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Fuzzy soft topological spaces and the related category FST

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Abstract: In this paper, we consider fuzzy soft sets with a different approach where we generalize the idea initially introduced by Šostak for fuzzy sets. Subsequently, we define the fuzzy set topology categories of fuzzy soft topological spaces and give certain properties of them. Furthermore, we define the initial and the final fuzzy soft topological spaces.

Key words: Fuzzy soft set, fuzzy soft topology, fuzzy soft continuity, initial fuzzy soft topolgy, final fuzzy soft topology

1. Introduction

Set theory, which was initiated by George Cantor in his seminal work, serves as a foundation to several branches of mathematics. Cantor defined a set as a collection of objects stated with the same property. In his definition, the sets are crisp and defined by their elements, and thus it is clear if an element belongs to a set or not. In other words, according to Cantor, a set and its elements are precisely determined. However, if we aim to model a concept in real life by using the mathematical properties of Cantor's set theory, then we might run into various difficulties due to vagueness that exists in problems related to economics, engineering, medicine, etc. In order to incorporate the vagueness into set theory, many theories have been introduced. The most successful one for these kinds of vague concepts is Zadeh's fuzzy sets [16]. The key idea behind this theory is to have a membership function for the elements of a set. The value of this function indicates up to which degree an element belongs to the set (see Definition 2.1 for details). Fuzzy set theory and its applications developed rapidly and gained the interest of many researchers. On the other hand, Molodtsov [9] defined soft set theory as a new approach for vagueness, which can be seen as a generalization of fuzzy set theory. Roughly speaking, instead of having only one membership function as originally introduced in fuzzy set theory, one can define the approximate elements of a set by using the parametrized subsets in soft set theory. Aktas and Cagman [2] showed that every fuzzy set is a soft set. Hence, it would not be wrong to say that fuzzy sets are a special class of soft sets. Furthermore, soft set theory can also be applied successfully to many areas of mathematics.

As a further improvement, fuzzy soft sets were first introduced by Maji et al. [8] as a combination of fuzzy and soft sets. This hybrid model gave rise to new scientific studies, papers, and applications. For instance, Ahmad and Kharal [1] enhanced the concepts given in [8] and supported the propositions by examples and counterexamples. Aktas and Cagman [2] defined soft groups. Feng et al. [5] defined soft semirings and gave some properties. Ozturk and Bayramov developed the category of soft modules [10]. Aygunoglu and Aygun [3] defined fuzzy soft groups. Kharal and Ahmad [6] also studied the mappings on fuzzy soft sets. Fuzzy soft

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topological spaces were first defined by Tanay and Kandemir [14]. Roy and Samanta [11] and Simsekler and Yuksel [12] developed fuzzy soft topology. In these papers fuzzy soft topology is a crisp set containing fuzzy soft sets. Kucuk and Ozturk defined the homology modules of fuzzy soft modules [7]. Aygunoglu et al. [4] observed the category of fuzzy soft topological spaces (FSTOP) and fuzzificated the fuzzy soft topology by grading the fuzzy soft open and fuzzy soft closed sets from 0 to 1. This idea was initially given by Šostak [13] for fuzzy topologies. In this paper, we study the category of FST between fuzzy soft topological spaces and define the fuzzy soft initial and fuzzy soft final topologies, respectively.

The paper is structured as follows. First, we define fuzzy soft inclusion, fuzzy soft equality, fuzzy soft union, and fuzzy soft intersection. Then we introduce fuzzy soft topology by using a different approach; namely, we generalize the idea of Šostak [13]. Accordingly, we define fuzzy soft continuous mappings and also the category of FST. Finally, we give the initial and final fuzzy soft topological spaces.

2. Preliminaries

In this section, we give several preliminary results that are essential for our own approach. Before starting, it is useful to fix the notations. Throughout the paper, U denotes the initial universe, E denotes the set of all possible parameters for U, 2^U denotes the power set of U, and I^U denotes the set of all fuzzy subsets of U. Additionally, (U,E) denotes the universal set U and the parameter set E and $F(U,E)$ denotes the family of fuzzy soft sets on (U,E).

Definition 2.1 *[16] A fuzzy set A in U is a set of ordered pairs*

$$A = \{(x, \mu_A(x)) : x \in U\},$$

where $\mu_A : U \mapsto [0,1] = I$ is a mapping and $\mu_A(x)$ (or $A(x)$) states the degree of belonging of x in A.

Definition 2.2 *[9] Let $A \subseteq E$. A pair (F,A) is called a soft set over U where F is a mapping given by $F : A \mapsto 2^U$.*

Definition 2.3 *[11] Let $A \subseteq E$. f_A is defined to be a fuzzy soft set on $U_{\widetilde{E}}$ if $f : A \mapsto I^U$ is a mapping defined by $f(e) = \mu_f^e$ such that*

$$f^e = \begin{cases} \mu_f^e = \bar{O} & \text{if } \ e \in E - A \\ \mu_f^e \neq \bar{O} & \text{if } \ e \in A, \end{cases}$$

where $\bar{O}(u) = 0$ for each $u \in U$.

Definition 2.4 *[11] The complement of a fuzzy soft set f_A is a fuzzy soft set on $U_{\widetilde{E}}$, which is denoted by f_A^c. Furthermore, $f^c : A \mapsto I^U$ is defined as follows:*

$$f^c = \begin{cases} \mu_{f^c}^e = 1 - \mu_f^e & \text{if } \ e \in A \\ \mu_{f_A}^e = \bar{1} & \text{if } \ e \in E \backslash A, \end{cases}$$

where $\bar{1}(u) = 1$ for each $u \in U$.

Definition 2.5 *[11] The fuzzy soft set f_Φ on $U_E^\sim$ is defined as a null fuzzy soft set and is denoted by Φ. Moreover, $\Phi(e) = \bar{O}$ for every $e \in E$.*

Definition 2.6 *[11] The fuzzy soft set f_A on $U_E^\sim$ is defined to be an absolute fuzzy soft set and is denoted by $U_E^\sim$. Further, $U(e) = f(e) = \bar{1}$ for every $e \in E$.*

Definition 2.7 *[11] The fuzzy soft set f_A on $U_E^\sim$ is defined to be an absolute fuzzy soft set and is denoted by $U_E^\sim$. Further, $U(e) = f(e) = \bar{1}$ for every $e \in E$.*

Definition 2.8 (Fuzzy soft inclusion) *Let f_A and g_A be two fuzzy soft sets on $U_E^\sim$. The following relation holds:*

$$(f_A \subseteq^\sim g_A)^e = \inf_U (f_A^c \vee g_A)^e(u) \quad \text{for all } e \in A.$$

Remark 2.9 *The real number $(f_A \subseteq^\sim g_A)^e$ denotes the degree of the inclusion of the fuzzy soft set g_A for each parameter.*

Definition 2.10 (Fuzzy soft equality) *Consider two fuzzy soft sets f_A and g_A. Then we have the following equation:*

$$(f_A =^\sim g_A)^e = (f_A \subseteq^\sim g_A)^e \wedge (g_A \subseteq^\sim f_A)^e \quad \text{for all } e \in A.$$

A crisp family can be defined as crisp subsets of U by a function $\mathcal{A} : 2^U \mapsto 2$ that indicates which subsets belong to this family. The intersection of all subsets of $\mathcal{A}$ can be considered as a function $\wedge\mathcal{A} : U \mapsto 2$ defined as follows:

$$\wedge\mathcal{A}(x) = 1 \quad \text{iff} \quad x \in A \quad \text{for all } A \in \mathcal{A}.$$

Šostak [13] formalized this idea for fuzzy sets and obtained the following:

$$\wedge\mathcal{A}(x) = \inf_{A \in 2^U} \left[\mathcal{A}(A)^c \vee A(x)\right].$$

Now we further generalize this formula for the fuzzy soft family of fuzzy soft subsets of (U, E) and we get the following:

Definition 2.11 *Let $\mathcal{A} : F(U,E) \mapsto I$ be a mapping, which will be understood as a fuzzy soft family of fuzzy soft subsets of (U,E). The intersection of this fuzzy soft family is a function $\wedge\mathcal{A}^e : U \mapsto I$ (where $\mathcal{A}^e$ denotes the degree of elements of U with respect to the parameter $e \in E$), defined by the equality*

$$(\wedge\mathcal{A})^e(x) = \inf_{f_A \in U_E^\sim} \left[(\mathcal{A}(f_A^e))^c \vee f_A^e(x)\right], \quad \text{for all } e \in E.$$

Definition 2.12 *Let $\mathcal{A} : F(U,E) \mapsto I$. The union of this family is a function $\vee\mathcal{A}^e : U \mapsto I$, which is defined by the equality*

$$(\vee\mathcal{A})^e(x) = \sup_{f_A \in F(U,E)} \left[\mathcal{A}(f_A^e(x)) \vee f_A^e(x)\right], \quad \text{for all } e \in E.$$

3. Fuzzy soft topological spaces and the related category FST

In this section, we proceed with the fuzzy soft topological spaces and their corresponding FST categories. Throughout this work, (U, E) denotes the universe and the parameter set, respectively, and f_A is considered as a fuzzy soft set on (U, E).

Definition 3.1 *[4] Let $\tau : F(U, E) \mapsto I$ be a function satisfying the following three axioms:*

(i) If $f_A, g_A \in F(U, E)$, then $\tau^e(f_A \wedge g_A) \geq \tau^e(f_A) \wedge \tau^e(g_A)$.

(ii) If $f_{A_i} \in F(U, E), \forall i \in I$, then $\tau^e(\vee_{i\in I} f_{A_i}) \geq \wedge_{i\in I}\tau^e(f_{A_i})$.

(iii) $\tau^e(\Phi) = \tau^e(U_{\tilde{E}}) = 1$.

Then τ^e is called a fuzzy soft topology on $U_{\tilde{E}}$. The pair $(U_{\tilde{E}}, \tau^e)$ is called a fuzzy soft topological space over $U_{\tilde{E}}$. The real number $\tau^e(f_A)$ will be called the degree of openness of the fuzzy soft set f_A.

Remark 3.2 *Axiom (i) shows that the intersection of two fuzzy soft sets is not less open than the minimum of openness of these sets. In addition, axiom (ii) states that the degree of openness of the union of any crisp family of fuzzy soft sets is not less than the smallest degree of openness of these sets. As understood from the last axiom, the null and absolute fuzzy soft sets are exactly open sets.*

Definition 3.3 *Let $(U_{\tilde{E}}, \tau^e)$ be a fuzzy soft topological space. We define the mapping $\delta^e : F(U, E) \mapsto I$ by the equality $\delta^e(f_A) = \tau^e(f_A^c)$ for every $f_A \in F(U, E)$. The number $\delta^e(f_A)$ will be called the degree of closedness of a fuzzy soft set f_A.*

With the help of the previous definitions, we give the following proposition.

Proposition 3.4 *The mapping $\delta^e : F(U, E) \mapsto I$ satisfies the following axioms:*

(i) If $f_A, g_A \in F(U, E)$, then $\delta^e(f_A \vee g_A) \geq \delta^e(f_A) \wedge \delta^e(g_A)$.

(ii) $f_{A_i} \in F(U, E), \forall i \in I$, then $\delta^e(\wedge_{i\in I} f_{A_i}) \geq \wedge_{i\in I}\delta^e(f_{A_i})$.

(iii) $\delta^e(\Phi) = \delta^e(U_{\tilde{E}}) = 1$.

Proof We utilize the mapping δ^e given by Definition 3.3.

(i) $\delta^e(f_A \vee g_A) = \tau^e((f_A \vee g_A)^c) = \tau^e(f_A^c \wedge g_A^c) \geq \tau^e(f_A^c) \wedge \tau^e(g_A^c) = \delta^e(f_A) \wedge \delta^e(g_A)$.

(ii) $\delta^e(\wedge_{i\in I} f_{A_i}) = \tau^e((\wedge_{i\in I} f_{A_i})^c) = \tau^e(\vee_{i\in I} f_{A_i}^c) \geq \wedge_{i\in I}\tau^e(f_{A_i}^c) = \wedge_{i\in I}\delta^e(f_{A_i})$.

□

Remark 3.5 *A fuzzy soft topology can be defined by the family of mappings δ^e satisfying the above axioms (i), (ii), and (iii). This topology is defined by the relation $\tau^e(f_A) = \delta^e(f_A^c)$.*

Definition 3.6 *[15] Let $F(U,E)$ and $F(V,P)$ be two families of fuzzy soft sets over U and V, respectively. Let $\varphi : U \mapsto V$ and $\psi : E \mapsto P$ be two mappings. Then the pair (φ, ψ) is called a fuzzy soft mapping and is denoted by*

$$\varphi_\psi = (\varphi, \psi) : U_E^{\sim} \mapsto V_P^{\sim}.$$

The image of f_A under the fuzzy soft mapping φ_ψ is a fuzzy soft set over $V_P^{\sim}$ and is given by

$$\varphi_\psi(f_A)^p(v) = \begin{cases} \vee_{\varphi(u)=v} \vee_{\psi(e)=p} f^e(u) & \text{if } u \in \varphi^{-1}(v), \\ \Phi & \text{otherwise} \end{cases}$$

for all $p \in \psi(e)$ and for all $v \in V$.
The preimage of a fuzzy soft set g_B over $V_P^{\sim}$ is then equal to

$$\varphi_\psi^{-1}(g_B)^e(u) = g^{\psi(e)}(\varphi(u)),$$

for all $e \in \psi^{-1}(p)$ and for all $u \in U$.

Definition 3.7 *Let $(U_E^{\sim}, \tau_1^e)$ and $(V_P^{\sim}, \tau_2^e)$ be fuzzy soft topological spaces and $\varphi_\psi : U_E^{\sim} \mapsto V_P^{\sim}$ be a mapping. If $\tau_1^e(\varphi_\psi^{-1}(g_B)) \geq \tau_2^e(g_B)$ for all g_B over $V_P^{\sim}$ then the mapping φ_ψ is called fuzzy soft continuous.*

Proposition 3.8 *$\varphi_\psi : (U_E^{\sim}, \tau_1^e) \mapsto (V_P^{\sim}, \tau_2^e)$ is fuzzy soft continuous if and only if $\delta_1^e(\varphi_\psi^{-1}(g_B)) \geq \delta_2^e(g_B)$ for all g_B over $V_P^{\sim}$.*

Proof

$\Rightarrow$: $\delta_1^e(\varphi_\psi^{-1}(g_B)) = \tau_1^e((\varphi_\psi^{-1}(g_B))^c) = \tau_1^e(\varphi_\psi^{-1}(g_B^c)) \geq \tau_2^e(g_B^c) = \delta_2^e(g_B)$.

$\Leftarrow$: $\tau_1^e(\varphi_\psi^{-1}(g_B^c) = \tau_1^e((\varphi_\psi^{-1}(g_B))^c)) = \delta_1^e(\varphi_\psi^{-1}(g_B)) \geq \delta_2^e(g_B) = \tau_2^e((g_B)^c)$.

This shows that φ_ψ is fuzzy soft continuous. □

Theorem 3.9 *Let $(U_E^{\sim}, \tau_1^e)$ and $(V_P^{\sim}, \tau_2^e)$ and $(W_R^{\sim}, \tau_3^e)$ be fuzzy soft topological spaces and $\varphi_{1_{\psi_1}} : U_E^{\sim} \mapsto V_P^{\sim}$ and $\varphi_{2_{\psi_2}} : V_P^{\sim} \mapsto W_R^{\sim}$ be fuzzy soft continuous mappings. Then the composition $\varphi_{2_{\psi_2}} \circ \varphi_{1_{\psi_1}}$ is fuzzy soft continuous.*

Proof Let $h_C \in F(W,R)$. Since $\varphi_{2_{\psi_2}}$ is fuzzy soft continuous, we have

$$\tau_2(\varphi_{2_{\psi_2}}^{-1}(h_C)) \geq \tau_3^e(h_C).$$

Since $\varphi_{1_{\psi_1}}$ is fuzzy soft continuous, the following inequality holds:

$$\tau_1^e(\varphi_{1_{\psi_1}}^{-1}(\varphi_{2_{\psi_2}}^{-1}(h_C))) \geq \tau_2^e(\varphi_{2_{\psi_2}}^{-1}(h_C) \geq \tau_3^e(h_C).$$

Hence, $\varphi_{2_{\psi_2}} \circ \varphi_{1_{\psi_1}}$ is fuzzy soft continuous. □

Theorem 3.10 *Let $\varphi : U \mapsto U$, $\psi : E \mapsto E$ be the identity mappings. Then $i = \varphi_\psi$ is called an identity fuzzy soft function and this function is fuzzy soft continuous.*

Since the composition is associative and the identity fuzzy soft mapping i is fuzzy soft continuous with respect to any fuzzy soft topology on $F(U,E)$, the following definition is justified:

Definition 3.11 *The category in which the objects are fuzzy soft topological spaces and the morphisms are fuzzy soft continuous mappings between fuzzy soft topological spaces is denoted by FST.*

3.1. The initial fuzzy soft topology for a mapping

Let U be a universal and E be a parameter set, $(V_P^{\sim}, \sigma)$ be a fuzzy soft topological space, and $\varphi_\psi : U_E^{\sim} \mapsto V_P^{\sim}$ be a fuzzy soft mapping. We mean the weakest fuzzy soft topology on $U_E^{\sim}$, which makes φ_ψ fuzzy soft continuous by the initial fuzzy soft topology. Now we construct this fuzzy soft topology.

Let $\mathcal{F} = \{f_A = \varphi_\psi^{-1}(g_B) : g_B \in F(V,P)\}$ be a family of fuzzy soft subsets of $F(U,E)$. For a fuzzy soft set $f_A \in \mathcal{F}$, we define $\mathcal{P}_{f_A} = \{g_B : g_B \in F(V,P),\ f_A = \varphi_\psi^{-1}(g_B)\}$ and $\tau^e(f_A) = \sup\{\sigma(g_B) : g_B \in \mathcal{P}_f\}$. It can be easily seen that

$$\bigcup\{\mathcal{P}_f : f_A \in U_E^{\sim}\} = V_P^{\sim} \text{ and } \tau_f^e(\varphi_\psi^{-1}(g_B)) \geq \sigma_f(g_B) \text{ for all } \ g_B \in V_P^{\sim}.$$

Suppose that we take $f_{1_A}, f_{2_A} \in \mathcal{F}$; then we have $f_A = f_{1_A} \wedge f_{2_A} \in \mathcal{F}$ and $\mathcal{P}_{f_A} \supset \{g_{1_B}, g_{2_B} : g_{1_B} \in \mathcal{P}_{f_{1_A}},\ g_{2_B} \in \mathcal{P}_{f_{2_A}}\}$. Eventually, we obtain the following:

$$\begin{aligned}
\tau^e(f_A) &= \sup\{\sigma(g_B) : g_B \in \mathcal{P}_{f_A}\} \\
&\geq \sup\{\sigma(g_{1_B} \wedge g_{2_B}) : g_{1_B} \in \mathcal{P}_{f_{1_A}}, g_{2_B} \in \mathcal{P}_{f_{2_A}}\} \\
&\geq \sup\{\sigma(g_{1_B}) : g_{1_B} \in \mathcal{P}_{f_{1_A}}\} \wedge \sup\{\sigma(g_{2_B}) : g_{2_B} \in \mathcal{P}_{f_{2_A}}\} \\
&= \tau^e(f_{1_A}) \wedge \tau^e(f_{2_A}).
\end{aligned}$$

Hence, $\tau(f_A) \geq \tau^e(f_{1_A}) \wedge \tau^e(f_{2_A})$ for $f_{1_A}, f_{2_A} \in \mathcal{F}$. Furthermore, for any subfamily f_{i_A} of $\mathcal{F}$, we have

$$\tau^e(f_A)(\bigvee_i f_{i_A}) \geq \bigwedge_i \sup\{\sigma(g_{i_B}) : g_{i_B} \in \mathcal{P}_{f_{i_A}}\} = \bigwedge_i \tau^e(f_{i_A}).$$

Finally, one can easily see that

$$\Phi_E = \varphi_\psi^{-1}(\Phi_P) \in \mathcal{F},\ U_E^{\sim} = \varphi_\psi^{-1}(V_P^{\sim}) \in \mathcal{F} \text{ and } \tau^e(\phi_E) = \tau^e(U_E^{\sim}) = 1.$$

As a result, τ^e is a fuzzy soft topology over $U_E^{\sim}$ and moreover it is the weakest fuzzy soft topology, which assures that the mapping $\varphi_\psi : (U_E^{\sim}, \tau^e) \mapsto (V_P^{\sim}, \sigma^e)$ is fuzzy soft continuous.

In addition, we give the following theorem regarding the initial topology of the family of mappings.

Theorem 3.12 *Let $\{(V_{P_a}^{\sim}, \sigma_a) : a \in \mathcal{A}\}$ be a family of fuzzy soft topological spaces and $(\varphi_\psi)_a : U_E^{\sim} \mapsto V_{P_a}$ be a mapping for each $a \in \mathcal{A}$. We define the initial fuzzy topology over $U_E^{\sim}$ by the equality $\tau^e(f_A) = \inf_a \tau_a^e(f_A)$ where $f_A \in F(U,E)$. Then τ^e is a fuzzy soft topology over $U_E^{\sim}$. Moreover, it is the initial fuzzy soft topology for the family of the mappings $\{(\varphi_\psi)_a : U_E^{\sim} \mapsto V_{P_a}^{\sim} : a \in \mathcal{A}\}$.*

Proof We prove that $\tau^e(f_A) = \inf_a \tau^e_a(f_A)$ generates a topology over $U^{\sim}_E$.

$$\tau^e(f_{1_A} \wedge f_{2_A}) = \inf_a \tau^e_a(f_{1_A} \wedge f_{2_A}) \geq \inf_a \tau^e_a(f_{1_A}) \wedge \tau^e_a(f_{2_A})$$
$$\geq \inf_a \tau^e_a(f_{1_A}) \wedge \inf_a \tau^e_a(f_{2_A}) = \tau^e(f_{1_A}) \wedge \tau^e(f_{2_A}).$$

Furthermore, we get

$$\tau^e(\vee_i f_{i_A}) = \inf_a \tau^e_a(\vee_i f_{i_A}) \geq \inf_a \wedge_i \tau^e_a(f_{i_A})$$
$$= \wedge_i \inf_a \tau^e_a(f_{i_A}) = \wedge_i \tau^e(f_{i_A}).$$

The proof of the last axiom is trivial. This completes the proof. □

Theorem 3.13 *FST is a complete category. Moreover, FST contains products and inverse limits.*

Proof It is obvious from the above theorem. □

3.2. The final fuzzy soft topology for a mapping

Let $(U^{\sim}_E, \tau^e)$ be a fuzzy soft topological space and $V^{\sim}_P$ be a fuzzy soft set, and let $\varphi_\psi : U^{\sim}_E \mapsto V^{\sim}_P$ be a mapping and $g_B \in V^{\sim}_P$. Assume that we construct a fuzzy soft topology on $V^{\sim}_P$ such that $\sigma^e(g_B) = \tau^e(\varphi^{-1}_\psi(g_B))$. Accordingly, σ^e is a fuzzy soft topology over $V^{\sim}_P$. Moreover, it is the strongest fuzzy soft topology, which makes the mapping $\varphi_\psi : (U^{\sim}_E, \tau^e) \mapsto (V^{\sim}_P, \sigma^e)$ fuzzy soft continuous.
Let $g_{1_B}, g_{2_B} \in V^{\sim}_P$. Then we have

$$\sigma^e(g_{1_B} \wedge g_{2_B}) = \tau^e(\varphi^{-1}_\psi(g_{1_B} \wedge g_{2_B}))$$
$$= \tau^e(\varphi^{-1}_\psi(g_{1_B}) \wedge \varphi^{-1}_\psi(g_{2_B}))$$
$$\geq \tau^e(\varphi^{-1}_\psi(g_{1_B})) \wedge \tau^e(\varphi^{-1}_\psi(g_{2_B}))$$
$$= \sigma^e(g_{1_B}) \wedge \sigma^e(g_{2_B}).$$

Let $\vee_i g_{i_B}$ be a family of fuzzy soft sets over $V^{\sim}_P$. Then we have

$$\sigma^e(\vee_i g_{i_B}) = \tau^e(\varphi^{-1}_\psi(\vee_i g_{i_B})) = \tau^e(\vee_i \varphi^{-1}_\psi(g_{i_B})$$
$$\geq \wedge_i \tau^e(\varphi^{-1}_\psi(g_{i_B})) = \wedge_i \sigma^e_{g_B}((g_{i_B}, P)).$$

It is clear that σ^e is a fuzzy soft topology over $V^{\sim}_P$.

Finally, we give the following theorem concerning the final topology of a family of mappings.

Theorem 3.14 *Let $\{(U^{\sim}_{a_E}, \tau^e_{a^\sim} : a \in \mathcal{A})\}$ be a family of fuzzy soft sets and $V^{\sim}_P$ be a fuzzy soft set $(\varphi_\psi)_a : U^{\sim}_{a_E} \mapsto V^{\sim}_P$ be a mapping. Let σ^e_a denote the final fuzzy soft topology on $V^{\sim}_P$ for $(\varphi_\psi)_a$. We define $\sigma^e : V^{\sim}_P \mapsto I$ by the equality $\sigma^e(g_B) = \inf_a \sigma^e_a(g_B)$. Then σ^e is a fuzzy soft topology over $V^{\sim}_P$. Moreover it is the strongest fuzzy soft topology, which ensures that all the mappings $(\varphi_\psi)_a : U^{\sim}_{a_E} \mapsto V^{\sim}_P$ are fuzzy soft continuous.*

We omit the proof of this theorem, since it follows an approach similar to that given in Theorem 3.12. Using this theorem, we also get the following result:

Theorem 3.15 *The category FST is cocomplete. Moreover, it contains coproducts and direct limits.*

References

[1] Ahmad B, Kharal A. On fuzzy soft sets. Adv Fuzzy Syst 2009; 2009: 586507. doi:10.1155/2009/586507.

[2] Aktas H, Cagman N. Soft sets and soft groups. Inform Sci 2007; 77: 2726-2735.

[3] Aygunoglu A, Aygun H. Some notes on soft topological spaces. Neural Computing and Applications 2011; 21: 113. doi: 10.1007/s00521-011-0722-3.

[4] Aygunoglu A, Cetkin V, Aygun H. An introduction to fuzzy soft topological spaces. Hacettepe Journal of Mathematics and Statistics 2014; 43: 197-208.

[5] Feng F, Jun YB, Zhao X. Soft semirings. Computers and Mathematics with Applications 2008; 56 (10): 2621-2628.

[6] Kharal A, Ahmad B. Mappings on fuzzy soft classes. Advances in Fuzzy Systems 2009; 2009: 407890. doi:10.1155/2009/407890.

[7] Kucuk A, Ozturk T. Homology modules of fuzzy soft modules. Annals of Fuzzy Mathematics and Informatics 2013; 5 (3): 607-619.

[8] Maji PK, Roy AR, Biswas R. Fuzzy soft sets. Journal of Fuzzy Mathematics 2001; 9 (3): 589-602.

[9] Molodtsov D. Soft set theory-First results. Computers and Mathematics with Applications 1999; 37 (4-5): 19-31.

[10] Ozturk T, Bayramov S. Category of chain complexes of soft modules. International Mathematical Forum 2012; 7 (20): 981-992.

[11] Roy S, Samanta TK. A note on fuzzy soft topological spaces. Annals of Fuzzy Mathematics and Informatics 2011; 3 (2): 305-311.

[12] Simsekler T, Yuksel S. Fuzzy soft topological spaces. Annals of Fuzzy Mathematics and Informatics 2013; 5 (1): 87-96.

[13] Šostak, A. On a fuzzy topological structure. In: Proceedings of the 13th Winter School on Abstract Analysis, Palermo, 1985, pp. 89-103.

[14] Tanay B, Kandemir MB. Topological structure of fuzzy soft sets. Computers and Mathematics with Applications 2011; 61: 2952-2957.

[15] Varol PB, Aygun H. Fuzzy soft topology. Hacettepe Journal of Mathematics and Statistics 2012; 41 (3): 407-419.

[16] Zadeh LA. Fuzzy sets. Information and Control 1965; 8: 338-353.