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Soft β-rough sets and their application to determine COVID-19

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Abstract: Soft rough set theory has been presented as a basic mathematical model for decision-making for many real-life data. However, soft rough sets are based on a possible fusion of rough sets and soft sets which were proposed by Feng et al. [20]. The main contribution of the present article is to introduce a modification and a generalization for Feng's approximations, namely, soft β-rough approximations, and some of their properties will be studied. A comparison between the suggested approximations and the previous one [20] will be discussed. Some examples are prepared to display the validness of these proposals. Finally, we put an actual example of the infections of coronavirus (COVID-19) based on soft β-rough sets. This application aims to know the persons most likely to be infected with COVID-19 via soft β-rough approximations and soft β-rough topologies.

Key words: COVID-19, information system, soft sets, soft topological spaces, decision-making

1. Introduction

Soft set theory, initiated by Molodtsov [34], is a new approach to dealing with uncertainty. Prior to the introduction of soft set theory, probability theory, fuzzy set theory, and rough set theory were common tools for dealing with uncertainty. Although these theories have been applied successfully to many problems, there are still some difficulties associated with these theories. For example, in probability theory, a large number of experiments are needed to check the stability of the system. Such experimentation is not affordable in economics and environmental sciences. Perhaps, the difficulties associated with these theories described above are due to the lack of resources for parameterization [34]. This theory has sufficient parameters so that it is free from the above-mentioned difficulties. Soft set theory deals, on one hand, with uncertainty and vagueness, while, on the other hand, it has enough tools for parameterization. These qualities of soft set theory make it popular among researchers and experts working in a variety of fields. For theoretical aspects and applications of soft set theory, we refer the reader to the references [3–12, 14, 17–23, 30–34, 36, 42, 43, 46–48] and their literatures.

Soft set theory and rough set theory have two different approaches to vagueness. Feng et al. [20] proposed a possible fusion of both rough sets and soft sets. They introduced the concept of soft rough sets where, instead of equivalence classes, the parameterized subsets of a set serve the purpose of finding lower and upper approximations of a subset. However, there are some unusual situations obtained in their approach. For example, the upper approximation of a nonempty set may be empty. Moreover, the upper approximation of a subset of the universe may not contain the set.

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These situations do not arise in the classical rough set theory. Accordingly, the first objective of the present paper is to define a soft rough set model where such situations may not occur. A new type of rough sets, namely, soft β-rough sets is proposed. We demonstrate that information granules are finer than soft rough sets in this new model, and thus, these new techniques will strengthen Feng models. Consequently, any soft exact set in Feng must be soft β-exact. The converse is not true in general so, we give some counterexamples to indicate these connections. The importance of the suggested approximations is not only that it is reducing the boundary regions, but also it is satisfying most of Pawlak's rough set properties without any restrictions. Hence, this technique can be useful in discovering the vagueness in the data, and thus, it can help us in decision-making. Some results that were not valid in soft rough sets can be proved by soft β-rough approximations. Therefore, we prove that our approach represents a modification and generalization to Feng et al. [20].

A novel coronavirus SARS-CoV-2 or COVID-19 has recently emerged from China with a total of 45171 cases of pneumonia. Accordingly, many researchers have published many papers to study this dangerous virus (for example, see the references [13, 24, 27, 28, 40, 41]) and their literatures. Together with severe acute respiratory syndrome (SARS coronavirus) and Middle East respiratory syndrome (MERS coronavirus) [28], this is the third highly pathogenic human coronavirus that has emerged over the last two decades. The person-to-person transmission has been described in both hospital and family settings [29]. Therefore, the utmost importance to prevent further spread in the public and healthcare settings. Coronavirus transmission from contaminated dry surfaces, including auto-inoculation of mucous membranes in the nose, eyes or mouth, has been postulated, stressing the importance of a detailed understanding of coronavirus persistence on inanimate surfaces [29]. There are therefore two factors affecting the transmission of infections, namely contact with infected surfaces and interactions with infected viruses. So, for these reasons, the second goal of the present paper is to introduce new tools to make an accurate decision in order to identify the people most likely to be infected with COVID-19. In fact, we used some data about six persons who may be infected by COVID-19 from a medical experiment in Hospitals. Using soft β-rough approximations and soft rough topologies, which are generated by soft β-rough approximations, we make a topological reduction for the information system and then we identify the impact factors for transmission of COVID-19 infection into human. Moreover, we can say that the suggested methods can be used to analyze data of COVID-19 with quantitative or qualitative data [35, 36]. By coding qualitative data of the information system in the application, we express "yes" and "no" with "1" and "0" values, respectively, and obtain soft β-rough sets. Finally, we present two algorithms in which soft β-rough approximations and their soft β-rough topologies are used for decision making in COVID-19 infections.

2. Preliminaries

In the following, we present the basic concepts that are used in this study.

2.1. Pawlak rough set theory

In 1982, Pawlak [37] introduced the theory of rough set as a new mathematical methodology to deal with the vagueness in knowledge-based systems, data analysis and information systems. This model has many applications in many fields such as economics, medical diagnosis and some other fields. In order to handle the vagueness and imprecision in the data, equivalence relations play an important role in this theory. This theory has been applied successfully to solve many problems, but in daily life, it is very difficult to find an equivalence relation amongst the elements of a set under consideration. Therefore, some other rough set models have been introduced, for instance the reader can see [1–3, 5, 11, 16, 25, 26, 35, 38, 39, 44, 45, 49–51].

Definition 2.1 *[37] Let U be a universe finite set and R be an equivalence relation on U. $U/R = \{[x]_R : x \in U\}$ will denote to the family of equivalence classes of R on U. Then, the pair (U, R) is said to be an approximation space. The lower and upper approximation of $X \subseteq U$ are defined respectively by*

$$\underline{R}(X) = \{x \in U : [x]_R \subseteq X\}, \text{ and}$$

$$\overline{R}(X) = \{x \in U : [x]_R \cap X \neq \emptyset\}.$$

According to Pawlak's definition, X is called a rough set if $\underline{R}(X) \neq \overline{R}(X)$.

Definition 2.2 *[37] Let (U, R) be an approximation space. Then, the boundary, positive, negative regions and accuracy for the approximations of $X \subseteq U$ are defined respectively by*

$$BND_R(X) = \overline{R}(X) - \underline{R}(X),$$

$$POS_R(X) = \underline{R}(X),$$

$$NEG_R(X) = U - \overline{R}(X), \text{ and}$$

$$\mu_R(X) = \frac{|\underline{R}(X)|}{|\overline{R}(X)|}, \text{ where } |\overline{R}(X)| \neq 0.$$

Obviously, $\mu_R(X) \leq 1$. If $\mu_R(X) = 1$, then X is called exact. Otherwise, it is called rough.

Remark 2.3 *(i) If $BND_R(X) = \emptyset$, that means $\underline{R}(X) = \overline{R}(X)$, then X is crisp or exact with respect to R. On the other side, if $BND_R(X) \neq \emptyset$, then X is said to be rough.*

(ii) The pair $(\underline{R}(X), \overline{R}(X))$ can be also referred to rough set of X.

(iii) If $X \subseteq U$ is defined by a predicate P, then another interpretations are given

$x \in POS_R(X)$ means x certainly has property P.

$x \in BND_R(X)$ means x possibly has property P.

$x \in NEG_R(X)$ means x definitely does not has property P, for $x \in U$.

Proposition 2.4 *[37] Let (U, R) be an approximation space. Then, the following properties are held*

(L1) $\underline{R}(X) \subseteq X$.
(L2) $\underline{R}(\emptyset) = \emptyset$.
(L3) $\underline{R}(U) = U$.
(L4) $\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$.
(L5) If $X \subseteq Y$, then $\underline{R}(X) \subseteq \underline{R}(Y)$.
(L6) $\underline{R}(X) \cup \underline{R}(Y) \subseteq \underline{R}(X \cup Y)$.
(L7) $\underline{R}(X^c) = (\overline{R}(X))^c$.
(L8) $\underline{R}(\underline{R}(X)) = \underline{R}(X)$.
(L9)If $X \in U/R$, then $\underline{R}(X) = X$.

(U1) $X \subseteq \overline{R}(X)$.
(U2) $\overline{R}(\emptyset) = \emptyset$.
(U3) $\overline{R}(U) = U$.
(U4) $\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$.
(U5) If $X \subseteq Y$, then $\overline{R}(X) \subseteq \overline{R}(Y)$.
(U6) $\overline{R}(X) \cap \overline{R}(Y) \supseteq \overline{R}(X \cap Y)$.
(U7) $\overline{R}(X^c) = (\underline{R}(X))^c$.
(U8) $\overline{R}(\overline{R}(X)) = \overline{R}(X)$.
(U9) If $X \in U/R$, then $\overline{R}(X) = X$.

Where X^c denotes to the complement of X with respect to U.

2.2. Soft set theory and soft rough sets

For soft set theory, let U be a universe set of objects and E_U (E, for short) be the set of parameters in U. The parameters may be attributes, characteristics, or properties of U. $P(U)$ will be denoted to the power set of U. Main concepts and results were found in [3–12, 14, 17–23, 30–34, 36, 42, 43, 46–48].

Definition 2.5 *[34] A pair (F, A) is said to be soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of U. For $a \in A$, $F(a)$ may be considered as the set of a-approximate elements of the soft set (F, A).*

Definition 2.6 *[20] Let $S = (F, A)$ be a soft set over U. Then, a pair $A_s = (U, S)$ is called a soft approximation space. The soft A_s-lower approximation and soft A_s-upper approximation for $X \subseteq U$ are defined respectively by*

$$\underline{S}(X) = \{u \in U : \exists e \in A, [u \in F(e) \subseteq X]\}, \text{ and}$$

$$\overline{S}(X) = \{u \in U : \exists e \in A, [u \in F(e), F(e) \cap X \neq \emptyset]\}.$$

In general, $\underline{S}(X)$ and $\overline{S}(X)$ are referred to soft rough approximations with respect to A_s. Moreover,

$$POS_{A_s}(X) = \underline{S}(X),$$

$$NEG_{A_s}(X) = U - \overline{S}(X) = (\overline{S}(X))^c, \text{ and}$$

$$BND_{A_s}(X) = \overline{S}(X) - \underline{S}(X).$$

are called soft A_s-positive region, soft A_s-negative region and soft A_s-boundary region of X. If $\underline{S}(X) = \overline{S}(X)$ means $BND_{A_s}(X) = \emptyset$, then X is said to be soft A_s-definable or soft A_s-exact set. Otherwise, X is called soft A_s-rough. Moreover, the accuracy of X is defined by

$$\mu_{A_s}(X) = \frac{|\underline{S}(X)|}{|\overline{S}(X)|}, \text{ where } |\overline{A_s}(X)| \neq 0. \text{ It is called soft } A_s\text{-accuracy of } X.$$

Proposition 2.7 *[20] Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ be a soft approximation space. Then, for each $X \subseteq U$*

$$\underline{S}(X) = \bigcup_{e \in A} \{F(e) : F(e) \subseteq X\}, \text{ and}$$

$$\overline{S}(X) = \bigcup_{e \in A} \{F(e) : F(e) \cap X \neq \emptyset\}.$$

Proposition 2.8 *[20] Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ be a soft approximation space. Then, for each $X, Y \subseteq U$, the followings are held*

(i) $\underline{S}(\emptyset) = \overline{S}(\emptyset) = \emptyset$.
(ii) $\underline{S}(U) = \overline{S}(U) = \bigcup_{e \in A} F(e)$.
(iii) $\underline{S}(X \cap Y) \subseteq \underline{S}(X) \cap \underline{S}(Y)$.
(iv) $\underline{S}(X \cup Y) \supseteq \underline{S}(X) \cup \underline{S}(Y)$.
(v) $\overline{S}(X \cup Y) = \overline{S}(X) \cup \overline{S}(Y)$.
(vi) $\overline{S}(X \cap Y) \subseteq \overline{S}(X) \cap \overline{S}(Y)$.
(vii) If $X \subseteq Y$, *then* $\underline{S}(X) \subseteq \underline{S}(Y)$.
(viii) If $X \subseteq Y$, *then* $\overline{S}(X) \subseteq \overline{S}(Y)$.

Proposition 2.9 *[20] Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ be a soft approximation space. Then, for each $X \subseteq U$, the followings are held*

(i) $\underline{S}(\underline{S}(X)) = \underline{S}(X)$.
(ii) $\overline{S}(\overline{S}(X)) \supseteq \overline{S}(X)$.
(iii) $\underline{S}(X) \subseteq \overline{S}(\underline{S}(X))$.
(iv) $\underline{S}(\overline{S}(X)) = \underline{S}(X)$.

Definition 2.10 *[20] Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ be a soft approximation space. Then, S is called a full soft set if $U = \bigcup_{e \in A} F(e)$. Clearly, if S is full soft, then $\forall x \in U, \exists\ e \in A$ s.t. $x \in F(e)$.*

Proposition 2.11 *[20] Let $S = (F, A)$ be a full soft set over U and $A_s = (U, S)$ be a soft approximation space. Then, the conditions are held:*

(i) $\underline{S}(U) = \overline{S}(U) = U$.

(ii) $X \subseteq \overline{S}(X)\ \forall\ X \subseteq U$.

(iii) $\overline{S}(\{x\}) \neq \emptyset\ \forall\ x \in U$.

According to Pawlak's [37] and Feng's [20] notions, we give Definition 2.12.

Definition 2.12 *Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ be a soft approximation space. Then, four types of soft rough sets are given by*

(i) X is roughly soft A_s-definable if $\underline{S}(X) \neq \emptyset$ and $\overline{S}(X) \neq U$.

(ii) X is internally soft A_s-indefinable if $\underline{S}(X) = \emptyset$ and $\overline{S}(X) \neq U$.

(iii) X is externally soft A_s-indefinable if $\underline{S}(X) \neq \emptyset$ and $\overline{S}(X) = U$.

(iv) X is totally soft A_s-indefinable if $\underline{S}(X) = \emptyset$ and $\overline{S}(X) = U$, for $X \subseteq U$.

Definition 2.13 *[11] Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ be a soft approximation. Then, a collection $\tau_{SR} = \{U, \emptyset, \underline{S}(X), \overline{S}(X), BND_{A_s}(X)\}$ forms a topology on U. It is called a soft rough topology with respect to X. Moreover, the base of τ_{SR} is $\mathcal{B}_{SR} = \{U, \underline{S}(X), BND_{A_s}(X)\}$, for $X \subseteq U$.*

3. Soft β-rough approximations in soft rough sets

In this section, we describe new generalized soft rough approximations, namely, soft β-rough approximations and some of their properties are examined. The relationship between the proposed approaches and Feng's approaches [19] is discussed. Moreover, we illustrate that the soft β-rough approach strengthen the concept of soft rough sets.

Definition 3.1 *Let $S = (F, A)$ be a soft set over U and $A_s = (U, S)$ be a soft approximation space. Then, soft β-lower approximation and soft β-upper approximation are defined by*

$$\underline{S}_\beta(X) = X \cap \overline{S}(\underline{S}(\overline{S}(X))), \text{ and}$$

$$\overline{S}_\beta(X) = X \cup \underline{S}(\overline{S}(\underline{S}(X))), \text{ respectively, for } X \subseteq U.$$

In general, $\underline{S}_\beta(X)$ and $\overline{S}_\beta(X)$ are referred to soft β-rough approximations of X with respect to A_s.

Definition 3.2 *Let $A_s = (U, S)$ be a soft approximation space. Then, soft β-positive region, β-negative region, β-boundary region and β-accuracy of A_s are defined by*

$$POS_\beta(X) = \underline{S_\beta}(X)\ ,$$

$$NEG_\beta(X) = U - \overline{S_\beta}(X),$$

$$BND_\beta(X) = \overline{S_\beta}(X) - \underline{S_\beta}(X), \text{ and}$$

$\mu_\beta(X) = \frac{|\underline{S_\beta}(X)|}{|\overline{S_\beta}(X)|}$, *where* $|\overline{S_\beta}(X)| \neq 0$, *respectively, for* $X \subseteq U$.

It is clear that, if $\underline{S_\beta}(X) = \overline{S_\beta}(X)$ *that means* $BND_\beta(X) = \emptyset$, *then* X *is said to be soft* β*-definable or soft* β*-exact set. Otherwise,* X *is called soft* β*-rough.*

Now, we introduce and superimpose some basic properties of soft β-rough approximations $\underline{S_\beta}$ and $\overline{S_\beta}$.

Proposition 3.3 *Let* $S = (F, A)$ *be a soft set over* U *and* $A_s = (U, S)$ *be a soft approximation space. Then, for each* $X, Y \subseteq U$, *the followings are held*

(i) $\underline{S_\beta}(\emptyset) = \overline{S_\beta}(\emptyset) = \emptyset$.

(ii) $\underline{S_\beta}(U) = \bigcup_{e\in A} F(e)$ *and* $\overline{S_\beta}(U) = U$.

(iii) If $X \subseteq Y$, *then* $\underline{S_\beta}(X) \subseteq \underline{S_\beta}(Y)$.

(iv) If $X \subseteq Y$, *then* $\overline{S_\beta}(X) \subseteq \overline{S_\beta}(Y)$.

(v) $\underline{S_\beta}(X \cap Y) \subseteq \underline{S_\beta}(X) \cap \underline{S_\beta}(Y)$.

(vi) $\underline{S_\beta}(X \cup Y) \supseteq \underline{S_\beta}(X) \cup \underline{S_\beta}(Y)$.

(vii) $\overline{S}(X \cap Y) \subseteq \overline{S_\beta}(X) \cap \overline{S_\beta}(Y)$.

(viii) $\overline{S_\beta}(X \cup Y) = \overline{S_\beta}(X) \cup \overline{S_\beta}(Y)$.

Proof (i) Since $\underline{S}(\emptyset) = \overline{S}(\emptyset) = \emptyset$, then $\underline{S_\beta}(\emptyset) = \emptyset$ and $\overline{S_\beta}(\emptyset) = \emptyset$.

(ii) Since $\underline{S}(U) = \overline{S}(U) = \bigcup_{e\in A} F(e)$, then $\underline{S_\beta}(U) = U \cap \overline{S}(\underline{S}(\overline{S}(U))) = U \cap \bigcup_{e\in A} F(e) = \bigcup_{e\in A} F(e)$ and $\overline{S}_\beta(U) = U \cup \underline{S}(\overline{S}(\underline{S}(U))) = U \cup \bigcup_{e\in A} F(e) = U$.

(iii) If $X \subseteq Y$, then $\underline{S}(X) \subseteq \underline{S}(Y)$ and $\overline{S}(X) \subseteq \overline{S}(Y)$. This implies to $\underline{S_\beta}(X) = X \cap \overline{S}(\underline{S}(\overline{S}(X))) \subseteq Y \cap \overline{S}(\underline{S}(\overline{S}(Y))) = \underline{S_\beta}(Y)$.

(iv) is similar to (iii).

(v) Since $X \cap Y \subseteq X$ and $X \cap Y \subseteq Y$, then, by (iii), $\underline{S_\beta}(X \cap Y) \subseteq \underline{S_\beta}(X) \cap \underline{S_\beta}(Y)$.

(vi) Since $X \subseteq X \cup Y$ and $Y \subseteq X \cup Y$, then, by (iv), $\underline{S_\beta}(X \cup Y) \supseteq \underline{S_\beta}(X) \cup \underline{S_\beta}(Y)$.

(vii) is similar to (v).

(viii) Obviously, directed by (v)-(vii). □

From Definition 3.1, it is easy to prove some properties of $\underline{S_\beta}$ and $\overline{S_\beta}$. So, we omit the proof.

Proposition 3.4 *Let* $S = (F, A)$ *be a soft set over* U *and* $A_s = (U, S)$ *be a soft approximation space. Then, for each* $X \subseteq U$, *the followings are held*

(i) $\underline{S_\beta}(\underline{S_\beta}(X)) = \underline{S_\beta}(X)$.

(ii) $\overline{S_\beta}(X) \subseteq \overline{S_\beta}(\overline{S_\beta}(X))$.

(iii) $\underline{S_\beta}(X) \subseteq \overline{S_\beta}(\underline{S_\beta}(X))$.

(iv) $\underline{S_\beta}(\overline{S_\beta}(X)) \subseteq \overline{S_\beta}(X)$.

The equality in Proposition 3.4 does not verify, in general, as shown in Examples 3.5 and 3.6.

Example 3.5 *Let* $U = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2, e_3, \cdots, e_6\}$ *and* $A = \{e_1, e_2, e_3\} \subseteq E$ *such that* $(F, A) = \{(e_1, \{x_1, x_4\}), (e_2, \{x_3\}), (e_3, \{x_2, x_3, x_4\})\}$. *If* $X = \{x_3, x_4\}$, *then* $\underline{S_\beta}(X) = X$ *and* $\overline{S_\beta}(X) = \{x_2, x_3, x_4\}$ *which implies* $\overline{S_\beta}(\overline{S_\beta}(X)) = U$. *Hence,* $\overline{S_\beta}(X) \neq \overline{S_\beta}(\overline{S_\beta}(X))$. *Also,* $\overline{S_\beta}(\underline{S_\beta}(X)) = \overline{S_\beta}(X) = \{x_2, x_3, x_4\} \neq \underline{S_\beta}(X)$.

Example 3.6 *Let* $U = \{x_1, x_2, x_3, \cdots, x_6\}$, *and* $A = \{e_1, e_2, e_3, e_4\}$ *such that* $(F, A) = \{(e_1, \{x_1, x_6\}), (e_2, \{x_3\}), (e_3, \emptyset), (e_4, \{x_1, x_2, x_5\})\}$. *If* $X = \{x_3, x_4, x_5\}$, *then* $\underline{S_\beta}(X) = \{x_3, x_5\}$ *and* $\overline{S_\beta}(X) = \{x_3, x_4, x_5\} = X$ *which implies* $\underline{S_\beta}(\overline{S_\beta}(X)) = \{x_3, x_5\} \neq \overline{S_\beta}(X)$. *Moreover, if* $Y = \{x_1, x_4\}$, *then* $\underline{S}(Y) = \emptyset$ *and* $\overline{S}(Y) = \{x_1, x_2, x_5, x_6\}$. *Obviously,* $Y \nsubseteq \overline{S}(Y)$ *but* $\underline{S_\beta}(Y) = \{x_1\}$ *and* $\overline{S_\beta}(Y) = Y$. *Therefore,* $\underline{S_\beta}(Y) \subseteq Y \subseteq \overline{S_\beta}(Y)$.

Proposition 3.7 *Let* $S = (F, A)$ *be a full soft set and* $A_s = (U, S)$ *be a soft approximation space. Then,*

(i) $\underline{S_\beta}(U) = U$. *(ii)* $\underline{S_\beta}(\overline{S_\beta}(X)) = \overline{S_\beta}(X), \forall\ X \subseteq U$.

Proof (i) Let $S = (F, A)$ be a full soft set. Then, by Proposition 2.11, $\underline{S_\beta}(U) = U \cap \overline{S}(\underline{S}(\overline{S}(U))) = U \cap \overline{S}(\underline{S}(U)) = U \cap \overline{S}(U) = U \cap U = U$.

(ii) Firstly, by Proposition 3.4, $\underline{S_\beta}(\overline{S_\beta}(X)) \subseteq \overline{S_\beta}(X)\ \forall\ X \subseteq U$. For the inverse inclusion, let $S = (F, A)$ be a full soft set. Then, by Proposition 2.11, $X \subseteq \overline{S}(X)$, $\forall\ X \subseteq U$ and by Proposition 2.9, $\underline{S}(\overline{S}(X)) = \overline{S}(X)\ \forall\ X \subseteq U$. Hence, $X \subseteq \underline{S}(\overline{S}(X))\ \forall\ X \subseteq U$ and since $\underline{S}(X) \subseteq X$, $\forall\ X \subseteq U$. Then, $\overline{S}(\underline{S}(X)) \subseteq \overline{S}(X)$ and implies to $\overline{S_\beta}(X) = X \cup \overline{S}(\underline{S}(X)) \subseteq \underline{S}\ (\overline{S}(X))$. Accordingly, $\overline{S_\beta}(X) \subseteq \underline{S}(\overline{S}(\overline{S_\beta}(X)))$ and $\overline{S_\beta}(X) \subseteq \underline{S_\beta}(\overline{S_\beta}(X))$. □

Remark 3.8 *Propositions 3.3, 3.4 and 3.7 represent differences between our approach and Feng's approach. The proposed approximations satisfied most of Pawlak's properties. Then, Table 1 summarizes some of these properties. Also, it gives the first comparison among our approach and a Feng's method. Some codes in Table 1, say values "1" and "0" for denoting "yes" and "no", respectively, to show whether these approximations satisfy properties* $L1 - L9$ *and* $U1 - U9$ *in Proposition 2.4.*

Table 1. Soft rough and soft β-rough approximations with codes.

	$\underline{S}$	$\underline{S_\beta}$		$\overline{S}$	$\overline{S_\beta}$
L1	1	1	**U1**	1	1
L2	1	1	**U2**	1	1
L3	0	0	**U3**	0	1
L4	0	0	**U4**	1	1
L5	1	1	**U5**	1	1
L6	1	1	**U6**	1	1
L7	1	1	**U7**	1	1
L8	1	1	**U8**	1	1
L9	1	1	**U9**	1	1

In the following results, the relationship between soft rough approximations and soft β-rough approximations is studied. In fact, it proves that our suggested method is the modification and generalization for Feng's approach [20].

Theorem 3.9 *Let* $A_s = (U, S)$ *be a soft approximation space. Then, for each,* $X \subseteq U$

(i) $\underline{S}(X) \subseteq \underline{S_\beta}(X)$. *(ii)* $\overline{S_\beta}(X) \subseteq \overline{S}(X)$.

Proof It is sufficient to prove (i) and (ii) follows by the same manner. If $x \notin \underline{S_\beta}(X)$, then either $x \notin X$ or $x \notin \underline{S}(\overline{S}(\underline{S}(X)))$. There are two cases

Case 1, if $x \notin X$, then $x \notin \underline{S}(X)$.

Case 2, if $x \notin \overline{S}(\underline{S}(\overline{S}(X)))$, then, by Proposition 2.7, $\underline{S}(\overline{S}(X)) = \overline{S}(X)$ and thus $x \notin \overline{S}(\overline{S}(X))$. Accordingly, $\exists\ e \in A$, such that $x \in F(e)$ and $F(e) \cap \overline{S}(X) \neq \emptyset$. Thus, $x \notin \overline{S}(X)$ and this implies $x \notin \underline{S}(X)$. Hence, $\underline{S}(X) \subseteq \underline{S_\beta}(X)$. □

Corollary 3.10 *Let $A_s = (U, S)$ be a soft approximation space. Then,*

(i) $BND_\beta(X) \subseteq BND_{A_s}(X)$. *(ii) $\mu_{A_s}(X) \leq \mu_\beta(X)$, $X \subseteq U$.*

Corollary 3.11 *Let $A_s = (U, S)$ be a soft approximation space. If X is a soft exact set, then it is soft β-exact.*

The converse of above results is not true, in general, as shown in Example 3.12.

Example 3.12 *Consider $X = \{x_3, x_4, x_5\}$ and $Y = \{x_3, x_6\}$ in Example 3.6. Then, $\underline{S}(X) = \{x_3\}$ and $\overline{S}(X) = \{x_1, x_2, x_3, x_5\}$. While, $\underline{S_\beta}(X) = \{x_3, x_5\}$ and $\overline{S_\beta}(X) = X$. It is clear that $\underline{S}(X) \subseteq \underline{S_\beta}(X)$. Moreover, $X \nsubseteq \overline{S}(X)$. But, $\underline{S_\beta}(X) \subseteq X \subseteq \overline{S_\beta}(X)$. Similarly, $\underline{S}(Y) = \{x_3\}$, $\overline{S}(Y) = \{x_1, x_3, x_6\}$ and then $BND(Y) = \{x_1, x_6\}$ and $\mu_{A_s}(Y) = \frac{1}{3}$. But $\underline{S_\beta}(Y) = \overline{S_\beta}(Y) = Y$ and then $BND_\beta(Y) = \emptyset$ and $\mu_\beta(Y) = 1$. Also, Y is soft β-exact, while, it is soft rough with respect to Feng's approach.*

According to Theorem 3.9, we give Definition 3.13.

Definition 3.13 *Let $S = (F, A)$ be a full soft set over U and $A_s = (U, S)$ be a soft approximation space. Then, four types of soft β-rough sets are given*

(i) X is roughly soft β-definable if $\underline{S_\beta}(X) \neq \emptyset$ and $\overline{S_\beta}(X) \neq U$.

(ii) X is internally soft β-undefinable if $\underline{S_\beta}(X) = \emptyset$ and $\overline{S_\beta}(X) \neq U$.

(iii) X is externally soft β-undefinable if $\underline{S_\beta}(X) \neq \emptyset$ and $\overline{S_\beta}(X) = U$.

(iv) X is totally soft β-undefinable if $\underline{S_\beta}(X) = \emptyset$ and $\overline{S_\beta}(X) = U$, for $X \subseteq U$.

The intuitive meanings of the classification in Definition 3.13 are

(i) If X is roughly soft β-definable, then there are some elements of U that belong to X. This means that X^c can be completely determined by the soft approximation space A_s.

(ii) If X is internally soft β-undefinable, then there are some elements of U that belong to X^c. In this case, X is completely determined by the soft approximation space A_s.

(iii) If X is externally soft β-undefinable, then there are some elements of U that belong to X. So, the elements of U that belong to X^c can be completely determined by the soft approximation space A_s.

(iv) If X is totally soft β-undefinable, then there are elements of U that belong to either X or X^c can be completely determined by the soft approximation space A_s.

From Theorem 3.9, it is easy to show that the second difference between rough soft β-sets and rough soft A_s-sets is given. So, the proof of Theorem 3.14 is omitted.

Theorem 3.14 *Let $A_s = (U, S)$ be a soft approximation space. Then,*
(i) If X is roughly soft β-definable, then X is roughly soft A_s-definable.
(ii) If X is internally soft β-undefinable, then X is internally soft A_s-definable.
(iii) If X is externally soft β-undefinable, then X is externally soft A_s-definable.
(iv) If X is totally soft β-undefinable, then X is totally soft A_s-undefinable, for $X \subseteq U$.

Remark 3.15 *(i) Theorem 3.14 illustrates that soft β-rough sets are stronger than soft rough sets. For instance, if X is totally soft A_s-undefinable i.e. $\underline{S}(X) = \emptyset$ and $\overline{S}(X) = U$, then any element of U belongs to either X or X^c. By applying soft β-rough approximations, it may to have $\underline{S_\beta}(X) \neq \emptyset$ and $\overline{S_\beta}(X) \neq U$. Then, X can be soft β-definable. Meanwhile, some elements of U belong to X^c from available knowledge for A_s (See Examples 3.6 and 4.1).*
(ii) The converse of Theorem 3.14 is not true, in general, as shown in Example 4.1.

4. Decision making for human-to-human transmissions of COVID-19

In this section, we introduce two practical examples as applications of our approach in decision making for information system about infections of COVID-19 on human. Example 4.1 illustrates that the suggested approximations are more accurate tools rather than Feng method. On the other hand, in Example 4.2, we apply the suggested approximations (soft beta approximations) to make a topological reduction using Definition 2.12 and so we identify deciding factors of infections for COVID-19 in humans. In this model, we find that gatherings, contact with injured people, and work in hospitals are the only deciding factors for infection transmission. While, staying at home and haven't been in contact with humans protect and against viral infection with coronavirus. Currently, the emergence of a novel human coronavirus SARS-CoV-2 or COVID-19 has become a global health concern causing severe respiratory tract infections in humans. Human-to-human transmissions have been described with incubation times between $2 - -10$ days, facilitating its spread via droplets, contaminated hands or surfaces. According to [13], human coronaviruses can remain infectious on inanimate surfaces for up to 9 days. Surface disinfection with 0:1% sodium hypochlorite or 62e71% ethanol significantly reduces coronavirus infectivity on surfaces within 1 min exposure time. Table 2 illustrates the persistence of coronaviruses on different types of inanimate surfaces. In the following examples, we explain the persons most vulnerable for COVID-19 via soft β-rough approximations.

Example 4.1 *Suppose that $U = \{p_1, p_2, p_3, \cdots, p_{10}\}$ consists of ten persons and $A = \{e_1, e_2, e_3, e_4, e_5\}$ is a set of attributes parameters, where $e_1 =$ (stay at home) , $e_2 =$ (go out the home and contact with infected people), $e_3 =$ (work at hospital), $e_4 =$ (study at home) and $e_5 =$ (study out the home). Let (F, A) be a soft set. Table 3 is a collection of qualitative data generating from a medical sample from some patients in hospitals.*

By coding Table 3, we have another form of information system in Table 4.

Table 4, represents the tabular form of the soft set $(F, A) = \{(e_1, \{p_1, p_4, p_5, p_8, p_{10}\}), (e_2, \{p_2, p_3, p_7, p_9\}), (e_3, \{p_2, p_3, p_4, p_7, p_9\}), (e_4, \{p_1, p_5, p_6, p_8\}), (e_5, \{p_1, p_2, p_3, p_5, p_6, p_9, p_{10}\})\}$. From Table 4, we have a set of infected patients with COVID-19 is $X = \{p_3, p_4, p_7, p_9\}$. Thus, we get the following comparison

*[i] **According to Feng's approach**, we have $\underline{S}(X) = \emptyset$, $\overline{S}(X) = U$, $BND_{A_s}(X) = U$ and $\mu_{A_s}(X) = 0$. Then, by Remark 3.15, X is totally soft A_s-undefinable. This can be interpreted that no patient has COVID-19 in X which gives a contradict with Table 3.*

Table 2. Persistence of coronaviruses on different types of inanimate surfaces [24].

Type of surface	Virus	Strain/isolate	Inoculum (viral titer)	Temperature	Persistence
Steel	MERS-CoV	Isolate HCoV-EMC/2012	10^5	20°C	48h
				30°C	8-24 h
	TGEV	Unknown	10^6	4°C	≥28 d
				20°C	3-28 d
				40°C	4-96 h
	MHV	Unknown	10^6	4°C	≥28 d
				20°C	3-28 d
				40°C	4-96 h
	HCoV	Strain 229E	10^3	21°C	5 d
Aluminium	HCoV	Strains 229E and OC43	5×10^3	21°C	2-8 h
Metal	SARS-CoV	Strain P9	10^5	RT	5 d
Wood	SARS-CoV	Strain P9	10^5	RT	4 d
Paper	SARS-CoV	Strain P9	10^5	RT	4-5 d
	SARS-CoV	Strain GVU6109	10^6	RT	24 h
			10^5		3 h
			10^4		< 5 min
Glass	SARS-CoV	Strain P9	10^5	RT	4 d
	SARS-CoV	Strain 229E	10^3	21°C	5 d
Plastic	SARS-CoV	Strain HKU39849	10^5	22°-25° C	≤ 5 d
	MERS-CoV	Isolate HCoV-EMC/2012	10^5	20° C	4-24 h
				30° C	48 h
	SARS-CoV	Strain P9	10^5	RT	4 d
	SARS-CoV	Strain FFM1	10^7	RT	6-9 d
	HCoV	Strain 229E	10^7	RT	2-6 d
PVC	HCoV	Strain 229E	10^3	21° C	5 d
Silicon rubber	HCoV	Strain 229E	10^3	21° C	5 d
Surgical glove (latex)	HCoV	Strains 229E and OC43	5×10^3	21° C	≤ 8 h
Disposable gown	SARS-CoV	Strain GVU6109	10^6	RT	2 d
			10^5		24 h
			10^4		1 h
Ceramic	HCoV	Strain 229E	10^3	21° C	5 d
Teflon	HCoV	Strain 229E	10^3	21° C	5 d

$MERS$ = Middle East respiratory syndrome; $HCoV$ = human coronavirus; $TGEV$ = transmissible gastroenteritis virus; MHV = mouse hepatitis virus;$SARS$ = Severe acute respiratory syndrome; RT = room temperature.

***[ii] According to proposed approach**, we have $\underline{S_\beta}(X) = \overline{S_\beta}(X) = X$, $BND_\beta(X) = U$ and $\mu_\beta(X) = 1$. Then, by Remark 3.15, X is totally soft β-definable. This can be interpreted that the patients in X are only COVID-19 infections. Therefore, the suggested approach is more useful than Feng's method in removing the vagueness of roughness.*

Now, we construct algorithm 1 in Table 5 to illustrate the results in Section 3 of soft β-rough approximations in soft rough sets as an information system.

Example 4.2 *Suppose that $U = \{p_1, p_2, p_3, \cdots, p_6\}$ consists of six persons and $B = \{e_1, e_2, e_3, e_4\}$ is a set*

Table 3. A decision of a given information system.

Persons	Parameters					Decision for COVID-19
	Stay at Home	Go out Home	Work at Hospital	Study at Home	Study out Home	
p_1	yes	no	no	yes	yes	no
p_2	no	yes	yes	no	yes	no
p_3	no	yes	yes	no	yes	yes
p_4	yes	no	yes	no	no	yes
p_5	yes	no	no	yes	yes	no
p_6	no	no	no	yes	yes	no
p_7	no	yes	yes	no	no	yes
p_8	yes	no	no	yes	no	no
p_9	no	yes	yes	no	yes	yes
p_{10}	yes	no	no	no	yes	no

Table 4. Table 2 with coding.

Persons	Attributes					Decision for COVID-19
	$\mathbf{e}_1$	$\mathbf{e}_2$	$\mathbf{e}_3$	$\mathbf{e}_4$	$\mathbf{e}_5$	
p_1	1	0	0	1	1	0
p_2	0	1	1	0	1	0
p_3	0	1	1	0	1	1
p_4	1	0	1	0	0	1
p_5	1	0	0	1	1	0
p_6	0	0	0	1	1	0
p_7	0	1	1	0	0	1
p_8	1	0	0	1	0	0
p_9	0	1	1	0	1	1
p_{10}	1	0	0	0	1	0

of attributes parameters, where $e_1 =$ *(Stay at home),* $e_2 =$ *(Go out home and contact with infected people),* $e_3 =$ *(Work out home) and* $e_4 =$ *(Take all careless). This is an experimental data and taken from COVID-19 patients. By the same manner, in Example 4.1, we obtain the soft set* (F, A)*, which is given by Table 6.*

Now, from Table 6, we have two cases:

Case 1. Persons infected with COVID-19

The set of infected patients with COVID-19 is $X = \{p_1, p_2, p_3\}$*. Thus,* $\underline{S_\beta}(X) = \{p_1, p_2, p_3\}$*,* $\overline{S_\beta}(X) = \{p_1, p_2, p_3, p_5, p_6\}$ *and* $BND_\beta(X) = \{p_5, p_6\}$*. Hence, by Definition 2.13 [11], soft rough topology generated by soft* β*-rough approximations is given by*

Table 5. Algorithm on soft β-rough approximations.

Algorithm 1	**A decision making via soft β-rough approximations.**
Step 1:	Input the soft set (F, E).
Step 2:	Input the set A of choice parameters of Mr.X which is a subset of E.
Step 3:	Investigate the soft β-upper approximation, say, $\overline{S_\beta}(X)$ and soft β-lower approximation, say, $\underline{S_\beta}(X)$, for every $X \subseteq U$. According to Definition 3.1.
Step 4:	Determine a boundary region, say, $BND_\beta(X)$ from Step 2, for every $X \subseteq U$. According to Definition 3.1.
Step 5:	Calculate the accuracy of the approximation, say, $\mu_\beta(X)$ by Step 2, for every $X \subseteq U$. According to Definition 3.1.
Step 6:	Decide, exactly, rough sets and exact sets. Using Definition 3.1.

Table 6. Tabular representation for a soft set (F, A).

U/B	e_1	e_2	e_3	e_4	**Infected with COVID-19**
p_1	0	1	1	0	yes
p_2	0	1	1	0	yes
p_3	0	1	0	0	yes
p_4	1	0	0	1	no
p_5	1	0	1	1	no
p_6	1	0	1	1	no

$$\tau_{SR_\beta} = \{U, \emptyset, \{p_5, p_6\}, \{p_1, p_2, p_3\}, \{p_1, p_2, p_3, p_5, p_6\}\}.$$

Accordingly, the soft β-base is given by $\mathcal{B}_{SR_\beta} = \{U, \{p_5, p_6\}, \{p_1, p_2, p_3\}\}$.

Step 1. *When the attribute e_1 is removed, we have $\underline{S_\beta - \{e_1\}}(X) = \{p_1, p_2, p_3\}$, $\overline{S_\beta - \{e_1\}}(X) = \{p_1, p_2, p_3, p_5, p_6\}$ and $BND_\beta - \{e_1\}(X) = \{p_5, p_6\}$. Hence, by Definition 2.13, soft rough topology generated by soft β-rough approximations is given by $\tau_{SR_{\beta-\{e_1\}}} = \{U, \emptyset, \{p_5, p_6\}, \{p_1, p_2, p_3\}, \{p_1, p_2, p_3, p_5, p_6\}\} = \tau_{SR_\beta}$. Accordingly, soft β-base is given by $\mathcal{B}_{SR_{\beta-\{e_1\}}} = \{U, \{p_5, p_6\}, \{p_1, p_2, p_3\}\} = \mathcal{B}_{SR_\beta}$.*

Step 2. *When the attribute the attribute e_2 is removed, we have $\underline{S_\beta - \{e_2\}}(X) = \{p_1, p_2\}$, $\overline{S_\beta - \{e_2\}}(X) = \{p_1, p_2, p_3\}$ and $BND_\beta - \{e_2\}(X) = \{p_3\}$. Hence, by Definition 2.13, soft rough topology generated by soft β-rough approximations is given by $\tau_{SR_{\beta-\{e_2\}}} = \{U, \emptyset, \{p_3\}, \{p_1, p_2\}, \{p_1, p_2, p_3\}\} \neq \tau_{SR_\beta}$. Accordingly, soft β-base is given by $\mathcal{B}_{SR_{\beta-\{e_2\}}} = \{U, \{p_3\}, \{p_1, p_2\}\} \neq \mathcal{B}_{SR_\beta}$.*

Step 3. *When the attribute e_3 is removed, we have $\underline{S_\beta - \{e_3\}}(X) = \overline{S_\beta - \{e_3\}}(X) = \{p_1, p_2, p_3\}$ and $BND_\beta - \{e_3\}(X) = \emptyset$. Hence, by Definition 2.13, soft rough topology generated by soft β-rough approximations is given by $\tau_{SR_{\beta-\{e_3\}}} = \{U, \emptyset, \{p_1, p_2, p_3\}\} \neq \tau_{SR_\beta}$. Accordingly, soft β-base is given by $\mathcal{B}_{SR_{\beta-\{e_3\}}} = \{U, \{p_1, p_2, p_3\}\} \neq \mathcal{B}_{SR_\beta}$.*

Step 4. *When the attribute e_4 is removed, we have $\underline{S_\beta - \{e_4\}}(X) = \{p_1, p_2, p_3\}$, $\overline{S_\beta - \{e_4\}}(X) = \{p_1, p_2, p_3, p_5, p_6\}$ and $BND_\beta - \{e_4\}(X) = \{p_5, p_6\}$. Hence, by Definition 2.13, soft rough topology generated by soft β-rough* *approximations is given by* $\tau_{SR_{\beta-\{e_4\}}} = \{U,\ \emptyset,\ \{p_5,p_6\},\ \{p_1,p_2,p_3\},\ \{p_1,p_2,p_3,p_5,p_6\}\} = \tau_{SR_\beta}$. *The soft* β*-base is given by* $\mathcal{B}_{SR_{\beta-\{e_4\}}} = \{U,\{p_5,p_6\},\ \{p_1,p_2,p_3\}\} = \mathcal{B}_{SR_\beta}$. *Moreover, if the attributes* e_2 *and* e_3 *are removed, then* $\underline{S_\beta - \{e_2,e_3\}}(X) = \overline{S_\beta - \{e_2,e_3\}}(X) = \{p_1,p_2,p_3\}$ *and so* $BND_\beta - \{e_2,e_3\}(X) = \emptyset$. *Hence, by Definition 2.13, soft rough* β*-topology generated by soft* β*-rough approximations is given by* $\tau_{SR_{\beta-\{e_2,e_3\}}} = \{U,\emptyset,\{p_1,p_2,p_3\}\} \neq \tau_{SR_\beta}$. *Accordingly, the soft* β*-base is given by* $\mathcal{B}_{SR_{\beta-\{e_2,e_3\}}} = \{U,\ \{p_1,p_2,p_3\}\} \neq \mathcal{B}_{SR_\beta}$. *Therefore,* $CORE(SR_\beta) = \{e_2,e_3\}$*, this means that both "Go out home" and "Work out home" are the key attributes which confirm the COVID-19 infection.*

Case 2. Persons who are not infected with COVID-19

By the same manner in Case 1, we also have $CORE(SR_\beta) = \{e_2,e_3\}$.

Observation. From the core in cases 1 and 2, to avoid the COVID-19 infection between people, we must give two advices: "stay at home" and "work at home". Actually, all countries applying these parameters have few number of patients.

Finally, we propose an Algorithm 2 in Table 7 to describe how to use the new soft rough topology τ_{SR_β} and its base in decision making for information system through soft β-approximations.

Table 7. Algorithm on soft rough topology and its base.

Algorithm 2	**A decision making via soft rough topology τ_{SR_β} and its base.**
Step 1:	Input the soft set (F,A), using a finite universe U and a finite set A of attributes (parameters) represent the data as an information table, rows of which are labeled by attributes (C), columns by objects and entries of the table are attribute values.
Step 2:	Compute the soft β-upper approximation, say, $\overline{S_\beta}(X)$, soft β-lower approximation, say, $\underline{S_\beta}(X)$, and soft β-boundary, say, $BND_\beta(X)$, for the decision set $X \subseteq U$. According to Definition 3.1.
Step 3:	Generate the soft rough topology τ_{SR_β} on U and its soft base $\mathcal{B}_{SR_\beta}$ by using Definition 2.13.
Step 4:	Remove an attribute e_i from conditions of attributes (C) and find the soft β-lower and soft β-upper approximations and the soft β-boundary region of X on $C - \{e_i\}$, for each $i \in \mathbb{N}$.
Step 5:	Generate the soft rough topology $\tau_{SR_{\beta-\{e_i\}}}$ on U and its soft base $\mathcal{B}_{SR_{\beta-\{e_i\}}}$ by using Definition 2.13.
Step 6:	Repeat steps 4 and 5 for all attributes in C.
Step 7:	Those attributes in C for which $\mathcal{B}_{SR_{\beta-\{e_i\}}} \neq \mathcal{B}_{SR_\beta}$ forms the $CORE(SR_\beta)$.

Conclusion

The present paper represents a starting point for a framework to modify and generalize soft rough sets for Feng et al. [20]. We have initiated new soft rough approximations called soft β-rough approximations. These approximations satisfied most of Pawlak's properties, which never hold in Feng's approach. Several examples were given to indicate the connections between the soft β-rough sets and the soft rough sets. Theorem 3.9 and its corollaries and Theorem 3.14 showed that soft β-rough sets are a modification and generalization to soft rough sets and so any soft exact set must be soft β-exact. The counterexamples and the medical application showed that the converse need not be held, in general. Therefore, this technique may be useful in discovering the vagueness of the data and help in decision-making. One of the main goals is to introduce a COVID-19 as a medical application to show the importance of the soft β-rough methodologies in some real-life solutions. This approach can be applied to analyze data of COVID-19 with quantitative or qualitative data [15, 47] by coding the qualitative data. Finally, the induced soft β-rough topologies are used to reduce the proposed information system of COVID-19. Therefore, this technique of study is used to protect people from COVID-19 via the viewpoint of soft β-rough topologies.

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