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Erratum to "Study on quasi-Γ-hyperideals in Γ-semihypergroups"

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Abstract: We wrote this note to show that the definition of Γ-hypersemigroups in [2] should be corrected, and that it is not enough to replace the hyperoperation $\circ$ of the hypersemigroup by Γ to pass from a hypersemigroup to a Γ-hypersemigroup. Care should be taken about the definitions of (m,n)-quasi-Γ-hyperideal, the m-left Γ-hyperideal and the n-right Γ-hyperideal as well.

Key words: Γ-semihypergroup, (m,n)-quasi-Γ-hyperideal

According to Definition 1.1 of the paper, "if H and Γ are two nonempty sets, any mapping $H\times\Gamma\times H \to \mathcal{P}^*(H)$ is called a Γ-hypermultiplication in H and denoted by $(\cdot)_\Gamma$. The result of this hypermultiplication for $a,b\in H$ and $\alpha\in\Gamma$ is denoted by $a\alpha b$. A Γ-semihypergroup S is an ordered pair $(H,(\cdot)_\Gamma)$ where H and Γ are nonempty sets and $(\cdot)_\Gamma$ is a Γ-hypermultiplication on H which satisfies the following property: For all $(a,b,c,\alpha,\beta)\in H^3\times\Gamma^2$, $(a\alpha b)\beta c = a\alpha(b\beta c)$. If every $\gamma\in\Gamma$ is an operation, then H is a Γ-semigroup. Let A and B are two nonempty subsets of H. Then, we define

$$A\Gamma B = \bigcup_{\gamma\in\Gamma} A\gamma B = \bigcup\{a\gamma b \mid a\in A,\ b\in B \text{ and } \gamma\in\Gamma\}.\text{"} \tag{$*$}$$

So $a\alpha b$ is defined as a nonempty subset of S (as it was expected to be). That being so, the expression of the form $(a\alpha b)\beta c$ should be corrected, there is no sense in the way is stated. According to this definition, "if every $\gamma\in\Gamma$ is an operation", but we have to clarify here what "γ" is an operation means. Is it an operation between elements or an operation between sets? The $A\gamma B$ in $(*)$ shows that γ is an operation between sets while $a\gamma b$ in $(*)$ shows that it is an operation between elements. As a result, the definition of Γ-semihypergroup is wrong and a correct definition of Γ-hypersemigroups is needed.

A Γ-hypersemigroup H is called "regular" [2] if for every $x\in H$ there exists $y\in H$ such that $x\in x\Gamma y\Gamma x$ without any explanation what the $x\Gamma y\Gamma x$ means. What is the $x\Gamma y\Gamma x$? What is the "Γ" in it? It seems like an operation between elements, but even in that case, what is the $x\Gamma y\Gamma x$? According to [2], the most possibly case concerning the concept of regularity is the following: A Γ-hypersemigroup H is called regular if for every $x\in S$, there exist $y\in H$ and $\alpha,\beta\in\Gamma$ such that $x\in x\alpha y\beta x$; but as we already said expression of the form $x\alpha y\beta x$ should be corrected.

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For the sake of simplicity, throughout the paper, $\underbrace{H\Gamma H\Gamma ... \Gamma H}_{n\text{-times}}$ is denoted by H^n. But what the $H\Gamma H\Gamma ... \Gamma H$ means? First of all, do we have the right to write $H\Gamma H\Gamma \ldots \Gamma H$ without using parentheses? There is nothing about it in the bibliography.

According to Definition 2.2 in [2], a nonempty subset Q of a Γ-semihypergroup H is called (m,n)-quasi-Γ-hyperideal of H is $H^m\Gamma Q \cap Q\Gamma H^n \subseteq Q$. This is true for $m = n = 1$, but what about arbitrary m, n? Considering that this definition is a generalization of the concept of the (m,n)-quasi-ideals of semigroups introduced by Lajos (see, for example [4]), an (m,n)-quasi-Γ-hyperideal should be defined as $Q^m\Gamma H \cap H\Gamma Q^n \subseteq Q$. The definition of the m-left Γ-hyperideal L is defined as $H^m\Gamma L \subseteq L$; the correct is $L^m\Gamma H \subseteq L$. The n-right Γ-hyperideal of H is defined as $R\Gamma H^n \subseteq R$; the correct is $R^n\Gamma H \subseteq R$. But we have to keep in mind that, for an arbitrary nonempty subset A of S, the A^n is the set $\underbrace{\Big(\Big((A\Gamma A)\Gamma A\Big)\Gamma A\Big) \cdots \Gamma A}_{n\text{-times}}$. We are not in a semigroup or a Γ-semigroup where this is simple. It might be mentioned here that an element q of a *poe*-groupoid S is called an (m,n)-quasi-ideal element of S if $q^m e \wedge eq^n$ exists in S and $q^m e \wedge eq^n \leq q$ [3] and it is called $(0,n)$ (resp. $(m,0)$)-ideal element of S if $ea^n \leq a$ (resp. $a^m e \leq a$) [3]. Every $(m,0)$-ideal element is a $(m,0)$-quasi-ideal and every $(0,n)$-ideal element is a $(0,n)$-quasi-ideal element.

In what follows, the aim is to show that is not enough to pass from a semigroup to a hypersemigroup by replacing the multiplication "$\cdot$" of the semigroup by the hyperoperation "$\circ$" and to pass from a hypersemigroup to a Γ-hypersemigroup replacing the "$\circ$" by "Γ".

The paper in [2] is the paper in [1] with the only difference that the hypeoperation $\circ$ in [1] has been replaced by Γ in [2].

In fact,

Lemma 2.1 in [2] is the Lemma 2.10 in [1];

Proposition 2.1 in [2] is the Proposition 2.11 in [1];

Theorem 2.1 in [2] is the Proposition 2.10 in [1];

Theorem 2.2 in [2] is the Theorem 2.14 in [1];

Theorem 2.3 in [2] is the Theorem 2.16 in [1];

Theorem 2.4 in [2] is the Theorem 2.17 in [1];

Theorem 2.5 in [2] is the Theorem 2.18 in [1];

Theorem 2.6 in [2] is the Theorem 2.19 in [1];

Proposition 2.3 in [2] is the Proposition 2.20 in [1];

Theorem 2.7 in [2] is the Theorem 2.21 in [1];

Proposition 2.3 in [2] is the Proposition 2.22 in [1];

Lemma 3.1 in [2] is the Lemma 3.4 in [1];

Theorem 3.1 in [2] is the Theorem 3.5 in [1];

Corollary 3.1 in [2] is the Corollary 3.6 in [1];

Theorem 3.2 in [2] is the Theorem 3.7 in [1];

Theorem 3.3 in [2] is the Theorem 3.11 in [1];

Theorem 3.4 in [2] is the Theorem 3.12 in [1].

The definition of quasi-hyperideals and the definitions of m-left and n-right hyperideals of semihypergroups in [1] should be also corrected; in addition, except of Theorem 2.14, Theorem 2.17 (and the Examples), the results of section 2 in [1] duplicates, without citation, the paper "A note on (m,n)-quasi-ideals in semigroup" by Moin A. Ansari, M. Rais Khan, J. P. Kaushik in International Journal of Mathematical Analysis 3 (2009), 1853-1858 (with the usual change), that is a further indication that is not enough to pass from a semigroup to a hypersemigroup just replacing the $\cdot$ by $\circ$; but this is out of the scope of the present note.

The definition of Γ-hypersemigroups, the regularity, and related information has been given by the author of the present note in the paper "Lattice ordered semigroups and Γ-hypersemigroups" submitted to Turkish Journal of Mathematics.

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