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On near soft sets

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Abstract: This study aims to contribute to the theoretical studies on near soft sets and near soft topological spaces. In addition, it presents basic concepts and constructs that will form the basis for a near theoretical set-up of near soft sets. These concepts and structures include near soft point, near soft interior, near soft closure, near soft neighborhood, near soft continuity, and near soft open (closed) function.

Key words: Near soft set, near soft point, near soft interior, near soft closure, near soft continuity, near soft open (closed) function

1. Introduction

The rough sets were presented by Pawlak in order to solve the problem of uncertainty in 1982. In this set, the universal set is presented by the lower and upper approaches. This theory aimed to introduce some approaches into the sets. The theory of near sets was presented by Peters [6, 7]. While Peters defines the nearness of objects, he is dependent on the nature of the objects, so he classifies the universal set according to the available information of the objects. Moreover, through entrenching the notions of rough sets, numerous applications of the near set theory have been enlarged and varied. Near sets and rough sets are like two sides of a coin; the only difference is the fact that what is focused on for rough sets is the approach of sets with nonempty boundaries.

The soft sets were presented by Molodtsov [5] as a mathematical tool for modeling uncertainty problems and examined by many scientists, and they presented a new approach to uncertainty [1, 2, 4, 9]. Later, Feng et al. [3] conducted research on the problem of joining soft sets with rough sets and presented the concept of rough soft sets. Tasbozan et al. [10] introduced the definition of the near soft set by obtaining the lower and upper approximations of a soft set in nearness approximation space (NAS). Moreover, they examined some properties of the near soft set.

The aim of this study is to complement the theoretical studies of near soft sets and near soft topological spaces. In addition, it is intended to provide basic concepts and constructs that will form the basis for a near theoretical set-up of near soft sets.

2. Preliminaries

In this section, we recollect some descriptions and results presented discussed in [10]. Moreover, we present the concepts of near soft sets, their fundamental properties, and operations such as near soft union, near soft

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intersection, near soft point, near soft interior, near soft closure, near soft neighborhood, near soft continuity, and near soft open (closed) function.

Henceforth, let O be an initial universe set and F be a set of parameters with respect to O unless otherwise specified. Let $N_r(B)(X)$ be a family of neighborhoods of a set X and $X \subseteq O$.

Definition 2.1 *[10] Let $NAS = (O, F, \sim_{Br}, N_r, V_{N_r})$ be a nearness approximation space and $\sigma = F_B$ be a soft set over O. The lower and upper near approximation of $\sigma = F_B$ with respect to the NAS are denoted by $N_{r^*}(\sigma) = F_{B^*}$ and $N_r^*(\sigma) = F_B^*$, which are soft sets over with the set-valued mappings given by*

$$F_*(\phi) = N_{r^*}(F(\phi)) = \cup_{a \in O} \{\overline{a}_{B_r} \subseteq F(\phi)\},$$
$$F^*(\phi) = N_r^*(F(\phi)) = \cup_{a \in O} \{\overline{a}_{B_r} \cap F(\phi) \neq \emptyset\},$$

where all $\phi \in B$. For two operators N_{r^*} and N_r^* on a soft set, we say that these are the lower and upper near approximation operators, respectively. If $BN_{N_r(B)}(X) \geq 0$, then the soft set σ is said to be a near soft set.

Example 2.2 *Let $X = \{a_1, a_2, a_3\} \subset O = \{a_1, a_2, a_3, a_4, a_5\}$, $B = \{\phi_1, \phi_2, \phi_3\} \subset F = \{\phi_1, \phi_2, \phi_3, \phi_4\}$ denote a set of perceptual objects and a set of functions, respectively. Sample values of the ϕ_i, $i = 1, 2, 3, 4$ functions are shown in Table 1.*

Table 1.

	a_1	a_2	a_3	a_4	a_5
ϕ_1	0	1	0	1	1
ϕ_2	2	2	3	2	3
ϕ_3	0,01	0,03	0,02	0,03	0,02
ϕ_4	1	1	0	1	0

Let $\sigma = F_B$, $B = \{\phi_1, \phi_2, \phi_3\}$ be a soft set defined by
$F_B = \{(\phi_1, \{a_2, a_3\}), (\phi_2, \{a_1, a_3, a_4\}), (\phi_3, \{a_1, a_2, a_3, a_4\})\}$.
Then for $r = 1$;
$\overline{a_1}_{\phi_1} = \{a_1, a_3\}$, $\overline{a_2}_{\phi_1} = \{a_2, a_4, a_5\}$,
$\overline{a_1}_{\phi_2} = \{a_1, a_2, a_4\}$, $\overline{a_3}_{\phi_2} = \{a_3, a_5\}$,
$\overline{a_1}_{\phi_3} = \{a_1\}$, $\overline{a_2}_{\phi_3} = \{a_2, a_4\}$, $\overline{a_3}_{\phi_3} = \{a_3, a_5\}$.
$N_*(\sigma) = \{(\phi_1, \emptyset), (\phi_2, \emptyset), (\phi_3, \{a_1, a_2, a_4\})\}$ *and*
$N^*(\sigma) = \{(\phi_1, O), (\phi_2, O), (\phi_3, O\}$. $BN_N(\sigma) \geq 0$, *and then σ is a near soft set.*
Now for $r = 2$,
$\overline{a_1}_{\phi_1,\phi_2} = \{a_1\}$, $\overline{a_2}_{\phi_1,\phi_2} = \{a_2, a_4\}$, $\overline{a_3}_{\phi_1,\phi_2} = \{a_3\}$, $\overline{a_5}_{\phi_1,\phi_2} = \{a_5\}$,
$\overline{a_1}_{\phi_1,\phi_3} = \{a_1\}$, $\overline{a_2}_{\phi_1,\phi_3} = \{a_2, a_4\}$, $\overline{a_3}_{\phi_1,\phi_3} = \{a_3\}$, $\overline{a_4}_{\phi_1,\phi_3} = \{a_5\}$,
$\overline{a_1}_{\phi_2,\phi_3} = \{a_1\}$, $\overline{a_2}_{\phi_2,\phi_3} = \{a_2, a_4\}$, $\overline{a_3}_{\phi_2,\phi_3} = \{a_3, a_5\}$.
$N_*(\sigma) = \{(\phi_1, \{a_3\}), (\phi_2, \{a_1, a_3\}), (\phi_3, \{a_1, a_2, a_3, a_4\})\}$ *and*
$N^*(\sigma) = \{(\phi_1, \{a_2, a_3, a_4\}), (\phi_2, O), (\phi_3, O\}$.
$BN_N(\sigma) \geq 0$, *and then σ is a near soft set.*

Finally, for $r = 3$,
$\overline{a_1}_{\phi_1,\phi_2,\phi_3} = \{a_1\}$, $\overline{a_2}_{\phi_1,\phi_2,\phi_3} = \{a_2, a_4\}$, $\overline{a_3}_{\phi_1,\phi_2,\phi_3} = \{a_3\}$,
$\overline{a_5}_{\phi_1,\phi_2,\phi_3} = \{a_5\}$.
$N_*(\sigma) = \{(\phi_1, \{a_3\}), (\phi_2, \{a_1, a_3\}), (\phi_3, \{a_1, a_2, a_3, a_4\})\}$ *and*
$N^*(\sigma) = \{(\phi_1, \{a_2, a_3, a_4\}), (\phi_2, \{a_1, a_2, a_3, a_4\}), (\phi_3, \{a_1, a_2, a_3, a_4\}\}$.
$BN_N(\sigma) \geq 0$, *and then* σ *is a near soft set.*

We consider only near soft sets F_B over a universe O in which all the parameter sets B are the same. The set of all near soft sets over O is denoted by $NSS(O_B)$.

Definition 2.3 *Let* O *be an initial universe set and* F *be a universe set of parameters. Let* F_A *and* G_B *be near soft sets over a common universe set* O *and* A, $B \subset F$.

1. The extended intersection of F_A *and* G_B *over* O *is the near soft set* H_C, *where* $C = A \cup B$, *and* $\forall \phi \in C$,

$$H(\phi) = \begin{cases} F(\phi) & \text{if } \phi \in A - B \\ G(\phi) & \text{if } \phi \in B - A \\ F(\phi) \cap G(\phi) & \text{if } \phi \in A \cap B. \end{cases}$$

We write $F_A \cap G_B = H_C$.

2. The restricted intersection of F_A *and* G_B *is the near soft set* H_C, *where* $C = A \cap B$, *and* $H(\phi) = F(\phi) \cap G(\phi)$ *for all* $\phi \in C$. *We write* $F_A \cap G_B = H_C$.

3. The extended union of F_A *and* G_B *is the near soft set* H_C, *where* $C = A \cup B$, *and* $\forall \phi \in C$,

$$H(\phi) = \begin{cases} F(\phi) & \text{if } \phi \in A - B \\ G(\phi) & \text{if } \phi \in B - A \\ F(\phi) \cup G(\phi) & \text{if } \phi \in A \cap B. \end{cases}$$

We write $F_A \cup G_B = H_C$.

4. The restricted union of F_A *and* G_B *is the near soft set* H_C, *where* $C = A \cap B$, *and* $H(\phi) = F(\phi) \cup G(\phi)$ *for all* $\phi \in C$. *We write* $F_A \cup G_B = H_C$.

Definition 2.4 *[10] Let* $F_B \in NSS(O_B)$. *Then* F_B *is called:*

1. a null near soft set if $F(\phi) = \emptyset$, *for all* $\phi \in B$.

2. a whole near soft set if $F(\phi) = O$, *for all* $\phi \in B$.

Definition 2.5 *[10] Let* $F_B \in NSS(O_B)$. *Then* F_B^c *is said to be the relative complement of* F_B, *where* $F^c(\phi) = O - F(\phi)$ *for all* $\phi \in B$.

Definition 2.6 *[10] Let* F_B, $G_B \in NSS(O_B)$. *Then* F_B *is a near soft subset of* G_B, *denoted by* $F_B \subseteq G_B$, *if* $N_*(F_B) \subseteq N_*(G_B)$ *for all* $\phi \in B$, *i.e.* $N_*(F(\phi), B) \subseteq N_*(G(\phi), B)$ *for all* $\phi \in B$.

F_A *is called a near* soft superset *of* G_B, *denoted by* $F_A \supseteq G_B$, *if* G_B *is a near soft subset of* F_A.

Definition 2.7 *Let* F_B, $G_B \in NSS(O_B)$. *If* F_B *and* G_B *near soft sets are subsets of each other, then they are called equal, denoted by* $F_B = G_B$.

Theorem 2.8 *[10] Let $F_B,\ G_B \in NSS(O_B)$. Then:*

1. $(F_B \cup G_B)^c = F_B^c \cap G_B^c$.
2. $(F_B \cap G_B)^c = F_B^c \cup G_B^c$.

We can prove that the following De Morgan laws hold in near soft set theory for the extended union and the intersection.

Theorem 2.9 *Let I be an index set and $F_{iB} \in NSS(O_B),\ \forall i \in I$. Then:*

1. $\left[\underset{i\in I}{\cup} F_{iB}\right]^c = \underset{i\in I}{\cap} F_{iB}^c$ *and*
2. $\left[\underset{i\in I}{\cap} F_{iB}\right]^c = \underset{i\in I}{\cup} F_{iB}^c$.

Proof 1. Let $\left[\underset{i\in I}{\cup} F_{iB}\right]^c = H_B^c$. We have $H^c(\phi) = O - H(\phi) = O - \underset{i\in I}{\cup} F_i(\phi) = \underset{i\in I}{\cap}(O - F_i(\phi))$ for all $\phi \in B$. In other words, $\underset{i\in I}{\cap} F_{iB}^c = K_B$. We have $K(\phi) = \underset{i\in I}{\cap} {F_i}^c(\phi) = \underset{i\in I}{\cap}(O - F_i(\phi))$ for all $\phi \in B$. The proof is completed.

2. Let $\left[\underset{i\in I}{\cap} F_{iB}\right]^c = H_B^c$. We have $H^c(\phi) = O - H(\phi) = O - \underset{i\in I}{\cap} F_i(\phi) = \underset{i\in I}{\cup}(O - F_i(\phi))$ for all $\phi \in B$. In other words, $\cup F_{iB}^c = K_B$. We have $K(\phi) = \underset{i\in I}{\cup} {F_i}^c(\phi) = \underset{i\in I}{\cup}(O - F_i(\phi))$ for all $\phi \in B$. The proof is completed. □

Definition 2.10 *[10] Let $\sigma = F_B$ be a near soft set over O_B, τ be the collection of near soft subsets of σ, and $B \subseteq F$ be the nonempty set of parameters; then τ is called a near soft topology on O_B if τ satisfies the following axioms:*

1. *$\emptyset_B,\ O_B \in \tau$ where $\emptyset(\phi) = \emptyset$ and $F(\phi) = F$, for all $\phi \in B$.*
2. *Finite intersections of near soft sets in τ belong to τ.*
3. *Arbitrary unions of near soft sets in τ belong to τ.*

The pair (O, τ) is called a near soft topological space.

We write nsts instead of near soft topological space.

Example 2.11 *Let $O = \{a_1, a_2, a_3, a_4, a_5\}$, $B = \{\phi_1, \phi_2, \phi_3\} \subset F = \{\phi_1, \phi_2, \phi_3, \phi_4\}$ where $\sigma,\ F_B^1, ..., F_B^4$ are near soft sets over O_B, defined as follows. Then:*

$\sigma = F_B = \{(\phi_1, \{a_2, a_3\}), (\phi_2, \{a_1, a_3, a_4\}), (\phi_3, \{a_1, a_2, a_3, a_4\})\}$,

$F_B^1 = \{(\phi_1, \{a_2\}), (\phi_2, \{a_1\}), (\phi_3, \{a_1\})\}$,

$F_B^2 = \{(\phi_1, \{a_3\}), (\phi_2, \{a_3\}), (\phi_3, \{a_1, a_2\})\}$,

$F_B^3 = \{(\phi_1, \{a_2, a_3\}), (\phi_2, \{a_1, a_3\}), (\phi_3, \{a_1, a_2\})\}$,

$F_B^4 = \{(\phi_1, \emptyset), (\phi_2, \emptyset), (\phi_3, \{a_1\})\}$.

τ defines a near soft topology on O_B and thus (O, τ) is a nsts over O_B.

Definition 2.12 *Letting (O, τ) be a nsts over O_B, then the members of τ are said to be near soft open sets O_B.*

Definition 2.13 *Let (O, τ) be a nsts over O_B. A near soft set F_B over O_B is called a near soft closed set in O_B, if its relative complement F_B^c belongs to τ.*

Theorem 2.14 *[10] Let (O, B, τ) be a nsts on O_B. Then:*

1. *$\emptyset_B^c$, O_B^c are near soft closed sets over O_B.*
2. *Arbitrary intersections of the near soft closed sets are near soft closeds over O_B.*
3. *Finite unions of the near soft closed sets are near soft closed sets over O_B.*

Now, for any near soft set in a nsts, we present the following sets.

Definition 2.15 *Let (O, B, τ) be a nsts and $F_B \in NSS(O_B)$. Then:*

1. The set $\cap\{G_B \supseteq F_B : G_B$ is a near soft closed set of $O_B\}$ is called the near soft closure of F_B in O_B, denoted by $cl_n(F_B)$.

2. The set $\cup\{C_B \subseteq F_B : C_B$ is a near soft open set of $O_B\}$ is called the near soft interior of F_B in O_B, denoted by $int_n(F_B)$.

Example 2.16 *Let us consider the near soft topology τ that is given in Example 2.11. Then:*

$F_B = \{(\phi_1, \{a_2, a_3\}), (\phi_2, \{a_1, a_3, a_4\}), (\phi_3, \{a_1, a_2, a_3, a_4\})\}$,

$int_n(F_B) = F_B \cup F_B^1 \cup F_B^2 \cup F_B^3 \cup F_B^4$

$\quad = \{(\phi_1, \{a_2, a_3\}), (\phi_2, \{a_1, a_3, a_4\}), (\phi_3, \{a_1, a_2, a_3, a_4\})\}$, *and*

for near soft set $F_C = \{(\phi_1, \{a_1, a_2, a_4\}), (\phi_2, \{a_2, a_4, a_5\}), (\phi_3, \{a_4, a_5\}),$

$cl_n(F_C) = (F_B^2)^c \cap (F_B^4)^c = (F_B^4)^c = \{(\phi_1, O), (\phi_2, O), (\phi_3, \{a_2, a_3, a_4, a_5\})\}$.

Proposition 2.17 *Let (O, B, τ) be a nsts and F_B, $G_B \in NSS(O_B)$. Then the following hold:*

1. *$int_n(int_n(F_B)) = int_n(F_B)$.*
2. *If $F_B \subseteq G_B$, then $int_n(F_B) \subseteq int_n(G_B)$.*
3. *$int_n(F_B) \cap int_n(G_B) = int_n(F_B \cap G_B)$.*
4. *$int_n(F_B) \cup int_n(G_B) \subseteq int_n(F_B \cap G_B)$.*
5. *$cl_n(cl_n(F_B)) = cl_n(F_B)$.*
6. *If $F_B \subseteq G_B$, then $cl_n(F_B) \subseteq cl_n(G_B)$.*
7. *$cl_n(F_B) \cap cl_n(G_B) \subseteq cl_n(F_B \cap G_B)$.*
8. *$cl_n(F_B) \cup cl_n(G_B) \subseteq cl_n(F_B \cap G_B)$.*

The relation between the near soft closure and near soft interior is given by the following theorem.

Theorem 2.18 *Let (O, B, τ) be a nsts and $F_B \in NSS(O_B)$. Then the following hold:*

1. *$cl_n(F_B^c) = O - int_n(F_B)$.*
2. *$int_n(F_B^c) = O - cl_n(F_B)$.*

Proof 1. Let near soft open set $G_B \subseteq F_B$ and near soft closed set $C_B \supseteq F_B^c$. Then:

$int_n(F_B) = \cup\{C_B^c : C_B$ is a near soft closed set and $C_B \supseteq F_B^c\}$

$\quad = O - \cap\{C_B : C_B$ is a near soft closed set and $C_B \supseteq F_B^c\}$

$= O - cl_n(F_B^c)$. Therefore, $cl_n(F_B^c) = O - int_n(F_B)$.

2. Let G_B be near soft open set. Then for a near soft closed set $G_B \supseteq F_B$, $G_B \subseteq F_B^c$.

$cl_n(F_B) = \cap\{G_B^c : G_B$ is a near soft open set and $G_B \subseteq F_B^c\}$

$= O - \cup\{G_B : G_B$ is a near soft open set and $G_B \subseteq F_B^c\}$

$= O - int_n(F_B^c)$. Therefore, $int_n(F_B^c) = O - cl_n(F_B)$. □

Corollary 2.19 *Let (O, B, τ) be a nsts and F_B, $G_B \in NSS(O_B)$. Then the following hold:*

1. F_B is near soft closed iff $F_B = cl_n(F_B)$.

2. G_B is near soft open iff $G_B = int_n(G_B)$.

Proof 1. Suppose $F_B = cl_n(F_B) = \cap\{C_B : C_B$ is a near soft closed set and $C_B \supseteq F_B\}$, which implies $F_B \in \cap\{C_B$ is a near soft closed set and $C_B \supseteq F_B\}$, which implies that F_B is a near soft closed set.

Conversely, F_B is a near soft closed set in O_B. We take $F_B \subseteq F_B$ and F_B is a near soft closed set. That is why $F_B \in \cap\{C_B : C_B$ is a near soft closed set and $C_B \supseteq F_B\}$. $F_B \subseteq C_B$ implies $F_B = \cap\{C_B : C_B$ is a near soft closed set and $C_B \supseteq F_B\} = cl_n(F_B)$. $F_B = cl_n(F_B)$.

2. We apply near soft interiors. □

In the following, we give the definition of a near soft point.

Definition 2.20 *Let $F_B \in NSS(O_B)$. If for the element $\phi \in B$, $F(\phi) \neq \varnothing$ and $F(\phi') = \varnothing$ for all $\phi' \in B - \{\phi\}$, then F_B is called a near soft point in O_B, denoted by ϕ_F^N.*

Definition 2.21 *Let (O, B, τ) be a nsts and $G_B \in NSS(O_B)$. The near soft point $\phi_F^N \in O_B$ is called a near soft interior point of a near soft set G_B if there exists a near soft open set H_B such that $\phi_F^N \in H_B \subseteq G_B$.*

Proposition 2.22 *Let $\phi_F^N \in O_B$ for all $\phi \in B$ and G_B be a near soft open set in a nsts (O, B, τ). Then every near soft point $\phi_F^N \in G_B$ is a near soft interior point.*

Proof This is clearly seen from Definition 2.21. □

Definition 2.23 *Let F_B, $G_B \in NSS(O_B)$. The near soft point ϕ_F^N is said to be in the near soft set G_B, denoted by $\phi_F^N \in G_B$, if for the element $\phi \in B$ and $F(\phi) \subseteq G(\phi)$.*

Proposition 2.24 *Let $\phi_F^N \in O_B$ and $G_B \subseteq O_B$. If $\phi_F^N \in G_B$, then $\phi_F^N \notin G_B^c$.*

Proof If $\phi_F^N \in G_B$, then for $\phi \in B$ and $F(\phi) \subseteq G(\phi)$. This implies $F(\phi) \nsubseteq O - G(\phi) = G^c(\phi)$. Therefore, we have $\phi_F^N \notin G_B^c$. □

Remark 2.25 *As shown in the next example, the converse of the above proposition is not true in general.*

Example 2.26 *According to Example 2.2, G_B is a near soft set and is defined as follows:*
$G_B = \{(\phi_1, \{a_2\}), (\phi_2, \{a_1, a_3\}), (\phi_3, \{a_2, a_4\})\}$. *Then for a near soft point,*

$\phi_{2_F}^N = \{(\phi_2, \{a_1, a_2, a_4\})\}$, $\phi_{2_F}^N \notin G_B$ *and also*

$\phi_{2_F}^N \notin G_B^c = \{(\phi_1, \{a_1, a_3, a_4, a_5\}), (\phi_2, \{a_2, a_4, a_5\}), (\phi_3, \{a_1, a_3, a_5\})\}$.

Definition 2.27 *Let (O, B, τ) be a nsts and $G_B \in NSS(O_B)$. If there exists a near soft open set H_B such that $\phi_F^N \in H_B \subseteq G_B$, then G_B is called a near* soft neighborhood *(written near soft nbd) of the near soft point $\phi_F^N \in O_B$.*

The near soft nbd system of near soft point ϕ_F^N, which is denoted by $N_\tau(\phi_F^N)$, is the set of all its near soft nbd.

Definition 2.28 *Let (O, B, τ) be a nsts and $G_B \in NSS(O_B)$. If there exists a near soft open set H_B such that $F_B \subseteq H_B \subseteq G_B$, then G_B is called a near* soft nbd *of the near soft set F_B.*

The important properties of near soft nbd are given by the following theorem.

Theorem 2.29 *Let (O, B, τ) be a nsts and $G_B,\ H_B \in NSS(O_B)$. The neighborhood system $N_\tau(\phi_F^N)$ at ϕ_F^N in (O, B, τ) has the following properties:*

1. *If $G_B \in N_\tau(\phi_F^N)$, then $\phi_F^N \in G_B$;*
2. *If $G_B \in N_\tau(\phi_F^N)$ and $G_B \subseteq H_B$, then $H_B \in N_\tau(\phi_F^N)$;*
3. *If $G_B, H_B \in N_\tau(\phi_F^N)$, then $G_B \cap H_B \in N_\tau(\phi_F^N)$;*
4. *If $G_B \in N_\tau(\phi_F^N)$, then there is $M_B \in N_\tau(\phi_F^N)$ such that $G_B \in N_\tau(\phi'_H)$, for each $\phi'_H \in M_B$.*

Proof 1. If $G_B \in N_\tau(\phi_F^N)$, then there is $H_B \in \tau$ such that $\phi_F^N \in H_B \subseteq G_B$. Therefore, we have $\phi_F^N \in G_B$.

2. Let $G_B \in N_\tau(\phi_F^N)$ and $G_B \subseteq H_B$. Since $G_B \in N_\tau(\phi_F^N)$, then there is $M_B \in \tau$ such that $\phi_F^N \in M_B \subseteq G_B$. Therefore, we have $\phi_F^N \in M_B \subseteq G_B \subseteq H_B$ and so $H_B \in N_\tau(\phi_F^N)$.

3. If $G_B,\ H_B \in N_\tau(\phi_F^N)$, then there exist $M_B,\ S_B \in \tau$ such that $\phi_F^N \in M_B \subseteq G_B$ and $\phi_F^N \in S_B \subseteq H_B$. Hence, $\phi_F^N \in M_B \cap S_B \subseteq G_B \cap H_B$. Since $M_B \cap S_B \in \tau$, we have $G_B \cap H_B \in N_\tau(\phi_F^N)$.

4. If $G_B \in N_\tau(\phi_F^N)$, then there is $S_B \in \tau$ such that $\phi_F^N \in S_B \subseteq G_B$. Put $M_B = S_B$. Then for every $\phi'_H \in M_B$, $\phi'_H \in M_B \subseteq S_B \subseteq G_B$. This implies $G_B \in N_\tau(\phi'_H)$. □

Theorem 2.30 *Let (O, B, τ) be a nsts and $F_B,\ G_B \in NSS(O_B)$. A near soft set G_B is near soft open if and only if for each near soft set F_B contained in G_B, G_B is a near soft nbd of F_B.*

Proof Suppose G_B is a near soft open set in O_B. Then $F_B \subseteq H_B \subseteq G_B$ implies that G_B is a near soft nbd of each F_B.

Conversely, since $G_B \subseteq G_B$, there is a near soft open set H_B such that $G_B \subseteq H_B \subseteq G_B$. Hence, $H_B = G_B$ and G_B is a near soft open set. □

3. Near soft continuous functions

In this section, we explore the fundamental features of family $NSS(O_B)$ and give many properties of near soft continuous functions. Throughout the paper, the spaces O and V stand for near soft topological spaces with the assumed ((O, A, τ) and (V, B, τ^*)) unless otherwise stated and a near soft mapping $f : O \to V$ stands for a mapping, where $f : (O, A, \tau) \to (V, B, \tau^*)$, $u : O \to V$, and $p : A \to B$ are assumed mappings unless otherwise stated and $A,\ B \subset F$.

Proposition 3.1 *Let $F_B, G_B, H_B, S_B \in NSS(O_B)$. Then the following hold:*

1. $F_B \cap \emptyset_B = O_B$.
2. $F_B \cap O_B = F_B$.
3. $F_B \cup \emptyset_B = F_B$.
4. $F_B \cup O_B = O_B$.
5. $F_B \subseteq G_B$ *iff* $F_B \cap G_B = F_B$, *for all* $\phi \in B$.
6. $F_B \subseteq G_B$ *iff* $F_B \cup G_B = G_B$, *for all* $\phi \in B$.
7. *If* $F_B \cap G_B = \emptyset_B$, *then* $F_B \subseteq G_B^c$.
8. $F_B \cup F_B^c = O_B$.
9. *If* $F_B \subseteq G_B$ *and* $G_B \subseteq H_B$, *then* $F_B \subseteq H_B$.
10. *If* $F_B \subseteq G_B$ *and* $H_B \subseteq S_B$, *then* $F_B \cap H_B \subseteq G_B \cap S_B$.
11. $F_B \subseteq G_B$ *iff* $G_B^c \subseteq F_B^c$.

Proof We just show 5, 6, 7, and 11. The other proofs follow similar lines.

5. Assume that $F_B \subseteq G_B$. Then $N_*(F(\phi)) \subseteq N_*(G(\phi))$ for all $\phi \in B$. Let $F_B \cap G_B = H_B$. Since $H(\phi) = F(\phi) \cap G(\phi) = F(\phi)$ for all $\phi \in B$, by definition $H_B = F_B$.

Suppose that $F_B \cap G_B = F_B$. Let $F_B \cap G_B = H_B$. Since $H(\phi) = F(\phi) \cap G(\phi) = F(\phi)$ for all $\phi \in B$, we know that $N_*(F(\phi)) \subseteq N_*(G(\phi))$ for all $\phi \in B$. Thus, $F_B \subseteq G_B$.

6. It is similar to the proof of 5.

7. Suppose that $F_B \cap G_B = O_B$. Then $F(\phi) \cap G(\phi) = \varnothing$ and so $N_*(F(\phi)) \subseteq N_*(O - G(\phi)) = N_*(G^c(\phi))$ for all $\phi \in B$. We have $F_B \subseteq G_B^c$.

11. It follows from the following: $F_B \subseteq G_B$ iff $N_*(F(\phi)) \subseteq N_*(G(\phi))$ for all $\phi \in B$ iff $G(\ \phi\ )^c \subseteq F(\ \phi\ )^c$ for all $\phi \in B$ iff $G^c(\phi) \subseteq F^c(\phi)$ for all $\phi \in B$ iff $G_B^c \subseteq F_B^c$. □

Next, we will set up several properties of near soft sets induced by mappings.

Definition 3.2 *Let $NSS(O_A)$ and $NSS(V_B)$ be the families of all near soft sets over O and V, respectively. The mapping f is called a near soft mapping from O to V, denoted by $f : NSS(O_A) \to NSS(V_B)$, where $u : O \to V$ and $p : A \to B$ are two mappings.*

1. Let F_A be a near soft set in $NSS(O_A)$. Then for all $\omega \in B$ the image of F_A under f, written as $f(F_A) = (f(F), p(A))$, is a near soft set in $NSS(V_B)$ defined as follows:

$$f(F)(\omega) = \begin{cases} \bigcup\limits_{\phi \in p^{-1}(\omega) \cap A} u(F(\phi)), & p^{-1}(\omega) \cap A \neq \varnothing \\ \varnothing, & \textit{otherwise.} \end{cases}$$

2. Let G_B be a near soft set in $NSS(V_B)$. Then for all $\phi \in A$ the near soft inverse image of G_B under f, written as $f^{-1}(G_B) = (f^{-1}(G), p^{-1}(B))$, is a near soft set in $NSS(O_A)$ defined as follows:

$$f^{-1}(G)(\phi) = \begin{cases} u^{-1}(G(p(\phi))), & p(\phi) \in B \\ \varnothing, & \textit{otherwise.} \end{cases}$$

Definition 3.3 *Let (O, A, τ) be a nsts and $F_A,\ G_A \in NSS(O_A)$ and let $f : O \to V$ be a mapping. Then for $\omega \in B$, the near soft union and intersection of near soft images of $F_A,\ G_A$ are defined as:*

1. $(f(F_A) \cup f(G_A))\,\omega = f(F_A)\omega \cup f(G_A)\omega$,
2. $(f(F_A) \cap f(G_A))\,\omega = f(F_A)\omega \cap f(G_A)\omega$.

Definition 3.4 *Let (V, B, τ^*) be a nsts and $F_B,\ G_B \in NSS(V_B)$ and let $f : O \to V$ be a mapping. Then for $\phi \in A$, near soft union and intersection of near inverse soft images of $F_B,\ G_B$ are defined as:*

1. $\left(f^{-1}\left(F_B\right) \cup f^{-1}(G_B)\right)\phi = f^{-1}(F_B)\phi \cup f^{-1}(G_B)\phi$,
2. $\left(f^{-1}\left(F_B\right) \cap f^{-1}(G_B)\right)\phi = f^{-1}(F_B)\phi \cap f^{-1}(G_B)\phi$.

Definition 3.5 *Let (O, A, τ) and (V, B, τ^*) be two nstss. Let u be a mapping from O to V and p be a mapping from A to B. Let f be a mapping from $NSS(O_A)$ to $NSS(V_B)$ and $\phi_F^N \in O_A$. Then:*

1. f is near soft continuous at $\phi_F^N \in O_A$ if for each $G_B \in N_{\tau^*}(f\left(\phi_F^N\right))$, there exists a $H_A \in N_\tau(\phi_F^N)$ such that $f(H_A) \subseteq G_B$.

2. f is near soft continuous on O_A if f is near soft continuous at each near soft point in O_A.

Theorem 3.6 *Let (O, A, τ) and (V, B, τ^*) be two nstss and $f : O \to V$. f is near soft continuous on O_A if and only if the inverse image of a near soft open set in V_B is a near soft open set in O_A.*

Proof Let $f : O \to V$ be near soft continuous on O_A. Let $G_B \in \tau^*$. To prove that $f^{-1}(G_B) \in \tau$, let $\phi_F^N \in f^{-1}(G_B)$. Then, by assumption, f is near soft continuous at ϕ_F^N. Thus, there exists a near soft open set G_A in O_A such that $f(G_A) \subseteq G_B$, but then $\phi_F^N \in G_A \subseteq f^{-1}(G_B)$ will imply ϕ_F^N is a near soft interior point of $f^{-1}(G_B)$. As any $\phi_F^N \in f^{-1}(G_B)$ is its near soft interior point it is a near soft open set in O_A.

Conversely, to prove that f is near soft continuous on O_A, fix up any $\phi_F^N \in O_A$. Select any near soft open set G_B in V_B containing $f(\phi_F^N)$. By assumption, $f^{-1}(G_B)$ is a near soft open set in O_A. $f(\phi_F^N) \in G_B \Rightarrow \phi_F^N \in f^{-1}(G_B)$. Define $G_A = f^{-1}(G_B)$. Then we get $\phi_F^N \in G_A$ and $f(G_A) = f[f^{-1}(G_B)] \subseteq G_B$. Therefore, f is near soft continuous at ϕ_F^N. As f is near soft continuous at each $\phi_F^N \in O_A$, we get that f is near soft continuous on O_A. □

Theorem 3.7 *Let (O, A, τ) and (V, B, τ^*) be two nstss and $f : O \to V$. f is near soft continuous on O_A if and only if the inverse image of a near soft closed set in V_B is a near soft closed set in O_A.*

Proof Let $f : O \to V$ be near soft continuous and let H_B be near soft closed set in V_B. Then $V_B - H_B$ is a near soft open set in V_B. Hence, by Theorem 3.6, $f^{-1}(V_B - H_B)$ is near soft open in O_A, i.e. $O_A - f^{-1}(H_B)$ is near soft open in O_A. Thus, $f^{-1}(H_B)$ is near soft closed in O_A.

Conversely, to prove that f is near soft continuous on O_A, let M_B be any near soft open set in V_B. Then $V_B - M_B$ is a near closed set in V_B. Thus, $f^{-1}(V_B - M_B)$ is a near soft closed set in O_A, by assumption. Hence, $O_A - f^{-1}(M_B)$ is a near soft closed set in O_A. Thus, $f^{-1}(M_B)$ is a near soft open set in O_A. Thus, the inverse image of a near soft open set in V_B is a near soft open set in O_A. Hence, by Theorem 3.6, f is near soft continuous on O_A. □

Example 3.8 *Let $O = \{a_1, a_2, a_3\}$, $V = \{b_1, b_2, b_3\}$, $A = \{\phi_1, \phi_2, \phi_3\}$,*

$K = \{\alpha_1, \alpha_2, \alpha_3\}$, and (O, B, τ), (V, K, τ^*), nsts, where $B,\ K \subseteq F$. Let us take the following information table.

Table 2.

	b_1	b_2	b_3	
α_1	1	1	1	
α_2	0,1	1	0,1	
α_3	1	2	1	

Let $\mu = F_K,\ K = \{\alpha_1, \alpha_2, \alpha_3\}$ *be a soft set defined by*
$F_K = \{(\alpha_1, \{b_1, b_2\}), (\alpha_2, \{b_2, b_3\}), (\alpha_3, \{b_1, b_3\})\}$.
Then for $r = 1$,
$\overline{b_1}_{\alpha_1} = \{b_1, b_2, b_3\}$,
$\overline{b_1}_{\alpha_2} = \{b_1, b_3\},\ \overline{b_2}_{\alpha_2} = \{b_2\}$,
$\overline{b_1}_{\alpha_3} = \{b_1, b_3\},\ \overline{b_2}_{\alpha_3} = \{b_2\}$.
$N_*(\mu) \neq \emptyset$, *and thus* $BN_N(\sigma) \geq 0$, *so* μ *is a near soft set.*
Let u *be a mapping from* O *to* V *and* p *be a mapping from* B *to* K.
$u(a_1) = \{b_1\},\ u(a_2) = \{b_3\},\ u(a_3) = \{b_2\}$,
$p(\phi_1) = \{\alpha_2\},\ p(\phi_2) = \{\alpha_1\},\ p(\phi_3) = \{\alpha_3\}$.
Take the near soft topology τ *on* O_B *that is given in Example 2.11, i.e.*
$\tau = \left\{\emptyset_B, F_B, F_B^1, F_B^2, F_B^3, F_B^4\right\}$, *and let* $\tau^* = \{\emptyset_K, F_K\}$.
$f : (O, B, \tau) \to (V, K, \tau^*)$ *is a near soft mapping. Then* F_K *is a near soft open set in* V_K *and*
$f^{-1}(F_K) = \{(\phi_1, \{a_2, a_3\}), (\phi_2, \{a_1, a_3\}), (\phi_3, \{a_1, a_2\})\}$ *is a near soft open set in* O_B.
Thus, f *is a near soft continuous function.*

Theorem 3.9 *Let* (O, A, τ) *and* (V, B, τ^*) *be two nstss and* $\phi_F^N \in O_A$. *For a function* $f : NSS(O_A) \to NSS(V_B)$, *the following are equivalent:*

1. f *is near soft continuous at* ϕ_F^N;
2. $\forall G_B \in N_{\tau^*}(f\left(\phi_F^N\right))$, *and there exists a* $H_A \in N_\tau(\phi_F^N)$ *such that* $H_A \subseteq f^{-1}(G_B)$;
3. *For each* $G_B \in N_{\tau^*}(f\left(\phi_F^N\right))$, $f^{-1}G_B \in N_\tau(\phi_F^N)$.

Proof It is open from Definition 3.5. □

Theorem 3.10 *Let* (O, A, τ) *and* (V, B, τ^*) *be two nstss. For a function* $f : NSS(O_A) \to NSS(V_B)$, *the following are equivalent:*

1. f *is near soft continuous;*
2. $\forall F_A \in NSS(O_A)$, *and the inverse image of every near soft nbd of* $f(F_A)$ *is a near soft nbd of* F_A;
3. $\forall F_A \in NSS(O_A)$ *and for each near soft nbd* H_B *of* $f(F_A)$, *there is a near soft nbd* G_A *of* F_A *such that* $f(G_A) \subseteq H_B$.

Proof 1.⇒2. We have that f is near soft continuous. If H_B is a near soft nbd of $f(F_A)$, then H_B contains a near soft open nbd G_B of $f(F_A)$. Since $f(F_A) \subseteq G_B \subseteq H_B$, $f^{-1}(f(F_A)) \subseteq f^{-1}(G_B) \subseteq f^{-1}(H_B)$. However, $F_A \subseteq f^{-1}(f(F_A))$ and $f^{-1}(G_B)$ is a near soft open set. Consequently, $f^{-1}(H_B)$ is a near soft nbd of F_A.

2.⇒1. We will use Theorem 3.6 and 3.7. Let $G_B \in \tau^*$. Then $f^{-1}(G_B)$ is a near soft subset of O_A. Let F_A be any near soft subset of $f^{-1}(G_B)$. Then G_B is a near soft open nbd of $f(F_A)$, and by 2, $f^{-1}(G_B)$ is a near soft near soft nbd of F_A. By Theorem 2.30, $f^{-1}(G_B)$ is a near soft open set.

2.⇒3. Let $F_A \in \tau$ and let H_B be any near soft nbd of $f(F_A)$. By 2, $f^{-1}(H_B)$ is a near soft nbd of F_A. Then there is a $G_A \in \tau$ such that $F_A \subseteq G_A \subseteq f^{-1}(H_B)$. Hence, we have a near soft open nbd G_A of F_A such that $f(F_A) \subseteq f(G_A) \subseteq H_B$.

3.⇒2. Let H_B be a near soft nbd of $f(F_A)$. Then there is a near soft nbd G_A of F_A such that $f(G_A) \subseteq H_B$. Hence, $f^{-1}(f(G_A)) \subseteq f^{-1}(H_B)$. Furthermore, since $G_A \subseteq f^{-1}(f(G_A))$, $f^{-1}(H_B)$ is a near soft nbd of F_A. □

Theorem 3.11 *Let (O, A, τ) and (V, B, τ^*) be two nstss and $f : O \to V$. f is near soft continuous if and only if $f[cl_n(G_A)] \subseteq cl_n^*[f(G_A)]$ for any $G_A \subseteq O_A$.*

($cl_n(G_A)$ =near soft closure of G_A in (O, A, τ) and $cl_n^*[f(G_A)]$ =near soft closure of $f(G_A)$ in (V, B, τ^*).)

Proof Let $f : O \to V$ be near soft continuous and let $G_A \subseteq O_A$. We know that $G_A \subseteq f^{-1}[f(G_A)]$. Hence, $G_A \subseteq f^{-1}[f(G_A)] \subseteq f^{-1}[cl_n^*(f(G_A))]$ (since $f(G_A) \subseteq cl_n^*(f(G_A))$ always). As $cl_n^*(f(G_A))$ is a near soft closed set in V_B and f is near soft continuous function on O_A, $f^{-1}[cl_n^*(f(G_A))]$ is a near soft closed set in O_A (see Theorem 3.7). Hence, $cl_n(G_A) \subseteq f^{-1}[cl_n^*(f(G_A))]$, i.e. $f[cl_n(G_A)] \subseteq cl_n^*[f(G_A)]$.

Conversely, to prove that is near soft continuous on O_A, let H_B be any near soft closed set in V_B. Define $G_A = f^{-1}(H_B)$. Then by assumption $f[cl_n(f^{-1}(H_B))] \subseteq cl_n^*[(f(f^{-1}(H_B))]$, but $f(f^{-1}(H_B)) \subseteq H_B$ always. Thus, $cl_n^*[f[f^{-1}(H_B)]] \subseteq cl_n^*(H_B) = H_B$. Therefore, we get $f[cl_n(f^{-1}(H_B))] \subseteq H_B$. Hence, $cl_n(f^{-1}(H_B)) \subseteq f^{-1}(H_B)$. As $f^{-1}(H_B) \subseteq cl_n(f^{-1}(H_B))$ always, we get $cl_n(f^{-1}(H_B)) = f^{-1}(H_B)$. Thus, $f^{-1}(H_B)$ is a near soft closed set in O_A. Thus, by Theorem 3.7, is a near soft continuous function. □

Theorem 3.12 *Let (O, A, τ) and (V, B, τ^*) be two nstss and $f : O \to V$. f is near soft continuous if and only if $f^{-1}[int_n^*(G_B)] \subseteq int_n[f^{-1}(G_B)]$ for any $G_B \subseteq V_B$.*

($int_n^*(G_B)$ =near soft interior of G_B in (V, B, τ^*) and $int_n[f^{-1}(G_B)]$ =near soft interior of $f^{-1}(G_B)$ in (O, A, τ).)

Proof Let $f : O \to V$ be near soft continuous and let $G_B \subseteq V_B$. Then $int_n^*(G_B)$ is a near soft set in V_B. Hence, by Theorem 3.6 $f^{-1}[int_n^*(G_B)]$ is near open set in O_A. As $int_n^*(G_B) \subseteq G_B$, $f^{-1}[int_n^*(G_B)] \subseteq f^{-1}(G_B)$. Thus, $f^{-1}[int_n^*(G_B)] \subseteq int_n[f^{-1}(G_B)]$.

Conversely, to prove that $f : O \to V$ is near soft continuous on O_A, let G_B be any near soft open set in V_B. Then $int_n^*(G_B) = G_B$. By assumption, $f^{-1}[int_n^*(G_B)] \subseteq int_n[f^{-1}(G_B)]$, i.e. $f^{-1}(G_B) \subseteq int_n[f^{-1}(G_B)]$, but always $int_n[f^{-1}(G_B)] \subseteq f^{-1}(G_B)$. Thus, $int_n[f^{-1}(G_B)] = f^{-1}(G_B)$. This implies that $f^{-1}(G_B)$ is a near soft open set in O_A. Therefore, by Theorem 3.6, it is a near soft continuous function on O_A. □

4. Near soft open and near soft closed functions

Definition 4.1 *Let (O, A, τ) and (V, B, τ^*) be two nstss and $f : O \to V$. f be a near soft mapping on O_A. If the image of each near soft open (resp. near soft closed) set in O_A is a near soft open (resp. near soft closed)* *set in V_B, then f is called a near soft open (resp. near soft closed) function.*

Theorem 4.2 *A near soft mapping $f : O \to V$ is near soft closed if f for each $\phi_F^N \in O_A$ and each near soft open set G_A in O_A there exists a near soft set G_B in V_B such that $G_B \subseteq f(G_A)$.*

Proof Follows directly from Definition 4.1. □

Theorem 4.3 *Let $f : O \to V$ be near soft continuous and $F_A \in \tau$. Then f is a near soft open function.*

Proof We have $F_A = int_n(F_A)$, for $F_A \in \tau$. Therefore, $f(F_A) = f(int_n(F_A)) = int_n(f(F_A)) \Rightarrow f(F_A)$ is a near soft open set. Thus, f is a near soft open function. □

Theorem 4.4 *A near soft mapping $f : O \to V$ is near soft closed if and only if $cl_n^*(f(G_A)) \subset f(cl_n(G_A))$ for each near soft set G_A in O_A.*

Proof Let $cl_n^*(f(G_A))\widetilde{\subset} f(cl_n(G_A))$. By Definition 2.15, we have $f(G_A) = f(cl_n(G_A))$ and so $f(cl_n(G_A))$ is a near soft closed set and f is a near soft closed mapping.

Conversely, if f is near soft closed then $f(cl_n(G_A))$ is a near soft closed set containing $f(G_A)$ and thus $cl_n^*(f(G_A)) \subset f(cl(G_A))$. □

Corollary 4.5 *Let $f : O \to V$ be near soft continuous. f is near soft closed if and only if $f(cl_n(G_A)) = cl_n^*(f(G_A))$ for all $G_A \subset O_A$.*

Proof Since f is near soft continuous, $f(G_A) \subset f(cl_n(G_A)) \subset cl_n^*(f(G_A))$. Since f is near soft closed, $f(cl_n(G_A))$ is near soft closed and thus these two inclusions imply $cl_n^*(f(G_A)) = f(cl_n(G_A))$.

Conversely, since $f(cl_n(G_A)) \subset cl_n^*(f(G_A))$ for all $G_A \subset O_A$, f is near soft continuous. If G_A is near soft closed, then $G_A = cl_n(G_A)$ so that $f(G_A) = f(cl_n(G_A)) = cl_n^*(f(G_A))$ is also near soft closed. This means that f is near soft closed. □

Theorem 4.6 *A near soft function $f : O \to V$ is near soft open if and only if $f(int_n^*(G_A)) \subset int_n(f(G_A))$ for every near soft set G_A in O_A.*

Proof If f is near soft open, then $f(int_n^*(G_A)) = int_n f(int_n^*(G_A)) \subset int_n(f(G_A))$.

Conversely, take a near soft open set G_A in O_A. Then, by hypothesis, $f(G_A) = f(int_n^*(G_A)) \subset int_n(f(G_A)) \Rightarrow f(G_A)$ is near soft open in V_B. □

Corollary 4.7 *Let $f : O \to V$ be a near soft continuous. f is near soft open if and only if $f^{-1}(int_n^*(G_B)) = int_n(f^{-1}(G_B))$ for all $G_B \subset V_B$.*

Proof Since f is near soft continuous, then $f^{-1}(int_n^*(G_B)) \subset int_n(f^{-1}(G_B))$. The other inclusion, $f^{-1}(int_n^*(G_B)) \supset int_n(f^{-1}(G_B))$, equivalent to $f(int_n(f^{-1}(G_B)) \subset int_n(G_B)$, follows because $f(int_n(f^{-1}(G_B))$ is near soft open and contained in $f((f^{-1}(G_B)) \subset G_B$.

Conversely, the inclusion $f^{-1}(int_n^*(G_B)) \subset int_n(f^{-1}(G_B))$, valid for all $G_B \subset V_B$, tells us that f is near soft continuous. Since $int_n(G_A) \subset f^{-1}(int_n^* f(G_A))$, we have $f(int_n^*(G_A)) \subset int_n(f(G_A))$ for all $G_A \subset O_A$. In particular, $f(G_A) = f(int_n^*(G_A)) \subset f(G_A)$ i.e. $f(G_A) = int_n(f(G_A))$ when G_A is near soft open. Therefore, f is near soft open. □

Theorem 4.8 *Let $f : O \to V$ be a near soft open (resp. near soft closed) mapping. If G_B is a near soft set in V_B and G_A is a near soft closed (resp. near soft open) set in O_A, containing $f^{-1}(G_B)$, then there exists a near soft closed (resp. near soft open) set H_B in V_B, such that $G_B \subset H_B$ and $f^{-1}(H_B) \subset G_B$.*

Proof Let $H_B = (f(G_A)^c)^c$. Since $f^{-1}(G_B) \subset G_A$, we have $f(G_A^c) \subset G_B^c$. Since f is near soft open (resp. near soft closed), then H_B is a near soft closed set (resp. near soft open set) if $f^{-1}(H_B) = (f^{-1}(f(G_A^c))^c \widetilde{\subset} (G_A^c)^c = G_A$ and hence $G_B \subset H_B$ and $f^{-1}(H_B) \subset G_A$. □

5. Conclusions

This paper introduces the concepts of near soft union, near soft intersection, near soft point, near soft interior, near soft closure, near soft neighborhood, near soft continuity, and near soft open (closed) function of a near soft set in near soft topological spaces and some of their features. The aim of these findings is to provide a theoretical foundation for further implementations of topology of near sets and also to contribute to the improvement of information system and various fields in engineering.

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