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On strongly Ozaki bi-close-to-convex functions

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Abstract: In this paper, we introduce and investigate a new subclass of strongly Ozaki bi-close-to-convex functions in the open unit disk. We have also found estimates for the first two Taylor–Maclaurin coefficients for functions belonging to this class. The results presented in this paper have been shown to generalize and improve the work of Brannan and Taha.

Key words: Bi-close-to-convex functions, coefficient estimates, bi-univalent

1. Introduction

Let $\mathcal{A}$ be the class of all analytic functions f in the open unit disk $\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$ and normalized by the conditions $f(0) = f'(0) - 1 = 0$ and having the following form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1.1}$$

Also denote by $\mathcal{S}$ the class of all functions in $\mathcal{A}$ that are univalent in $\mathbb{U}$ (see, for details, [5]).

By the Koebe one-quarter theorem, we know that the range of every function in $\mathcal{S}$ contains the disk $\left\{w : |w| < \frac{1}{4}\right\}$ [5]. Therefore, every univalent function f has an inverse f^{-1} so that $f^{-1}\big(f(z)\big) = z \quad \big(z \in \mathbb{U}\big)$ and $f\big(f^{-1}(w)\big) = w \quad (|w| < r_0(f);\ r_0(f) \geq 1/4)$. In fact, the inverse function $g = f^{-1}$ is given by the power series

$$\begin{aligned} g(w) = f^{-1}(w) &= w + \sum_{n=2}^{\infty} b_n w^n \\ &= w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots . \end{aligned} \tag{1.2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and its inverse map $g = f^{-1}$ are univalent in $\mathbb{U}$. Let Σ be the class of all bi-univalent functions in $\mathbb{U}$ having the series expansion (1.1). For a brief history of functions in the class Σ, see the work of Srivastava et al. [16] (see also [4, 9, 20]).

Coefficient bounds for various subclasses of bi-univalent functions were obtained by several authors including Ali et al. [1], Srivastava et al. [17], and Sümer Eker [19]. Judging by the remarkable flood of

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papers on the subject, the pioneering work of Srivastava et al. [16] appears to have revived the study of analytic and bi-univalent functions in recent years.

A function $f \in \mathcal{S}$ is said to be *starlike of order* α $(0 \le \alpha < 1)$, denoted $f \in \mathcal{S}^*(\alpha)$, if

$$Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha \ (0 \le \alpha < 1, \ z \in \mathbb{U}),$$

and is said to be *convex of order* α $(0 \le \alpha < 1)$, denoted $f \in \mathcal{K}(\alpha)$, if

$$Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha, \ (0 \le \alpha < 1, \ z \in \mathbb{U}).$$

For $\alpha = 0$, these classes reduce well-known classes $\mathcal{S}^*$ and $\mathcal{K}$, the class of *starlike functions* and the class of *convex functions*, respectively.

Furthermore, a function $f \in \mathcal{S}$ is said to be *strongly starlike of order* α, if for all $z \in \mathbb{U}$

$$\left|arg\frac{zf'(z)}{f(z)}\right| \le \alpha\frac{\pi}{2}, \ (0 \le \alpha \le 1),$$

and is said to be *strongly convex of order* α, if for all $z \in \mathbb{U}$

$$\left|arg\left(1 + \frac{zf''(z)}{f'(z)}\right)\right| \le \alpha\frac{\pi}{2}, \ (0 \le \alpha \le 1).$$

We denote by $\tilde{\mathcal{S}}^*(\alpha)$ and $\tilde{\mathcal{K}}(\alpha)$ the class strongly starlike of order α and the class strongly convex of order α, respectively. These classes were introduced by Brannan and Kirwan [3] and Stankiewicz [18], independently.

It is well known that, as α increases, the sets $\mathcal{S}^*(\alpha)$ and $\mathcal{K}(\alpha)$ become smaller but the sets $\tilde{\mathcal{S}}^*(\alpha)$ and $\tilde{\mathcal{K}}(\alpha)$ become larger. For more information about the classes $\mathcal{S}^*(\alpha)$, $\mathcal{K}(\alpha)$, $\tilde{\mathcal{S}}^*(\alpha)$, and $\tilde{\mathcal{K}}(\alpha)$, see [6].

Theorem 1 *[7, 10] If f and ϕ are analytic in any domain D, ϕ is univalent and convex in D, and*

$$Re\left(\frac{f'(z)}{\phi'(z)}\right) > 0 \tag{1.3}$$

in D, then f is also univalent in D.

Kaplan gave the name "close-to-convex in D" to functions f that satisfy the conditions of Theorem 1. Now we take $D = \mathbb{U}$. In Theorem 1, the usual normalization plays no role. Since we can multiply f and ϕ by a positive constant without harm to the inequality (1.3), we may assume without loss of generality that f has the form given by (1.1), $\phi(0) = 0$, and $|\phi'(0)| = 1$. (A normalization $\phi'(0) = 1$ will cause a serious loss to the set of close-to-convex functions.) Therefore, we add a factor $e^{i\beta}$ in (1.3) and use the form

$$Re\left(\frac{f'(z)}{e^{i\beta}\phi'(z)}\right) > 0, \tag{1.4}$$

where $\phi(z) = z + ...$ is in $\mathcal{K}$. Thus, if the inequality (1.4) is satisfied in $\mathbb{U}$ for some β and some $\phi \in \mathcal{K}$, then f is univalent in $\mathbb{U}$ and so is a close-to-convex function in $\mathbb{U}$. Now we can give the following definition.

Definition 1 *[7] A function f, analytic in $\mathbb{U}$, is said to be close-to-convex in $\mathbb{U}$ if there is a function $\phi \in \mathcal{K}$ and a real β such that the inequality (1.4) holds in $\mathbb{U}$. We let $\mathcal{C}$ denote the set of all functions of the form (1.4) that are close-to-convex in $\mathbb{U}$.*

By the Alexander theorem, we know that ϕ is convex if and only if $\varphi(z) = z\phi'(z)$ is starlike. Then we can rewrite the inequality (1.4) by the following condition: there is $\varphi \in \mathcal{S}^*$ such that

$$Re\left(\frac{zf'(z)}{e^{i\beta}\varphi(z)}\right) > 0.$$

Geometrically, f is close-to-convex if and only if the image of $|z| = R$ has no "hairpin turns"; that is, there are no sections of the curve $f(C_R)$ in which the tangent vector turns backward through an angle $\geq \pi$.

Although the class of close-to-convex functions was introduced by Kaplan [7] in 1952, in 1935 Ozaki [10, 11] had already considered the functions in $\mathcal{A}$ satisfying the following condition:

$$Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > -\frac{1}{2}, \ (z \in \mathbb{U}). \tag{1.5}$$

The functions satisfying the inequality (1.5) are close-to-convex and therefore they are in $\mathcal{S}$ by the definition of Kaplan [7].

Recently, Kargar and Ebadian [8] generalized Ozaki's condition as follows:

Definition 2 *[8] Let $\mathcal{F}(\lambda)$ denote the class of locally univalent normalized analytic functions f in the unit disk satisfying the condition*

$$Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \frac{1}{2} - \lambda, \quad (z \in \mathbb{U}), \tag{1.6}$$

for some $-1/2 < \lambda \leq 1$. The class $\mathcal{F}(1)$ was studied by Ponnusamy et al. [13]. Also, $\mathcal{F}(\frac{1}{2}) = \mathcal{K}$. Clearly, $\mathcal{F}(\lambda) \subset \mathcal{K} \subset \mathcal{S}^*$ for all $\lambda \in (-1/2, 1/2)$.

Recently, Allu et al. extended the class $\mathcal{F}(\lambda)$ as follows:

Definition 3 *[2, 21] Let $f \in \mathcal{A}$. Then f is called strongly Ozaki-close-to-convex if and only if*

$$\left|arg\left(\frac{2\lambda - 1}{2\lambda + 1} + \frac{2}{2\lambda + 1}\left(1 + \frac{zf''(z)}{f'(z)}\right)\right)\right| < \frac{\alpha\pi}{2}, \quad (0 < \alpha \leq 1, \ 1/2 \leq \lambda \leq 1 \ z \in \mathbb{U}).$$

This class is denoted by $\mathcal{F}_O(\lambda, \alpha)$.

The object of the present paper is to introduce a new subclass of the function class Σ, namely strongly Ozaki bi-close-to-convex functions, and find an estimate on the coefficients $|a_2|$ and $|a_3|$ for functions in this class.

2. Main results

Definition 4 *A function f given by (1.1) is said to be in the class $\mathcal{F}_{O,\Sigma}(\lambda,\alpha)$, $(0<\alpha\leq 1;\ 1/2\leq\lambda\leq 1)$, if the following conditions are satisfied:*

$$f\in\Sigma \ \ and \ \ \left|arg\left(\frac{2\lambda-1}{2\lambda+1}+\frac{2}{2\lambda+1}\left(1+\frac{zf''(z)}{f'(z)}\right)\right)\right|<\frac{\alpha\pi}{2} \qquad (0<\alpha\leq 1, z\in\mathbb{U}) \tag{2.1}$$

and

$$\left|arg\left(\frac{2\lambda-1}{2\lambda+1}+\frac{2}{2\lambda+1}\left(1+\frac{wg''(w)}{g'(z)}\right)\right)\right|<\frac{\alpha\pi}{2} \qquad (0<\alpha\leq 1, w\in\mathbb{U}), \tag{2.2}$$

where the function g is given by (1.2).

We first state and prove the estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in the class $\mathcal{F}_{O,\Sigma}(\lambda,\alpha)$.

Theorem 2 *If f given by (1.1) is in the class $\mathcal{F}_{O,\Sigma}(\lambda,\alpha)$, then*

$$|a_2|\leq\frac{\alpha(2\lambda+1)}{\sqrt{2\alpha(2\lambda+1)+4(1-\alpha)}} \tag{2.3}$$

and

$$|a_3|\leq\frac{\alpha(2\lambda+1)}{2}. \tag{2.4}$$

Proof For f given by (1.1), we can write from (2.1) and (2.2)

$$\frac{2\lambda-1}{2\lambda+1}+\frac{2}{2\lambda+1}\left(1+\frac{zf''(z)}{f'(z)}\right)=[p(z)]^{\alpha}, \tag{2.5}$$

$$\frac{2\lambda-1}{2\lambda+1}+\frac{2}{2\lambda+1}\left(1+\frac{wg''(w)}{g'(z)}\right)=[q(w)]^{\alpha}, \tag{2.6}$$

where $p(z)$ and $q(w)$ are in Carathéodory class $\mathcal{P}$. Thus, $p(z)$ and $q(w)$ have the following series expansions:

$$p(z)=1+p_1z+p_2z^2+p_3z^3+\cdots \tag{2.7}$$

and

$$q(w)=1+q_1w+q_2w^2+q_3w^3+\cdots. \tag{2.8}$$

Now, equating coefficients (2.5) and (2.6), we find that

$$\frac{4a_2}{2\lambda+1}=\alpha p_1, \tag{2.9}$$

$$\frac{12a_3}{2\lambda+1}=\frac{8a_2^2}{2\lambda+1}+\alpha p_2+\frac{\alpha(\alpha-1)}{2}p_1^2, \tag{2.10}$$

$$-\frac{4a_2}{2\lambda+1}=\alpha q_1, \tag{2.11}$$

and

$$\frac{16a_2^2}{2\lambda+1} = \frac{12a_3}{2\lambda+1} + \alpha q_2 + \frac{\alpha(\alpha-1)}{2}q_1^2. \tag{2.12}$$

From (2.9) and (2.11), we get

$$p_1 = -q_1 \tag{2.13}$$

and

$$\frac{32a_2^2}{(2\lambda+1)^2} = \alpha^2(p_1^2+q_1^2). \tag{2.14}$$

Also, from (2.10), (2.12), and (2.14), we get

$$a_2^2 = \frac{\alpha^2(2\lambda+1)^2(p_2+q_2)}{8\alpha(2\lambda+1)+16(1-\alpha)}. \tag{2.15}$$

It is well known that from the Caratheódory lemma, the coefficients of $|p_n| \leq 2$ and $|q_n| \leq 2$ for $n \in \mathbb{N}$ (see [5]). If we take the absolute value of both sides of a_2^2 and if we apply the Carathéodory lemma to coefficients p_2 and q_2, we obtain

$$|a_2^2| \leq \frac{\alpha^2(2\lambda+1)^2}{2\alpha(2\lambda+1)+4(1-\alpha)}.$$

This gives the desired bound for $|a_2|$, as asserted in (2.3).

Now, in order to find the bound on $|a_3|$, from (2.10) and (2.12) and (2.13), we can write

$$\frac{12a_3}{2\lambda+1} = \alpha\left(2p_2+q_2\right) + \frac{3\alpha(\alpha-1)}{2}p_1^2. \tag{2.16}$$

If we take $\alpha = 1$ and apply the Caratheódory lemma, then

$$|a_3| \leq \frac{(2\lambda+1)}{2}.$$

Now we consider the case $0 < \alpha < 1$. From (2.16), we can write

$$\frac{12}{2\lambda+1}Re(a_3) = \alpha Re\left\{(2p_2+q_2) + \frac{3(\alpha-1)}{2}p_1^2\right\}. \tag{2.17}$$

From Herglotz's representation formula [12] for the functions $p(z)$ and $q(w)$, we have

$$p(z) = \int_0^{2\pi} \frac{1+ze^{-it}}{1-ze^{-it}}d\mu_1(t)$$

and

$$q(w) = \int_0^{2\pi} \frac{1+we^{-it}}{1-we^{-it}}d\mu_2(t),$$

where $\mu_i(t)$ are increasing on $[0, 2\pi]$ and $\mu_i(2\pi) - \mu_i(0) = 1$, $i = 1, 2$.

We also have

$$p_n = 2\int_0^{2\pi} e^{-int} d\mu_1(t), \qquad n = 1, 2, \ldots$$

$$q_n = 2\int_0^{2\pi} e^{-int} d\mu_2(t), \qquad n = 1, 2, \ldots.$$

Now (2.17) can be written as follows :

$$\frac{12}{2\lambda+1} Re(a_3) = 4\alpha\int_0^{2\pi} cos2td\mu_1(t) + 2\alpha\int_0^{2\pi} cos2td\mu_2(t) - 6\alpha(1-\alpha)\left[\left(\int_0^{2\pi} costd\mu_1 t\right)^2 - \left(\int_0^{2\pi} sintd\mu_1(t)\right)^2\right]$$

$$\leq 4\alpha\int_0^{2\pi} cos2td\mu_1(t) + 2\alpha\int_0^{2\pi} cos2td\mu_2(t) + 6\alpha(1-\alpha)\left(\int_0^{2\pi} sintd\mu_1(t)\right)^2$$

$$= 2\alpha\left\{2\int_0^{2\pi}(1-2sin^2t)d\mu_1(t) + \int_0^{2\pi}(1-2sin^2t)d\mu_2(t) + 3(1-\alpha)\left(\int_0^{2\pi} sintd\mu_1(t)\right)^2\right\}.$$

By Jensen's inequality [14], we have

$$\left(\int_0^{2\pi} |sint| d\mu(t)\right)^2 \leq \left(\int_0^{2\pi} sin^2td\mu(t)\right).$$

Hence,

$$\frac{12}{2\lambda+1} Re(a_3) \leq 2\alpha\left\{3 - 2\int_0^{2\pi} sin^2td\mu_2(t) - (1+3\alpha)\int_0^{2\pi} sin^2td\mu_1(t)\right\}$$

and thus

$$Re(a_3) \leq \frac{\alpha(2\lambda+1)}{2},$$

which implies

$$|a_3| \leq \frac{\alpha(2\lambda+1)}{2}.$$

This completes the proof of the theorem. □

3. Coefficient estimates for the functions class $\mathcal{F}_{O,\Sigma}(\lambda,\beta)$

Definition 5 *A function f given by (1.1) is said to be in the class $\mathcal{F}_{O,\Sigma}(\lambda,\beta)$, $(0 \leq \beta < 1;\ 1/2 \leq \lambda \leq 1)$ if the following conditions are satisfied:*

$$f \in \Sigma \ \text{and} \ Re\left(\frac{2\lambda-1}{2\lambda+1} + \frac{2}{2\lambda+1}\left(1 + \frac{zf''(z)}{f'(z)}\right)\right) > \beta \qquad (z \in \mathbb{U}) \tag{3.1}$$

and

$$Re\left(\frac{2\lambda-1}{2\lambda+1} + \frac{2}{2\lambda+1}\left(1 + \frac{wg''(w)}{g'(z)}\right)\right) > \beta \qquad (w \in \mathbb{U}), \tag{3.2}$$

where the function g is given by (1.2).

For $\lambda = \frac{1}{2}$, the class of $\mathcal{F}_{O,\Sigma}(\lambda,\beta)$ is reduced to $C_\Sigma(\beta)$ of biconvex order β $(0 \leq \beta < 1)$, which was introduced by Brannan and Taha [4].

Theorem 3 *If f given by (1.1) is in the class $\mathcal{F}_{O,\Sigma}(\lambda,\beta)$, then*

$$|a_2| \leq \sqrt{\frac{(1-\beta)(2\lambda+1)}{2}} \tag{3.3}$$

and

$$|a_3| \leq \frac{(1-\beta)(2\lambda+1)}{2}. \tag{3.4}$$

Proof We can write the inequalities in (3.1) and (3.2) as follows:

$$\frac{2\lambda-1}{2\lambda+1} + \frac{2}{2\lambda+1}\left(1+\frac{zf''(z)}{f'(z)}\right) = \beta + (1-\beta)p(z) \tag{3.5}$$

and

$$\frac{2\lambda-1}{2\lambda+1} + \frac{2}{2\lambda+1}\left(1+\frac{wg''(w)}{g'(z)}\right) = \beta + (1-\beta)q(w), \tag{3.6}$$

where $p(z)$ and $q(w)$ are given by (2.7) and (2.8), respectively. Like the proof of Theorem 1, equating coefficients of (3.5) and (3.6) yields

$$\frac{4a_2}{2\lambda+1} = (1-\beta)p_1, \tag{3.7}$$

$$\frac{12a_3}{2\lambda+1} - \frac{8a_2^2}{2\lambda+1} = (1-\beta)p_2, \tag{3.8}$$

$$-\frac{4a_2}{2\lambda+1} = (1-\beta)q_1, \tag{3.9}$$

and

$$\frac{16a_2^2}{2\lambda+1} - \frac{12a_3}{2\lambda+1} = (1-\beta)q_2. \tag{3.10}$$

From (3.7) and (3.9) we get

$$p_1 = -q_1 \tag{3.11}$$

and

$$\frac{32a_2^2}{(2\lambda+1)^2} = (1-\beta)^2(p_1^2+q_1^2). \tag{3.12}$$

Also, from (3.8) and (3.10), we obtain

$$\frac{8a_2^2}{2\lambda+1} = (1-\beta)(p_2+q_2). \tag{3.13}$$

Thus, clearly we have

$$|a_2|^2 \leq \frac{(1-\beta)(2\lambda+1)}{8}\left(|p_2|+|q_2|\right). \tag{3.14}$$

If we apply the Carathéodory lemma to coefficients of p_2 and q_2, we find the upper bound on $|a_2|$ as given in (3.3).

In order to find the bound on $|a_3|$, we multiply (3.8) by 2 and add it to (3.10), and we obtain:

$$\frac{12a_3}{2\lambda+1} = (1-\beta)(2p_2+q_2). \tag{3.15}$$

Now let us take the absolute value of the both sides of (3.15). After that, if we apply the Carathéodory lemma to coefficients of p_2 and q_2, we find

$$|a_3| \leq \frac{(1-\beta)(2\lambda+1)}{2},$$

which is asserted in (3.4). □

If we take $\lambda = \frac{1}{2}$, in Theorem 3, we obtain the following corollary due to the result of Brannan and Taha:

Corollary 1 *[4] Let f given by (1.1) belong to $C_\sigma(\beta)$ $(0 \leq \beta < 1)$. Then*

$$|a_2| \leq \sqrt{1-\beta} \qquad \textit{and} \qquad |a_3| \leq 1-\beta.$$

References

[1] Ali RM, Lee SK, Ravichandran V, Supramaniam S. Coefficient estimates for bi-univalent Ma-Minda starlike and convex functions. Applied Mathematics Letters 2012; 25: 344-351.

[2] Allu V, Thomas DK, Tuneski N. On Ozaki close-to-convex functions. Bulletin of the Australian Mathematical Society 2019; 99: 89-100. doi: 10.1017/S0004972718000989

[3] Brannan DA, Kirwan WE. On some classes of bounded univalent functions. Journal of the London Mathematical Society 1969; 2: 431–443.

[4] Brannan DA, Taha TS. On some classes of bi-univalent functions. In: Mazhar SM, Hamoui A, Faour NS (editors). KFAS Proceedings Series, Vol. 3. Oxford, UK: Pergamon Press, 1988, pp. 53-60.

[5] Duren PL. Univalent Functions. Grundlehren der Mathematischen Wissenschaften. New York, NY, USA: Springer-Verlag, 1983.

[6] Goodman AW. Univalent Functions. New York, NY, USA: Mariner Publishing Company, 1983.

[7] Kaplan W. Close-to-convex schlicht functions. Michigan Math J 1952; 1 (2): 169-185.

[8] Kargar R, Ebadian A. Ozaki's conditions for general integral operator. Sahand Communications in Mathematical Analysis 2017; 5 (1): 61-67.

[9] Lewin M. On a coefficient problem for bi-univalent functions. P Am Math Soc 1967; 18: 63-68.

[10] Ozaki S. On the theory of multivalent functions. Science Reports of the Tokyo Bunrika Daigaku Section A 1935; 2 (40): 167-188.

[11] Ozaki S. On the theory of multivalent functions II. Science Reports of the Tokyo Bunrika Daigaku Section A 1941; 4 (77/78): 45-87.

[12] Pommerenke C. Univalent Functions. Göttingen, Germany: Vandenhoeck Ruprecht, 1975.

[13] Ponnusamy S, Sahoo SK, Yanagihara H. Radius of convexity of partial sums of functions in the close-to-convex family. Nonlinear Analysis 2014; 95: 219–228.

[14] Rudin W. Real and Complex Analysis. 3rd ed. New York, NY, USA: McGraw-Hill Education, 1998.

[15] Srivastava HM. Some inequalities and other results associated with certain subclasses of univalent and bi-univalent analytic functions. In: Pardalos PM, Georgiev PG, Srivastava HM (editors). Nonlinear Analysis: Stability, Approximation, and Inequalities. Springer Series on Optimization and Its Applications, Vol. 68. Berlin, Germany: Springer-Verlag, 2012, pp. 607-630.

[16] Srivastava HM, Mishra AK, Gochhayat P. Certain subclasses of analytic and bi-univalent functions. Appl Math Lett 2010; 23: 1188-1192.

[17] Srivastava HM, Sümer Eker S, Ali RM. Coefficient bounds for a certain class of analytic and bi-univalent functions. Filomat 2015; 29: 1839-1845.

[18] Stankiewicz J. Quelques problemes extremaux dansles classes des fonctions α- angulairement etoilees. Ann Univ Mariae Curie-Skłodowska Sect A 1966; 20: 59–75 (in French).

[19] Sümer Eker S. Coefficient bounds for subclasses of m-fold symmetric bi-univalent functions. Turk J Math 2016; 40: 641-646.

[20] Taha TS. Topics in univalent function theory. PhD, University of London, London, UK, 1981.

[21] Thomas D, Tuneski N, Vasudevarao A. Univalent Functions. A Primer. Berlin, Germany: De Gruyter, 2018.