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Multiple-group adjustment method in indirect adjustment

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Abstract: In view of the low accuracy of traditional adjustment, an improved multiple-group adjustment method in indirect adjustment is proposed in this paper. In the process of improved multiple-group adjustment, the first group is first adjusted, and then the adjustment result of the first group and the observation value of the second group are adjusted together, which makes the results in the first-group adjustment meet the overall adjustment results. Finally, the posteriori unit-weight variance value, the coordinated factor matrix of the adjustment result, and the unknown function weight reciprocal are calculated. The experimental results show that the accuracy of multiple-group adjustment in the indirect grouping adjustment will be more accurate than the traditional indirect adjustment method. Moreover, this work provides important ideas and techniques for handling the goniometric triangular network of control surveys.

Key words: Multiple-group adjustment, indirect adjustment, normal equations, error equations, accuracy evaluation

1. Introduction

The error equation of indirect adjustment is [1, 4, 5]:

$$\begin{aligned} \underset{n\times 1}{V} &= \underset{n\times u}{B}\,\underset{n\times 1}{\hat{x}} - f \\ -f &= BX^0 - B^0 - L, \end{aligned} \tag{1.1}$$

where $\hat{X} = X^0 + \hat{x}$ if the observed value X is divided into separate groups, namely:

$$\underset{n\times 1}{L} = \begin{bmatrix} \underset{n_1\times 1}{L_1} \\ \underset{n_2\times 1}{L_2} \end{bmatrix}, \underset{n\times n}{P} = \begin{bmatrix} \underset{n_1\times n_1}{P_1} & 0 \\ 0 & \underset{n_2\times n_2}{P_2}. \end{bmatrix}$$

The equation (1.1) can be written as [2, 6]:

$$\begin{bmatrix} \underset{n_1\times 1}{V_1} \\ \underset{n_2\times 1}{V_2} \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}\hat{x} - \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}, or \begin{cases} \underset{n_1\times 1}{V_1} = B_1\hat{x} - f_1 \\ \underset{n_2\times 1}{V_2} = B_2\hat{x} - f_2, \end{cases} \tag{1.2}$$

where $\begin{cases} -f_1 = B_1X^0 + B_1^0 - L_1 \\ -f_2 = B_2X^0 + B_2^0 - L_2 \end{cases}$

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The above equations are the adjustment of the function; if equation (1.1) is divided into two groups (see formula (2.1)), the first group will be individually adjusted, and then the adjustment results of the first group and the observation values of the second group will be adjusted together, which will make them consistent with the overall adjustment results. The above process is called indirect adjustment in groups [2, 7].

2. Method of indirect grouping adjustment

2.1. First adjustment

First, the error equations of the first group in equation (1.2) will be adjusted individually [3, 5]. The error equation is $V_1 = B_1\hat{x} - f_1$, and the stochastic model is $D_{L_1} = s_o^2 P_1^{-1}$. Then the indirect adjustment can be expressed as [2, 4, 8]:

$$\begin{cases} \hat{x}' = (B_1^T P_1 B_1)^{-1} B_1^T f_1 = N_1^{-1} f_{e1} \\ V_1' = B_1 \hat{x}' - f \\ Q_{\hat{x}'} = P_{\hat{x}}^{-1} = (B_1^T P_1 B_1)^{-1} = N_1^{-1} \end{cases} \tag{2.1}$$

From equation (2.1), the adjustment values of the unknowns $\hat{X}'$ in the first adjustment are:

$$\hat{X}' = X^0 + \hat{x}'. \tag{2.2}$$

2.2. Second adjustment

The adjustment values of the unknowns in the first adjustment are treated as the approximation of the unknowns; from error equations (1.2) and (2.1), the new error equation is obtained as follows [2, 9, 12]:

$$\begin{cases} V_1 = B_1 \hat{x}'' - f_1' \\ V_2 = B_2 \hat{x}'' - f_2' \end{cases} \tag{2.3}$$

Where $-f_1' = B_1\hat{X}' + B_1^0 - L_1 and - f_2 = B_2\hat{X}' + b_2^0 - L_2$, equation (2.3) is adjusted by overall indirect adjustment, and the normal equation is:

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} B_1 & B_2 \end{bmatrix} \hat{x}'' - \begin{bmatrix} B_1 \\ B_3 \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} f_1' \\ f_2' \end{bmatrix} = 0. \tag{2.4}$$

Equation (2.4) is treated as:

$$(B_1^T P_1 B_1 + B_2^T P_2 B_2)\hat{x}'' - (B_1^T P_1 f_1' + B_2^T P_2 f_2') = 0. \tag{2.5}$$

Taking into account equation (2.3),

$$B_1^T P_1 f_1' = B_1^T P_1 V_1' = 0. \tag{2.6}$$

Inserting equation (2.6) into equation (2.5),

$$(B_1^T P_1 B_1 + B_2^T P_2 B_2)\hat{x}'' - B_1^T P_2 f_2' = 0. \tag{2.7}$$

Based on equation (2.7),

$$\hat{x}'' = (B_1^T P_1 B_1 + B_2^T P_2 B_2)^{-1} B_2^T P_2 f_2'. \tag{2.8}$$

According to equation (2.1), equation (2.8) can be rewritten as

$$\hat{x}'' = (P_x + B_2^T P_2 B_2)^{-1} B_2^T P_2 f_2'. \tag{2.9}$$

The adjustment value of the unknown is:

$$\hat{X} = \hat{X}' + \hat{x}'' = X^0 + \hat{x}' + \hat{x}''. \tag{2.10}$$

Equation (2.9) can be expressed by the following equation:

$$(P_{\hat{x}'} + B_2^T P_2 B_2)\hat{x}'' - B_2^T P_2 f_2' = 0. \tag{2.11}$$

Equation (2.11) comprises the following normal equation of indirect adjustment:

$$\begin{cases} V_x'' = \hat{x}'' \\ V_2'' = B_2 \hat{x}'' - f_2' \end{cases} \tag{2.12}$$

Here, the weight of V_x'' is $P_{\hat{x}'}$, and the weight of V_2'' is P_2.

For indirect adjustment, when the second adjustment in the group adjustment is processed, the error equations (2.12) in the second set will be based on the first-adjustment results; at the same time, the parameter of $\hat{X}'$, the unknowns in the first adjustment, needs to take part in the adjustment as the virtual correlation observation value. Based on equation (2.12), the normal equation of (2.11) can be expressed as:

$$\hat{x}'' = (P_{\hat{x}'} + B_2^T P_2 B_2)^{-1} B_2^T P_2 f_2'. \tag{2.13}$$

According to Eqs. (2.11), (2.12), and (2.13), V_2'' can be obtained, which is the second solution method in the indirect group adjustment. The observations of the second sets were not considered in the first adjustment; thus, the error is $V_2' = 0$, and we can obtain:

$$V_1 = V_1' + V_1'' = B_1 \hat{x} - f_1 = B_1(\hat{x}' - \hat{x}'') - f_1. \tag{2.14}$$

Based on Eqs. (2.13) and (2.14),

$$\begin{cases} V_1' = B_1 \hat{x}'' \\ \hat{X} = X^0 + \hat{x}' + \hat{x}'' = \hat{X}' + \hat{x}' \\ V_2 = V_2' + V_2'' \end{cases} \tag{2.15}$$

Through the above deduction, it can be known that the calculation formula in the second adjustment only uses the first-adjustment result and observation values of the second sets. Obviously, the calculation and the storage in the group adjustment are better than the overall adjustment [9, 10, 13].

3. Accuracy evaluation

3.1. Estimation of posteriori unit-weight variance

The equation of posteriori unit-weight variance is:

$$\hat{\sigma}_0^2 = \frac{V^T P V}{n_1 + n_2 - t}, \tag{3.1}$$

where $V_T PV = (V')^T PV' = (V')^T PV''$ and $(V')^T PV' = (V_1'')^T P_1 V_1'' + (V_2'')^T P_2 V_2''$.

3.2. Coordinated factor matrix of adjustment result

Above all, the coordinated factor matrix of the second-adjustment result is the coordinated factor matrix of the ultimate-adjustment result, so the coordinated factor matrix of the unknown-adjustment value array is:

$$Q_{\hat{X}} = (B_1^T P_1 B_1 + B_2^T P_2 B_2)^{-1} = (P_{\hat{X}'} + B_2^T P_2 B_2)^{-1}. \tag{3.2}$$

3.3. Weight reciprocal of unknown function

Let the function of the unknown-adjustment value be $\hat{\varphi} = \phi(\hat{X})$, and after applying the complete differential, the weight function is obtained [2, 11]:

$$\hat{dj} = (\frac{dF}{d\hat{X}})_0 d\hat{X} = G_X^T d\hat{X}, \tag{3.3}$$

where $G_X^T = \left[\begin{array}{cccc} (\frac{\partial\phi}{\partial\hat{X}_1})_0 & (\frac{\partial\phi}{\partial\hat{X}_2})_0 & \cdots & (\frac{\partial\phi}{\partial\hat{X}_t})_0 \end{array} \right]$; according to the propagation rule coordinated factor,

$$Q_{\hat{\varphi}} = G_X^T Q_{\hat{X}G_X}. \tag{3.4}$$

The variance estimation of the unknown function is:

$$\hat{\sigma}_{\hat{\varphi}^2} = \hat{\sigma}_0^2 Q_{\hat{\varphi}}. \tag{3.5}$$

4. Recursive equations of multiple-group adjustment

For multiple-group adjustment, the result of the indirect grouping adjustment can always be taken as the relative observation value to the next group. The adjustment method is exactly the same as the principle of the two-group adjustment described above. Thus, it is not difficult to derive the recurrence formula of the multiple-group adjustment under different conditions. Only considering the uncorrelated observation vectors in each group, the recurrence equations of multiple-group adjustment are expressed as follows.

Set the error equation $V = B\hat{x} - f$ to be divided into m groups:

$$\left\{ \begin{array}{c} V_1 = B_1\hat{x} - f_1 \\ V_2 = B_2\hat{x} - f_2 \\ \cdots \\ V_m = B_m\hat{x} - f_m \end{array} \right. \tag{4.1}$$

The posteriori weight inverse matrix and the weight matrix of the observation vector are:

$$\left\{ \begin{array}{l} Q = diag[Q_1, Q_2, ..., Q_m] \\ P = Q^{-1} = diag[P_1, P_2, ..., P_m] \end{array} \right. \tag{4.2}$$

if

$$V_{m-1} = \left[\begin{array}{c} V_1 \\ V_2 \\ \cdots \\ V_{m-1} \end{array} \right], B_{m-1} = \left[\begin{array}{c} B_1 \\ B_2 \\ \cdots \\ B_{m-1} \end{array} \right], f_{m-1} = \left[\begin{array}{c} V_1 \\ V_2 \\ \cdots \\ V_{m-1} \end{array} \right]. \tag{4.3}$$

Then equation (4.3) can be rewritten as:

$$\begin{cases} V_{m-1} = B_{m-1}\hat{x} - f_{m-1} \\ V_m = B_m\hat{x} - f_m \end{cases} \tag{4.4}$$

According to Eqs. (3.2) and (4.4),

$$\begin{cases} Q_{\hat{X}^{m-1}} = (\sum_{i=1}^{m-1} B_i^T P_i B_i)^{-1} \\ P_{\hat{X}^{m-1}} = Q_{\hat{X}^{m-1}}^{-1} \end{cases} \tag{4.5}$$

The error equation at the m time adjustments is:

$$V_m^{(m)} = B_m^{\hat{x}^{(m)}} - f'_m, \tag{4.6}$$

where $-f'_m = F_m(\hat{x}^{m-1}) - L_m = B_m\hat{x}^{m-1} - f_m$.

If using the second method, the normal equation is:

$$(P_{\hat{X}^{(m-1)}} + B_m^T P_m B_m)\hat{x}^{(m)} - B_m^T P_m f'_m = 0, \tag{4.7}$$

which can be used to obtain

$$\begin{cases} \hat{x}^{(m)} = (P_{\hat{X}^{(m-1)}} + B_m^T P_m B_m)^{-1} B_m^T P_m f'_m \\ V_m^{(m)} = B_m\hat{x}^(m) - f'_m \\ V_{m-1}^{(m)} = B_{m-1}\hat{x}^{(m)} \end{cases} \tag{4.8}$$

The results by the m time adjustment are:

$$\begin{cases} \dot{L}_{m-1}^{(m)} = \dot{L}_{m-1}^{(m-1)} + \dot{V}_{m-1}^{(m)} \\ \hat{L}_m^{(m)} = L_m + V_m^{(m)} \\ \hat{X}^{(m)} = \hat{x}^{(m-1)} + \hat{x}^{(m)} \\ Q_{\hat{X}^{(m)}} = (P_{\hat{X}^{(m)}} + B_m^T P_m B_m)^{-1} \end{cases} \tag{4.9}$$

From the above recursion results, it is clearly known that the recursion equations need to make a new adjustment when adding a set of new observations every time an adjustment is made. However, the coefficient matrix of the normal equation and the constant term are simultaneously scalar, which makes the solution of the normal equation easy and feasible. Thus, the advantage of multiple-group adjustment is obvious.

5. Discussion and application analysis

In order to analyze and verify the improved multiple-group adjustment method in indirect grouping adjustment, the goniometric triangular network of a bridge control survey is taken as the research object (Figure 1). Points A, B, and C are the known points; initial data are listed in Table 1 and angle observations are listed in Table 2. The angles L_1 to L_4 are the first-period observed values, angles L_5 and L_6 are second-period observations, and the variance-covariance matrix of observations is:

$$Q = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}.$$

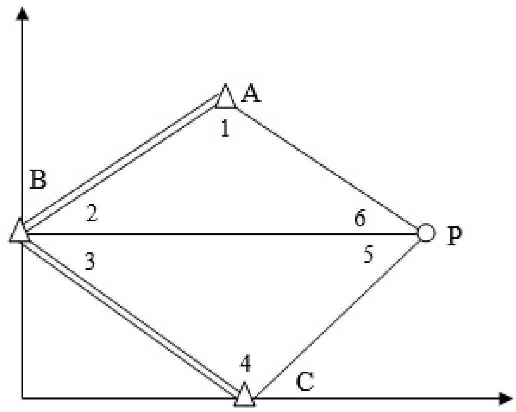

Figure 1. The goniometric triangular network.

Table 1. The known point information.

Point	X/m	Y/m	S/m	$\alpha/^{\circ\prime\prime\prime}$	To point
A	2000.00	1000.00	$\sqrt{2}$	225 00 00	B
B	1000.00	0.00	$\sqrt{2}$	135 00 00	C
C	0.00	1000.00			

Table 2. The observation data information.

Angle	$L/^{\circ\prime\prime\prime}$	Angle	$L/^{\circ\prime\prime\prime}$
1	99 00 06	4	89 59 57
2	44 59 57	5	44 59 58
3	45 00 02	6	45 00 04

The coordinates' approximate value of point P is $X_p^0 = 1000.0(m), Y_p^0 = 2000.0(m)$. Based on the method of indirect group adjustment, the calculation process of the coordinate adjustment of point P and the observation correction is as follows:

1) First adjustment: Based on the coordinate approximate values of point P, approximate coordinate azimuths can be obtained for each edge (see equation (5.1)) and coefficients a_{ij} and b_{ij} are as shown in Table 3.

$$\begin{cases} \alpha_{PA}^0 = 315°00'00'' \\ \alpha_{PB}^0 = 270°00'00'' \\ \alpha_{PC}^0 = 225°00'00'' \end{cases} \tag{5.1}$$

Table 3. The coefficients of error equations.

Point	$\Delta Y^0/m$	$\Delta Y^0/m$	$(S^0)^2 \times 100$	a_{ij}	b_{ij}
P-A	–1000	1000	2×10^8	–1.0313	–1.0313
P-B	–2000	0	4×10^8	–1.0313	0
P-C	–1000	–1000	2×10^8	–1.0313	–1.0313

Setting the error equation of observations $V_1' = B_1^T \hat{x}' - f_1$, the coefficients and constants are listed in Table 4.

Table 4. The coefficients and constants of error equation for observations.

Angle	$a/\hat{X}_p$	$b/\hat{X}_p$	$f_1/''$	$V_1'/''$	$V_1''/''$	$V_1/''$
1	1.0313	1.0313	6.0	–1.60	–1.67	–3.3
2	–1.0313	0	–3.0	0.10	0	0.1
3	1.0313	0	2.0	0.90	0	0.9
4	–1.0313	1.0313	–3.0	1.60	–1.67	–0.1

2) Second adjustment: the normal equation of the second adjustment is expressed as:

$$(N_X + B_2^T P_2 B_2)\hat{x}'' - B_2^T P_2 \overline{f_2} = 0, \tag{5.2}$$

where $N_X = B_1^T P_1 B_1 = \begin{bmatrix} 1.7728 & 0 \\ 0 & 1.9636 \end{bmatrix}, B_2^T P_2 B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 2.1272 \end{bmatrix}, B_2^T P_2 \overline{f_2} = \begin{bmatrix} 0 \\ -5.1565 \end{bmatrix}, \overline{f_2} = f_2 - B_2 \hat{x}' = \begin{bmatrix} -0.50 \\ 5.50 \end{bmatrix}$.

By $\hat{x}''$ and the equation $V_1'' = B_1 \hat{x}''$, we can obtain V_1'', and the results are shown in Table 4. From the preceding calculation results, the coordinate result of point P is:

$$\begin{cases} \hat{X}_P = 1000.028(m) \\ \hat{Y}_P = 1999.998(m) \end{cases} \tag{5.3}$$

In order to facilitate a comparison with the results of multiple-group adjustment, the coordinates of point P are calculated with the indirect adjustment method, giving:

$$\begin{cases} \hat{X}_P = 1000.032(m) \\ \hat{Y}_P = 1999.994(m) \end{cases} \tag{5.4}$$

From the above results for point P, it is known that the errors of approximate value and indirect adjustment results in X and Y, respectively, are $0.032m$ and $-0.006m$, and the errors of approximate value and multiple-group adjustment in the indirect grouping adjustment in X and Y, respectively, are $0.028m$ and $-0.002m$; the error curves are shown in Figure 2 and Figure 3. It can be seen from the figures that the accuracy of the multiple-group adjustment in the indirect group adjustment (the method proposed in this paper) is higher than that of the indirect adjustment method and approximate values. Thus, it can be seen that the proposed multiple-group adjustment method is more feasible and advantageous.

6. Conclusions

In this paper, we propose a multiple-group adjustment in indirect grouping adjustment. It is not difficult to see from the recursion formula presented above that the coefficient matrix of the normal equation and the constant term are simultaneously scalar, which makes the solution of the normal equation easy and feasible. Compared with the traditional indirect adjustment method, the method of multiple-group adjustment in indirect grouping adjustment has greater advantages. It is also worth noting that the accuracy of multiple-group adjustment

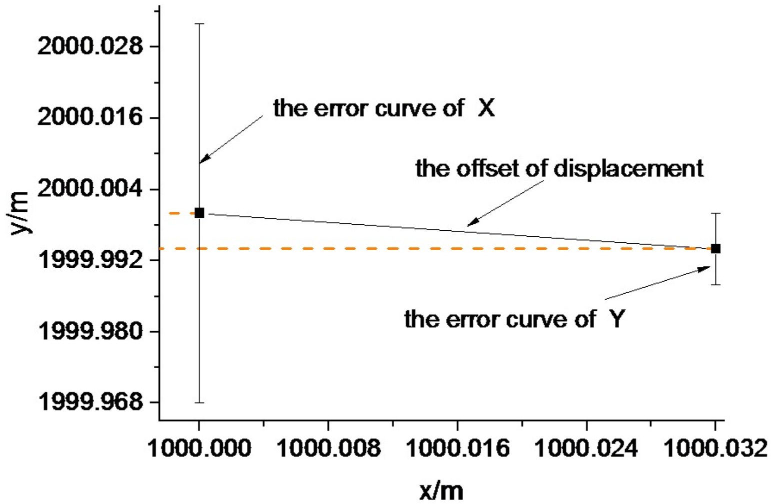

Figure 2. Error curve of approximate value and indirect adjustment result.

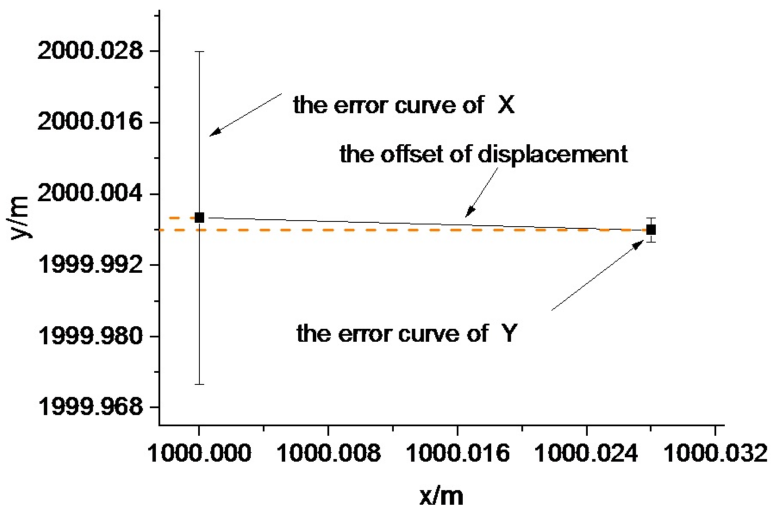

Figure 3. Error curve of approximate value and multiple adjustment in indirect grouping adjustment result.

in the indirect grouping adjustment will be more accurate than the traditional indirect adjustment method. Moreover, this paper provides important ideas and techniques for handling the goniometric triangular network of control surveys.

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