting $\tau=1, \xi=2, \lambda=0, \beta=\alpha$ in Theorem 3.2 we obtain following consequence.

Corollary 2. [5] For $0 \leq \lambda \leq 1$ and $m \in \mathcal{N}$ let the function $f \in \mathcal{S}_\Sigma(0,1,\alpha,2)$ be given by (1.3), then one has the following:

$$|a_{m+1}| \leq \begin{cases} \frac{\sqrt{2(1-\alpha)}}{m^2}, & 0 \leq \alpha < \frac{1}{2} \\ \frac{2(1-\alpha)}{m}, & \frac{1}{2} \leq \alpha < 1, \end{cases}$$

$$|a_{2m+1}| \leq min\left\{\frac{(m+1)(1-\alpha)}{m^2}, \frac{2(m+1)(1-\alpha)^2}{m^2} + \frac{1-\alpha}{m}\right\},$$

$$\left|a_{2m+1} - \frac{2m+1}{2}a_{m+1}^2\right| \leq \frac{1-\alpha}{m}.$$

Remark 2. For one-fold case of Corollary 2, we obtain the following consequence.

$$|a_2| = \begin{cases} \sqrt{2(1-\alpha)}, & 0 \leq \alpha < \frac{1}{2} \\ 2(1-\alpha), & \frac{1}{2} \leq \alpha < 1, \end{cases}$$

$$|a_3| \leq min\left\{2(1-\alpha), (1-\alpha)(5-4\alpha)\right\},$$

$$|a_3 - \frac{3}{2}a_2^2| \leq 1-\alpha.$$

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