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Compact matrix operators on Banach space of absolutely k-summable series

Fadime GÖKÇE*
Department of Statistics, University of Pamukkale, Denizli, Turkey

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Abstract: In a more recent paper [23], Banach spaces of absolutely k-summable series $|T_\varphi|_k$ have been introduced and studied by Sarıgöl and Agarwal. In this paper, we characterize matrix and compact operators from the space $|T_\varphi|_k$ to the classical spaces c_0, c and l_∞, by determining identities or estimates for operator norms and the Hausdorff measure of noncompactness.

Key words: Absolute summability, matrix transformations, Hausdorff measure of noncompactness, compact operator, bounded operators, operator norm, sequence spaces

1. Introduction

The sequence spaces play important roles in summability theory, a wide field of mathematics, which have several applications in approximation theory, calculus, and essentially in functional analysis. The classical theory deals with the generalization of the concept of convergence for sequences and series. The aim is to assign a limit for nonconvergent sequences and series by making use of an operator defined by infinite matrices. The reason why matrices are used for a general linear operator is that a linear operator from a sequence space to another one can be given by an infinite matrix. A large literature has grown up concerned with characterizing completely all matrices which transform one given sequence space into another, and also, many sequence spaces and related matrix operators have been studied by several authors (see, [1, 3–14, 16, 21, 27]). In a more recent paper [23], the Banach space $|T_\varphi|_k$ of absolutely k-summable series with an arbitrary triangle matrix T has been introduced and some matrix operators on this space have been investigated by Sarıgöl and Agarwal.

In the present paper, we characterize matrix and compact operators from the space $|T_\varphi|_k, 1 \leq k < \infty$, to the classical spaces c_0, c and l_∞, by determining certain identities or estimates for operator norms and the Hausdorff measure of noncompactness.

Firstly we recall some known concepts. A vector subspace of ω, the space of all sequences of real or complex numbers, is called a *sequence space*. In this respect, c_0, c and l_∞ are examples of sequence spaces, which stand for the sets of all null, convergent and bounded sequences, respectively. Also, b_s and c_s represent the spaces of all bounded and convergent series, respectively.

Let Λ, Γ be two sequence spaces and $U = (u_{nv})$ be an infinite matrix of complex numbers. The transform $U(\lambda)$ of a sequence $\lambda = (\lambda_v) \in \omega$ is given by the usual matrix product and its terms are written as

$$U_n(\lambda) = \sum_{v=0}^{\infty} u_{nv}\lambda_v,$$

*Correspondence: fgokce@pau.edu.tr
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provided that the series converges for all $n \geq 0$. If $U(\lambda) \in \Gamma$ for all $\lambda \in \Lambda$, then, U is a matrix operator from Λ into Γ, and the collection of all such infinite matrices is denoted by (Λ, Γ).

Let Λ be a normed space. Then, the unit sphere S_Λ in Λ is stated by

$$S_\Lambda = \{\lambda \in \Lambda : \|\lambda\| = 1\}.$$

If Λ and Γ are Banach spaces, then the set $\mathcal{B}(\Lambda, \Gamma)$ of all bounded (continuous) linear operators L from Λ to Γ is a Banach space according to the norm

$$\|L\| = \sup_{\lambda \in S_\Lambda} \|L(\lambda)\|_\Gamma .$$

Also, the operator L is called compact if its domain is all of Λ and the sequence $(L(\lambda_n))$ has a convergent subsequence in the space Γ for each bounded sequence (λ_n) in Λ. The set of such compact operators is denoted by $\mathcal{C}(\Lambda, \Gamma)$.

If $u \in \omega$ and $\Lambda \supset \Phi$ is a BK-space, Banach space on which all coordinate functionals $p_n(\lambda) = \lambda_n$ are continuous for all n, then

$$\|u\|_\Lambda^* = \sup_{\lambda \in S_\Lambda} \left| \sum_{k=0}^{\infty} u_k \lambda_k \right|$$

where Φ is the set of all finite sequences.

The domain of an infinite matrix U in the sequence space Λ and the β-dual of Λ are, respectively, defined by

$$\Lambda_U = \{\lambda = (\lambda_n) \in \omega : U(\lambda) \in \Lambda\}$$

and

$$\Lambda^\beta = \left\{ u = (u_v) : \sum_{v=0}^{\infty} u_v \lambda_v \text{ converges for all } \lambda = (\lambda_v) \in \Lambda \right\}.$$

A triangle matrix $T = (t_{nv})$ is stated as $t_{nn} \neq 0$ for all n and $t_{nv} = 0$ for $n < v$.

Throughout the paper, let T denote an arbitrary infinite triangle matrix of complex numbers and $\varphi = (\varphi_n)$ be a sequence of positive constants. Also, k^* is the conjugate of k, i.e. $1/k + 1/k^* = 1$ for $k > 1$, and $1/k^* = 0$ for $k = 1$.

Let $\sum \lambda_n$ be an infinite series with its partial sum $s = (s_n)$. The series $\sum \lambda_v$ is said to be summable $|U, \varphi_n|_k$, $1 \leq k < \infty$, if (see [25])

$$\sum_{n=0}^{\infty} \varphi_n^{k-1} |\Delta U_n(s)|^k < \infty, \tag{1.1}$$

where $\Delta U_n(s) = U_n(s) - U_{n-1}(s)$, $U_{-1}(s) = 0$.

In a more recent paper [23], the space $|T_\varphi|_k$ $(|T_\varphi|_1 = |T_\varphi|)$ has been defined by the set of all series summable by $|T, \varphi_n|_k$, i.e. the set of sequences (λ_v) satisfying

$$\sum_{n=0}^{\infty} \varphi_n^{k-1} \left| \sum_{v=0}^{n} \left\{ \sum_{r=v}^{n} (t_{nr} - t_{n-1,r}) \right\} \lambda_v \right|^k < \infty$$

where T is a triangle matrix. Also, note that, according to notion of domain, $|T_\varphi|_k = (l_k)_{\tilde{T}}$, i.e.

$$|T_\varphi|_k = \left\{ \lambda = (\lambda_n) \in \omega : \sum_{n=0}^{\infty} \left| \tilde{T}_n(\lambda) \right|^k < \infty \right\},$$

where the matrices $\tilde{T}, \hat{T}$ and $\bar{T}$ are given by

$$\widetilde{t}_{nv} = \begin{cases} \varphi_n^{1/k^*} \widehat{t}_{nv}, & 0 \le v \le n \\ 0, & v > n, \end{cases} \tag{1.2}$$

$$\bar{t}_{nv} = \sum_{i=v}^{n} t_{ni}, \ \hat{t}_{nv} = \bar{t}_{nv} - \bar{t}_{n-1,v}, \ (\bar{t}_{n-1,n} = 0). \tag{1.3}$$

On other hand, since each triangle matrix has a unique inverse [28], the triangle matrices $\tilde{T}, \hat{T}$ and $\bar{T}$ have the inverses stated by

$$\tilde{t}_{nv}^{-1} = \varphi_v^{\frac{-1}{k^*}} \hat{t}_{nv}^{-1}, \ \ \bar{t}_{nv}^{-1} = t_{nv}^{-1} - t_{n-1,v}^{-1}, \ \ t_{-1,0}^{-1} = 0. \tag{1.4}$$

However, since $\hat{T} = E \circ \bar{T}$, it follows that $\left(\hat{T}\right)^{-1} = \bar{T}^{-1} \circ E^{-1}, i.e.$

$$(\hat{t})_{nv}^{-1} = \begin{cases} \sum\limits_{i=v}^{n} (t_{ni}^{-1} - t_{n-1,i}^{-1}), & 0 \le v \le n \\ 0, & v > n, \end{cases} \tag{1.5}$$

where $E_n(\lambda) = \lambda_n - \lambda_{n-1}, \ \lambda_{-1} = 0$ for $n \ge 0$.

We require the following lemmas in proving our theorems:

Lemma 1.1 *[23] Let $1 \le k < \infty$. The space $|T_\varphi|_k$ is a BK-space with the norm $\|\lambda\|_{|T_\varphi|_k} = \left\| \tilde{T}(\lambda) \right\|_{l_k}$. Also, $|T_\varphi|_k$ is linearly isomorphic to the space l_k.*

Lemma 1.2 *[17] Let Λ and Γ be two arbitrary BK-spaces. Then,*
(a) $(\Lambda, \Gamma) \subset \mathcal{B}(\Lambda, \Gamma)$, i.e. every matrix $U \in (\Lambda, \Gamma)$ defines an operator $L_U \in \mathcal{B}(\Lambda, \Gamma)$ by $L_U(\lambda) = U(\lambda)$ for all $\lambda \in \Lambda$.
(b) If Λ has AK property, then $\mathcal{B}(\Lambda, \Gamma) \subset (\Lambda, \Gamma)$, i.e. for every operator $L \in \mathcal{B}(\Lambda, \Gamma)$ there exists a matrix $U \in (\Lambda, \Gamma)$ such that $L(\lambda) = U(\lambda)$ for all $\lambda \in \Lambda$.

Lemma 1.3 *[19] Let T be a triangle. Then, we have*

(a) For $\Lambda, \Gamma \subset \omega$, $U \in (\Lambda, \Gamma_T)$ if and only if $B = TU \in (\Lambda, \Gamma)$.

(b) If Λ and Γ are BK-spaces and $U \in (\Lambda, \Gamma_T)$, then $\|L_U\| = \|L_B\|$.

Lemma 1.4 *[26] The following statements hold:*

1. $U \in (l,c) \Leftrightarrow$ *(i)* $\lim\limits_n u_{nv}$ *exists for all* $v \geq 0$, *(ii)* $\sup\limits_{n,v} |u_{nv}| < \infty$ *and* $U \in (l,l_\infty) \Leftrightarrow$ *(ii) holds.*

2. *For* $1 < k < \infty$, *then,* $U \in (l_k,c) \Leftrightarrow (i)$*holds*, $(iii) \sup\limits_n \sum\limits_{v=0}^{\infty} |u_{nv}|^{k^*} < \infty$ *and* $U \in (l_k,l_\infty) \Leftrightarrow (iii)$ *holds.*

3. $U \in (l,c_0) \Leftrightarrow$ *(iv)* $\lim\limits_n u_{nv} = 0$ *for all* $v \geq 0$, *(ii) hold.*

4. *For* $1 < k < \infty$, *then,* $U \in (l_k,c_0) \Leftrightarrow (iii)$ *and* (iv) *hold.*

Lemma 1.5 *[19] Let* $1 < k < \infty$ *and* k^* *denote the conjugate of* k. *Then, we have* $l_k^\beta = l_{k^*}$, $l_\infty^\beta = c^\beta = c_0^\beta = l$ *and* $l^\beta = l_\infty$. *Also,* $\Lambda \in \{l_\infty, c, c_0, l, l_k\}$, *then, for all* $u \in \Lambda^\beta$,

$$\|u\|_\Lambda^* = \|u\|_{\Lambda^\beta}$$

where $\|.\|_{\Lambda^\beta}$ *is the natural norm on the dual space* Λ^β.

Lemma 1.6 *[2] Let* $\Lambda \supset \Phi$ *be a* BK*-space and* $\Gamma \in \{\ell_\infty, c, c_0\}$. *If* $U \in (\Lambda,\Gamma)$, *then we have*

$$\|L_U\| = \|U\|_{(\Lambda,l_\infty)} = \sup_n \|U_n\|_\Lambda^* < \infty.$$

2. Hausdorff measure of noncompactness

If H and R are subsets of a metric space (Λ,d) and, for every $r \in R$, there exists an $h \in H$ such that $d(r,h) < \varepsilon$ then, H is called an ε-net of R; if H is finite, then the ε-net H of R is called a finite ε-net of R. Let Q be a bounded subset of the metric space Λ. Then the Hausdorff measure of noncompactness of Q is defined by

$$\chi(Q) = \inf\{\varepsilon > 0 : Q \text{ has a finite } \varepsilon - \text{net in } \Lambda\},$$

and χ is called the Hausdorff measure of noncompactness.

The linear operator $L : \Lambda \to \Gamma$ is said to be (χ_1,χ_2)- bounded if $L(Q)$ is a bounded subset of Γ and there exists a positive constant M such that $\chi_2(L(Q)) \leq M\chi_1(Q)$ for every bounded subset Q of Λ, where χ_1 and χ_2 are Hausdorff measures on Λ and Γ. Also, the number

$$\|L\|_{(\chi_1,\chi_2)} = \inf\{M > 0 : \chi_2(L(Q)) \leq M\chi_1(Q) \text{ for all bounded sets } Q \subset \Lambda\}$$

is called the (χ_1,χ_2)-measure noncompactness of L. In particular, $\|L\|_{(\chi,\chi)} = \|L\|_\chi$.

Also, the following lemmas play important roles in our results.

Lemma 2.1 *[15]* $L \in \mathcal{B}(\Lambda,\Gamma)$ *and* S_Λ *be the unit sphere in* Λ. *Then,*

$$\|L\|_\chi = \chi(L(S_\Lambda)),$$

$$L \text{ is compact iff } \|L\|_\chi = 0.$$

Lemma 2.2 *[18] Let* Λ *be a normed sequence space,* $T = (t_{nv})$ *be an infinite triangle matrix,* χ_T *and* χ *denote the Hausdorff measures of noncompactness on* M_{Λ_T} *and* M_Λ, *the collections of all bounded sets in* Λ_T *and* Λ, *respectively. Then,* $\chi_T(Q) = \chi(T(Q))$ *for all* $Q \in M_{\Lambda_T}$.

Lemma 2.3 *[22] Let Λ, containing all finite sequences, be a BK-space with AK or $\Lambda = l_\infty$. If $U \in (\Lambda, c)$, then we have*

$$\lim_{n\to\infty} u_{nk} = \alpha_k \quad \textit{exists for all } k,$$

$$\alpha = (\alpha_k) \in \Lambda^\beta,$$

$$\sup_n \|U_n - \alpha\|_\Lambda^* < \infty,$$

$$\lim_{n\to\infty} U_n(\lambda) = \sum_{k=0}^{\infty} \alpha_k \lambda_k \textit{ for every } \lambda = (\lambda_k) \in \Lambda.$$

Lemma 2.4 *[22] Let Λ, containing all finite sequences, be a BK-space. Then, we have*
(a) If $U \in (\Lambda, c_0)$, then

$$\|L_U\|_\chi = \lim_{r\to\infty}\left(\sup_{n>r} \|U_n\|_\Lambda^*\right).$$

(b) If Λ has AK or $\Lambda = l_\infty$ and $U \in (\Lambda, c)$, then

$$\tfrac{1}{2}\lim_{r\to\infty}\left(\sup_{n\geq r} \|U_n - \alpha\|_\Lambda^*\right) \leq \|L_U\|_\chi \leq \lim_{r\to\infty}\left(\sup_{n\geq r} \|U_n - \alpha\|_\Lambda^*\right)$$

where $\alpha = (\alpha_k)$ is defined by $\alpha_k = \lim_{n\to\infty} u_{nk}$, for all $n \in \mathbb{N}$.
(c) If $U \in (\Lambda, l_\infty)$, then

$$0 \leq \|L_U\|_\chi \leq \lim_{r\to\infty}\left(\sup_{n>r} \|U_n\|_\Lambda^*\right).$$

3. Matrix and compact operators on space $|T_\varphi|_k$

In this section, computing operator norms we characterize the classes of infinite matrices $\left(|T_\varphi|_k, c_0\right)$, $\left(|T_\varphi|_k, c\right)$, $\left(|T_\varphi|_k, l_\infty\right)$ and also $\mathcal{C}\left(|T_\varphi|_k, c_0\right), \mathcal{C}\left(|T_\varphi|_k, c\right), \mathcal{C}\left(|T_\varphi|_k, l_\infty\right), 1 \leq k < \infty$. Then, we determine certain identities or estimates for the Hausdorff measures of noncompactness for these matrices.

We begin with a lemma providing ease in calculations.

Lemma 3.1 *Let $1 < k < \infty$. Then,*
(i) If $u = (u_v) \in \left\{|T_\varphi|_k\right\}^\beta$, then, $\tilde{u}^{(k)} = (\tilde{u}_v^{(k)}) \in l_{k^*}$
(ii) If $u = (u_v) \in \{|T_\varphi|\}^\beta$, then, $\tilde{u}^{(1)} = (\tilde{u}_v^{(1)}) \in l_\infty$
and, the equality

$$\sum_{v=0}^{\infty} u_v \lambda_v = \sum_{v=0}^{\infty} \widetilde{u}_v^{(k)} y_v \tag{3.1}$$

for all $\lambda \in |T_\varphi|_k$ (or $\lambda \in |T_\varphi|$) holds, where $y = \widetilde{T}(\lambda)$ and

$$\widetilde{u}_v^{(k)} = \varphi_v^{-1/k^*} \sum_{j=v}^{\infty} \widehat{t}_{jv}^{-1} u_j.$$

Proof (i) Let $u = (u_v) \in \left\{|T_\varphi|_k\right\}^\beta$. By the inverse mapping of $\tilde{T}$, Equation (3.1) is obtained at once. Also, it follows from Lemma 1.5 that $\tilde{u}^{(k)} \in l_{k^*}$ whenever $u \in \left\{|T_\varphi|_k\right\}^\beta$, which complete the proof.
The proof of (ii) is left to the reader. □

Lemma 3.2 *Let $1 < k < \infty$. Then, we have $\|u\|^*_{|T_\varphi|_k} = \left\|\tilde{u}^{(k)}\right\|_{l_{k^*}}$ for all $u \in \left\{|T_\varphi|_k\right\}^\beta$ and $\|u\|^*_{|T_\varphi|} = \left\|\tilde{u}^{(1)}\right\|_\infty$ for all $u \in \{|T_\varphi|\}^\beta$.*

Proof Let $u \in \left\{|T_\varphi|_k\right\}^\beta$. It is seen from Lemma 3.1 that $\tilde{u}^{(k)} \in l_{k^*}$. Also, by considering Lemma 1.5 and Lemma 3.1, we have

$$\|u\|^*_{|T_\varphi|_k} = \sup_{\lambda \in S_{|T_\varphi|_k}} \left|\sum_{v=0}^{\infty} u_v \lambda_v\right| = \sup_{y \in S_{l_k}} \left|\sum_{v=0}^{\infty} \tilde{u}_v^{(k)} y_v\right| = \left\|\tilde{u}^{(k)}\right\|^*_{l_k} = \left\|\tilde{u}^{(k)}\right\|_{l_{k^*}}.$$

The proof is similar for $u \in \{|T_\varphi|\}^\beta$. So it is left to the reader. □

Theorem 3.3 *Let $1 \le k < \infty$, Λ be arbitrary sequence space and $U = (u_{nv})$ be an infinite matrix of complex numbers. Further, let the infinite matrix $B = (b_{nj})$ be defined by*

$$b_{nj} = \varphi_n^{1/k^*} \sum_{v=0}^{n} \widehat{t}_{nv} u_{vj}. \tag{3.2}$$

Then, $U \in \left(\Lambda, |T_\varphi|_k\right)$ if and only if $B \in (\Lambda, l_k)$.

Proof Let $\lambda \in \Lambda$. Then, it follows from (3.2) that

$$\sum_{j=0}^{\infty} b_{nj}\lambda_j = \varphi_n^{1/k^*} \sum_{v=0}^{n} \widehat{t}_{nv} \sum_{j=0}^{\infty} u_{vj}\lambda_j = \sum_{v=0}^{n} \widetilde{t}_{nv} \sum_{j=0}^{\infty} u_{vj}\lambda_j,$$

which implies that $B_n(\lambda) = \tilde{T}_n(U(\lambda))$. This gives that $U(\lambda) \in |T_\varphi|_k$ for all $\lambda \in \Lambda$ if and only if $B(\lambda) \in l_k$ for all $\lambda \in \Lambda$. So, the proof of the theorem is completed. □

Theorem 3.4 *Let $1 \le k < \infty$ and $U = (u_{nv})$ be an infinite matrix of complex numbers. Further let Λ be any sequence space. Then, $U \in \left(|T_\varphi|_k, \Lambda\right)$ if and only if*

$$V^{(n)} \in (l_k, c) \ \textit{for all } n \ge 0$$

and

$$\tilde{U}^{(k)} \in (l_k, \Lambda),$$

where the matrices $V^{(n)}$ and $\tilde{U}$ are given by

$$\tilde{u}_{nj}^{(k)} = \varphi_j^{-1/k^*} \sum_{v=j}^{\infty} \widehat{t}_{vj}^{-1} u_{nv}$$

and

$$v_{mj}^{(n)} = \begin{cases} \varphi_j^{-1/k^*} \sum_{v=j}^{m} \widehat{t}_{vj}^{-1} u_{nv}, & 0 \le j \le m \\ 0, & j > m. \end{cases}$$

Proof Let $U \in \left(|T_\varphi|_k, \Lambda\right)$. Take $\lambda \in |T_\varphi|_k$. Since $|T_\varphi|_k = (l_k)_{\widetilde{T}}$ and $y = \tilde{T}(\lambda)$, it is immediately obtained that, for $n, m \geq 0$,

$$\sum_{j=0}^{m} u_{nj}\lambda_j = \sum_{j=0}^{m} v_{mj}^{(n)} y_j. \tag{3.3}$$

Hence, $U\lambda$ is well defined for all $\lambda \in |T_\varphi|_k$ if and only if $V^{(n)} \in (l_k, c)$. Also, letting $m \to \infty$, (3.3) gives that $U\lambda = \tilde{U}^{(k)}y$. Since $U\lambda \in \Lambda$, $\tilde{U}^{(k)}y$ is in Λ. So, $\tilde{U}^k \in (l_k, \Lambda)$.

Conversely, let $V^{(n)} \in (l_k, c)$ and $\tilde{U}^{(k)} \in (l_k, \Lambda)$. Then, by (3.3), it is seen that $U_n \in \left\{|T_\varphi|_k\right\}^\beta$ for all n, which gives that $U\lambda$ exists. Also, by $\tilde{U}^{(k)} \in (l_k, \Lambda)$ and (3.3), letting $m \to \infty$ implies $U \in \left(|T_\varphi|_k, \Lambda\right)$. □

Now, we list the following conditions and the tables:

1. $\lim\limits_{n\to\infty} \tilde{u}_{nv}^{(k)}$ exists for all $v \in \mathbb{N}$
2. $\lim\limits_{n\to\infty} \tilde{u}_{nv}^{(k)} = 0$ for all $v \in \mathbb{N}$
3. $\sup\limits_{n} \sum_{v=0}^{\infty} \left|\tilde{u}_{nv}^{(k)}\right|^{k^*} < \infty$
4. $\sup\limits_{n,v} \left|\tilde{u}_{nv}^{(k)}\right| < \infty$
5. $\sup\limits_{m,v} \left|\sum_{r=v}^{m} u_{nr}\widetilde{t}_{rv}^{-1}\right| < \infty,$ for all $n \in \mathbb{N}$
6. $\sup\limits_{m} \sum_{v=0}^{\infty} \left|\sum_{r=v}^{m} u_{nr}\widetilde{t}_{rv}^{-1}\right|^{k^*} < \infty,$ for all $n \in \mathbb{N}$
7. $\sum_{r=v}^{\infty} u_{nr}\widetilde{t}_{rv}^{-1}$ exists for all $v, n \in \mathbb{N}$

Table 1. Characterizations of the matrix class from $|T_\varphi|_k$ to $\{l_\infty, c_0, c\}$ $(1 < k < \infty)$.

From to	c	c_0	l_∞
$\lvert T_\varphi\rvert_k$	1,3,6,7	2,3,6,7	3,6,7
$\lvert T_\varphi\rvert$	1,4,5,7	2,4,5,7	4,5,7

By Theorem 3.4, we have the followings.

Theorem 3.5 *Let* $1 < k < \infty$ *and* U *be an infinite matrix. Then, Table 1 gives us necessary and sufficient conditions for* $U \in (|T_\varphi|_k, \Lambda)$ *or* $U \in (|T_\varphi|, \Lambda)$*, where* $\Lambda \in \{c, c_0, l_\infty\}$.

Table 2. Characterizations of the matrix class from $|T_\varphi|_k$ to $\{c_s, b_s\}$.

From to	c_s	b_s
$\lvert T_\varphi\rvert_k$	1,3,6,7	3,6,7
$\lvert T_\varphi\rvert$	1,4,5,7	4,5,7

Take the matrix $D = (d_{nj})$ as

$$d_{nj} = \begin{cases} 1, & 0 \leq j \leq n \\ 0, & j > n. \end{cases}$$

Then, since $b_s = \{l_\infty\}_D$ and $c_s = \{c\}_D$, the matrix classes $\left(|T_\varphi|_k, \Lambda\right)$ can be characterized as follows, where $\Lambda \in \{c_s, b_s\}$ with Lemma 1.3.

Corollary 3.6 *Put* $u(n, v) = \sum_{j=0}^{n} u_{jv}$ *instead of* u_{nv} *in the above conditions. Then, Table 2 gives us necessary and sufficient conditions for* $U \in (|T_\varphi|_k, \Lambda)$ *or* $U \in (|T_\varphi|, \Lambda)$*, where* $\Lambda \in \{c_s, b_s\}$.

Theorem 3.7 *Let* $1 < k < \infty$ *and* $\Lambda \in \{c_0, c, l_\infty\}$. *Then, for* $U \in \left(|T_\varphi|_k, \Lambda\right)$,

$$\|L_U\| = \sup_n \left\|\tilde{U}_n^{(k)}\right\|_{l_{k^*}} = \sup_n \left(\sum_{j=0}^{\infty} \left|\tilde{u}_{nj}^{(k)}\right|^{k^*}\right)^{1/k^*},$$

and, for $U \in (|T_\varphi|, \Lambda)$,

$$\|L_U\| = \sup_n \left\|\tilde{U}_n^{(1)}\right\|_{l_\infty} = \sup_{n,j} \left|\tilde{u}_{nj}^{(1)}\right|.$$

Proof The proof of the theorem is obtained from Lemma 1.6 and Lemma 3.2. □

Theorem 3.8 *Let* $1 < k < \infty$ *and* U *be an infinite matrix. Then,*

(*a*) *If* $U \in \left(|T_\varphi|_k, c_0\right)$, *then*

$$\|L_U\|_\chi = \limsup_{n\to\infty} \left\|\tilde{U}_n^{(k)}\right\|_{l_{k^*}} = \limsup_{n\to\infty} \left(\sum_{j=0}^{\infty} \left|\tilde{u}_{nj}^{(k)}\right|^{k^*}\right)^{1/k^*},$$

and

$$L_U \text{ is compact if and only if } \limsup_{n\to\infty} \sum_{j=0}^{\infty} \left|\tilde{u}_{nj}^{(k)}\right|^{k^*} = 0.$$

(*b*) *If* $U \in \left(|T_\varphi|_k, c\right)$, *then*

$$\frac{1}{2}\limsup_{n\to\infty} \left(\sum_{j=0}^{\infty} \left|\tilde{u}_{nj}^{(k)} - \alpha_j\right|^{k^*}\right)^{1/k^*} \leq \|L_U\|_\chi \leq \limsup_{n\to\infty} \left(\sum_{j=0}^{\infty} \left|\tilde{u}_{nj}^{(k)} - \alpha_j\right|^{k^*}\right)^{1/k^*}$$

and L_U *is compact if and only if* $\limsup_{n\to\infty}\sum_{j=0}^{\infty}\left|\tilde{u}_{nj}^{(k)}-\alpha_j\right|^{k^*}=0$, *where* $\alpha_j=\lim_{n\to\infty}\tilde{u}_{nj}^{(k)}$, *for all* $j\in\mathbb{N}$.

(c) *If* $U\in\left(\left|T_\varphi\right|_k, l_\infty\right)$, *then*

$$0\leq\|L_U\|_\chi\leq\limsup_{n\to\infty}\left(\sum_{j=0}^{\infty}\left|\tilde{u}_{nj}^{(k)}\right|^{k^*}\right)^{1/k^*},$$

and if $\limsup_{n\to\infty}\sum_{j=0}^{\infty}\left|\tilde{u}_{nj}^{(k)}\right|^{k^*}=0$, *then* L_U *is compact.*

Proof To avoid repetition, only the proof of the part (b) is given and the proofs of the others are left to the reader.

(b) Let $U\in\left(\left|T_\varphi\right|_k, c\right)$. To compute the Hausdorff measure of noncompactness of L_U, take the unit sphere $S_{\left|T_\varphi\right|_k}$ in the space $\left|T_\varphi\right|_k$. It is written from Lemma 2.1 that

$$\|L_U\|_\chi=\chi(US_{\left|T_\varphi\right|_k}).$$

Since $\left|T_\varphi\right|_k\cong l_k$, $U\in\left(\left|T_\varphi\right|_k, c\right)$ if and only if $\tilde{U}^{(k)}\in(l_k,c)$, and so

$$\begin{aligned}\|L_U\|_\chi &= \chi(US_{\left|T_\varphi\right|_k})=\chi(\tilde{U}^{(k)}\tilde{T}S_{\left|T_\varphi\right|_k})\\ &= \|L_{\tilde{U}^{(k)}}\|_\chi\end{aligned}$$

which implies, by Lemma 2.4,

$$\frac{1}{2}\lim_{r\to\infty}\left(\sup_{n\geq r}\left\|\tilde{U}_n^{(k)}-\alpha\right\|_{l_k}^{*}\right)\leq\|L_U\|_\chi\leq\lim_{r\to\infty}\left(\sup_{n\geq r}\left\|\tilde{U}_n^{(k)}-\alpha\right\|_{l_k}^{*}\right),\tag{3.4}$$

where $\alpha_j=\lim_{n\to\infty}\tilde{u}_{nj}^{(k)}$, for all $j\geq 0$.

By Lemma 1.5, $\left\|\tilde{U}_n^{(k)}-\alpha\right\|_{l_k}^{*}=\left\|\tilde{U}_n^{(k)}-\alpha\right\|_{l_{k^*}}$. The last equality completes the first part of the proof of (b) with (3.4). Also, the compactness of L_U is immediate by Lemma 2.1, which completes the proof of (b). □

Also, by following the above lines, we have the following theorems.

Theorem 3.9 (a) *If* $U\in\left(\left|T_\varphi\right|, c_0\right)$. *Then*

$$\|L_U\|_\chi=\limsup_{n\to\infty}\left\|\tilde{U}_n^{(1)}\right\|_{l_\infty}=\limsup_{n\to\infty}\sup_j\left|\tilde{u}_{nj}^{(1)}\right|,$$

and

$$L_U \text{ is compact if and only if } \limsup_{n\to\infty}\sup_j\left|\tilde{u}_{nj}^{(1)}\right|=0.$$

(b) *If* $U\in\left(\left|T_\varphi\right|, c\right)$, *then*

$$\tfrac{1}{2}\limsup_{n\to\infty}\sup_j\left|\tilde{u}_{nj}^{(1)}-\alpha_j\right|\leq\|L_U\|_\chi\leq\limsup_{n\to\infty}\sup_j\left|\tilde{u}_{nj}^{(1)}-\alpha_j\right|$$

and

L_U *is compact if and only if* $\limsup_{n\to\infty} \sup_j \left| \tilde{u}^{(1)}_{nj} - \alpha_j \right| = 0$ *where* $\alpha_j = \lim_{n\to\infty} \tilde{u}^{(1)}_{nj}$, *for all* $j \in \mathbb{N}$.

(c) *If* $U \in (|T_\varphi|, l_\infty)$, *then*

$$0 \le \|L_u\|_\chi \le \limsup_{n\to\infty} \sup_j \left| \tilde{u}^{(1)}_{nj} \right|,$$

and

$$L_U \text{ is compact if } \limsup_{n\to\infty} \sup_j \left| \tilde{u}^{(1)}_{nj} \right| = 0.$$

4. Applications

In this section, we give some consequences and applications of our theorems. For simplicity of notation, in what follows, we use the following:

$$\sigma_{nv} = \Delta u_{nv} \frac{P_v}{p_v} + u_{n,v+1}, \quad \eta_{iv} = \sum_{j=v}^{i} C_{i-j} P_j, \quad \Omega_{iv} = \frac{A^{-\lambda-1}_{i-v}}{i A^{\mu}_i} A^{\lambda+\mu}_v.$$

If we take the weighted mean matrix instead of $T = (t_{nv})$, i.e. $t_{nv} = p_v/P_n$ for $0 \le v \le n$, otherwise $t_{nv} = 0$, then, it is clearly seen that the space $|T_\varphi|_k$ is reduced to

$$\left|\overline{N}^{\varphi}_p\right|_k = \left\{ a = (a_v) : \sum_{n=1}^{\infty} \varphi_n^{k-1} \left| \frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n} P_{v-1} a_v \right|^k < \infty \right\},$$

which is studied by Sarıgöl and Mohapatra [20] and Sarıgöl [24]. Thus, the following results follow from Theorem 3.5, Theorem 3.7 and Theorem 3.8.

Corollary 4.1 *Let* $1 < k < \infty$. *Then,*

a-) $U \in (\left|\bar{N}^{\varphi}_p\right|_k, c_0)$ *iff, for all* j,

$$\lim_{n\to\infty} {\varphi_v}^{-1/k^*} \sigma_{nv} = 0 \text{ for all } v \tag{4.1}$$

$$\sup_n \sum_{v=0}^{\infty} \frac{1}{\varphi_v} |\sigma_{nv}|^{k^*} < \infty \tag{4.2}$$

$$\sup_m \left\{ \frac{1}{\varphi_v} \sum_{v=0}^{m-1} |\sigma_{jv}|^{k^*} + \frac{1}{\varphi_m} \left| u_{jm} \frac{P_m}{p_m} \right|^{k^*} \right\} < \infty \tag{4.3}$$

hold. Also,

$$\|L_U\|_\chi = \limsup_{n\to\infty} \left(\sum_{v=0}^{\infty} \left| {\varphi_v}^{-1/k^*} \sigma_{nv} \right|^{k^*} \right)^{1/k^*}$$

and $U \in \mathcal{C}(\left|\bar{N}^{\varphi}_p\right|_k, c_0)$ *if and only if* $\limsup_{n\to\infty} \sum_{v=0}^{\infty} \left| {\varphi_v}^{-1/k^*} \sigma_{nv} \right|^{k^*} = 0.$

b-) $U \in (|\bar{N}_p^\varphi|_k, c)$ *iff* (4.2), (4.3) *hold and*

$$\lim_{n\to\infty} {\varphi_v}^{-1/k^*}\sigma_{nv} \text{ exists for all } v.$$

Moreover,

$$\frac{1}{2}\limsup_{n\to\infty}\left(\sum_{v=0}^{\infty}\left|{\varphi_v}^{\frac{-1}{k^*}}\sigma_{nv}-\alpha_v\right|^{k^*}\right)^{1/k^*} \le \|L_U\|_\chi \le \limsup_{n\to\infty}\left(\sum_{v=0}^{\infty}\left|{\varphi_v}^{\frac{-1}{k^*}}\sigma_{nv}-\alpha_v\right|^{k^*}\right)^{1/k^*},$$

and $U \in \mathcal{C}(|\bar{N}_p^\varphi|_k, c_0)$ *if and only if* $\limsup_{n\to\infty}\sum_{v=0}^{\infty}\left|{\varphi_v}^{-1/k^*}\sigma_{nv}-\alpha_v\right|^{k^*}=0$ *where* $\alpha_v=\lim_{n\to\infty}{\varphi_v}^{\frac{-1}{k^*}}\sigma_{nv}$.

c-) $U \in (|\bar{N}_p^\varphi|_k, l_\infty)$ *iff* (4.2), (4.3) *hold and,*

$$0 \le \|L_U\|_\chi \le \limsup_{n\to\infty}\left(\sum_{v=0}^{\infty}\left|{\varphi_v}^{-1/k^*}\sigma_{nv}\right|^{k^*}\right)^{1/k^*}$$

also, if $\limsup_{n\to\infty}\sum_{v=0}^{\infty}\left|{\varphi_v}^{-1/k^*}\sigma_{nv}\right|^{k^*}=0$, *then* $U \in \mathcal{C}(|\bar{N}_p^\varphi|_k, l_\infty)$.

Proof Prove only the part (a). Take the weighted mean matrix instead of T in Theorem 3.5, Theorem 3.8. Then, the conditions 1,3,6,7 (see Table 1) are reduced to the conditions (4.1),(4.2) and (4.3), the measure noncompactness of L_A is easily obtained since

$$\frac{P_v}{p_v}u_{nv}-\frac{P_{v-1}}{p_v}u_{n,v+1}=\Delta u_{nv}\frac{P_v}{p_v}+u_{n,v+1}=\sigma_{nv}$$

where $\Delta u_{nv}=u_{nv}-u_{n,v+1}$ when $U \in (|\bar{N}_p^\varphi|_k, c_0)$. □

Corollary 4.2 *Let* $1<k<\infty$ *and* $\Lambda \in \{c_0, c, l_\infty\}$. *Then, for* $U \in \left(|\bar{N}_p^\varphi|_k, \Lambda\right)$,

$$\|L_U\| = \sup_n\left(\sum_{v=0}^{\infty}\left|{\varphi_v}^{-1/k^*}\sigma_{nv}\right|^{k^*}\right)^{1/k^*},$$

and, for $U \in \left(|\bar{N}_p^\varphi|, \Lambda\right)$,

$$\|L_U\| = \sup_{n,v}|\sigma_{nv}|.$$

Also, if we take the Nörlund matrix instead of T, i.e.

$$t_{nv}=\begin{cases} p_{n-v}/P_n, & 0\le v\le n \\ 0, & v>n, \end{cases}$$

then the space $|T_\varphi|_k$ is reduced to the absolute Nörlund series space

$$|N_p^\varphi|_k=\left\{a\in\omega : \sum_{n=1}^{\infty}\varphi_n^{k-1}\left|\sum_{v=1}^{n}\left(\frac{P_{n-v}}{P_n}-\frac{P_{n-v-1}}{P_{n-1}}\right)a_v\right|^k<\infty\right\}$$

studied by Hazar and Sarıgöl [10]. So Theorem 3.5, Theorem 3.7 and Theorem 3.8 are reduced to the following results.

Corollary 4.3 *Assume that* $1 < k < \infty$, (c_n) *is a sequence satisfying* $c_0P_0 = 1, c_1P_1 + c_2P_2 + ...c_nP_n = 0$ *and* $C_n = c_n - c_{n-1}$. *Then,*

a) $U \in (\left|N_p^{\varphi}\right|_k, c_0)$ *iff*

$$\lim_{n\to\infty} \varphi_v{}^{-1/k^*} \sum_{i=v}^{\infty} \eta_{iv} u_{ni} = 0 \text{ for all } v \tag{4.4}$$

$$\sup_n \sum_{v=0}^{\infty} \frac{1}{\varphi_v} \left| \sum_{i=v}^{\infty} \eta_{iv} u_{ni} \right|^{k^*} < \infty \tag{4.5}$$

$$\sum_{i=v}^{\infty} \eta_{iv} u_{ni} \text{ exists for all } n, v \tag{4.6}$$

$$\sup_m \left\{ \sum_{v=0}^{m} \frac{1}{\varphi_v} \left| \sum_{i=v}^{m} \eta_{iv} u_{ni} \right|^{k^*} \right\} < \infty, \text{ for all } n. \tag{4.7}$$

Also,

$$\|U\|_{\chi} = \limsup_{n\to\infty} \left(\sum_{v=0}^{\infty} \frac{1}{\varphi_v} \left| \sum_{i=v}^{\infty} \eta_{iv} u_{ni} \right|^{k^*} \right)^{1/k^*}$$

and $U \in \mathcal{C}(\left|N_p^{\varphi}\right|_k, c_0)$ *if and ony if* $\limsup_{n\to\infty} \sum_{v=0}^{\infty} \frac{1}{\varphi_v} \left| \sum_{i=v}^{\infty} \eta_{iv} u_{ni} \right|^{k^*} = 0$.

b) $U \in (\left|N_p^{\varphi}\right|_k, c)$ *iff* (4.5), (4.6), (4.7) *hold and*

$$\lim_{n\to\infty} \varphi_v{}^{-1/k^*} \sum_{i=v}^{\infty} \eta_{iv} u_{ni} \text{ exists for all } v.$$

Further,

$$\frac{1}{2} \limsup_{n\to\infty} \left(\sum_{v=0}^{\infty} \left| \varphi_v{}^{\frac{-1}{k^*}} \sum_{i=v}^{\infty} \eta_{iv} u_{ni} - \alpha_v \right|^{k^*} \right)^{1/k^*} \leq \|L_U\|_{\chi} \leq \limsup_{n\to\infty} \left(\sum_{v=0}^{\infty} \left| \varphi_v{}^{\frac{-1}{k^*}} \sum_{i=v}^{\infty} \eta_{iv} u_{ni} - \alpha_v \right|^{k^*} \right)^{1/k^*}$$

and $U \in \mathcal{C}(\left|N_p^{\varphi}\right|_k, c_0)$ *if and only if* $\limsup_{n\to\infty} \sum_{v=0}^{\infty} \left| \varphi_v{}^{\frac{-1}{k^*}} \sum_{i=v}^{\infty} \eta_{iv} u_{ni} - \alpha_v \right|^{k^*} = 0$, *where* $\alpha_v = \lim_{n\to\infty} \varphi_v{}^{\frac{-1}{k^*}} \sum_{i=v}^{\infty} \eta_{iv} u_{ni}$.

c) $U \in (\left|N_p^{\varphi}\right|_k, l_\infty)$ *iff* (4.5), (4.6), (4.7) *hold and*

$$0 \leq \|L_U\|_{\chi} \leq \limsup_{n\to\infty} \left(\sum_{v=0}^{\infty} \left| \varphi_v{}^{\frac{-1}{k^*}} \sum_{i=v}^{\infty} \eta_{iv} u_{ni} \right|^{k^*} \right)^{1/k^*}.$$

Moreover, if $\lim\limits_{n\to\infty}\sup \sum\limits_{v=0}^{\infty} \left| \varphi_v^{\frac{-1}{k^*}} \sum\limits_{i=v}^{\infty} \eta_{iv} u_{ni} \right|^{k^*} = 0$, *then* $U \in \mathcal{C}(\left|N_p^{\varphi}\right|_k, l_\infty)$.

Corollary 4.4 *Let* $1 < k < \infty$ *and* $X \in \{c_0, c, l_\infty\}$. *Then, for* $U \in \left(\left|N_p^{\varphi}\right|_k, X\right)$,

$$\|L_U\| = \sup_n \left(\sum_{v=0}^{\infty} \frac{1}{\varphi_v} \left| \sum_{i=v}^{\infty} \eta_{iv} u_{ni} \right|^{k^*} \right)^{1/k^*}$$

and, for $U \in \left(\left|N_p^{\varphi}\right|, X\right)$,

$$\|L_U\| = \sup_{n,v} \left| \sum_{i=v}^{\infty} \eta_{iv} u_{ni} \right|.$$

Now, consider the generalized Cesaro matrix $T^{\lambda,\mu} = (t_{nv}^{\lambda,\mu})$ defined by

$$t_{nv}^{\lambda,\mu} = \begin{cases} 1, & n, v = 0 \\ n^{1/k^*} \frac{A_{n-v}^{\lambda-1} A_v^{\mu}}{A_n^{\lambda+\mu}}, & 1 \le v \le n \\ 0, & v > n \end{cases}$$

where $\lambda + \mu \neq -1, -2$,

$$A_n^{\lambda} = \frac{(\lambda+1)(\lambda+2)...(\lambda+n)}{n!},$$

$$A_0^{\lambda} = 1, \ \ A_{-n}^{\lambda} = 0, \ \ n \ge 1.$$

If we take $T^{\lambda,\mu} = (t_{nv}^{\lambda,\mu})$ instead of the matrix T with $\varphi_n = n$ for all n, then $|T_\varphi|_k = |C_{\lambda,\mu}|_k$ and so the following results given by Güleç [8] are immediately obtained:

Corollary 4.5 *Let* $1 < k < \infty$. *Then,*

a) $U \in (|C_{\lambda,\mu}|_k, c_0)$ *iff*

$$\lim_{n\to\infty} \sum_{i=v}^{\infty} v^{1/k} \Omega_{iv} u_{ni} = 0 \text{ for all } v \tag{4.8}$$

$$\sup_n \sum_{v=1}^{\infty} \left| \sum_{i=v}^{\infty} v^{1/k} \Omega_{iv} u_{ni} \right|^{k^*} < \infty \tag{4.9}$$

$$\sum_{i=v}^{\infty} v^{1/k} \Omega_{iv} u_{ni} \text{ exists for all } n, v \tag{4.10}$$

$$\sup_m \sum_{v=1}^{m} \left| \sum_{i=v}^{m} v^{1/k} \Omega_{iv} u_{ni} \right|^{k^*} < \infty, \text{ for all } n \tag{4.11}$$

hold, and

$$\|U\|_\chi = \limsup_{n\to\infty} \left(\sum_{v=1}^{\infty} \left| \sum_{i=v}^{\infty} v^{1/k} \Omega_{iv} u_{ni} \right|^{k^*} \right)^{1/k^*}.$$

Also, $U \in \mathcal{C}(|C_{\lambda,\mu}|_k, c_0)$ *if and only if* $\limsup_{n\to\infty} \sum_{v=0}^{\infty} \left| \sum_{i=v}^{\infty} v^{1/k} \Omega_{iv} u_{ni} \right|^{k^*} = 0.$

b) $U \in (|C_{\lambda,\mu}|_k, c)$ *iff* (4.9), (4.10), (4.11)

$$\lim_{n\to\infty} \sum_{i=v}^{\infty} v^{1/k} \Omega_{iv} u_{ni} \text{ exists for all } v.$$

Further,

$$\tfrac{1}{2} \limsup_{n\to\infty} \left(\sum_{v=1}^{\infty} \left| \sum_{i=v}^{\infty} v^{1/k} \Omega_{iv} u_{ni} - \alpha_v \right|^{k^*} \right)^{1/k^*} \le \|L_U\|_\chi \le \limsup_{n\to\infty} \left(\sum_{v=1}^{\infty} \left| \sum_{i=v}^{\infty} v^{1/k} \Omega_{iv} u_{ni} - \alpha_v \right|^{k^*} \right)^{1/k^*},$$

and $U \in \mathcal{C}(|C_{\lambda,\mu}|_k, c)$ *if and only if* $\limsup_{n\to\infty} \sum_{v=1}^{\infty} \left| \sum_{i=v}^{\infty} v^{1/k} \Omega_{iv} u_{ni} - \alpha_v \right|^{k^*} = 0$*, where* $\alpha_v = \lim_{n\to\infty} \sum_{i=v}^{\infty} v^{1/k} \Omega_{iv} u_{ni}$.

c) $U \in (|C_{\lambda,\mu}|_k, l_\infty)$ *iff* (4.9),(4.10), (4.11) *hold and*

$$0 \le \|U\|_\chi \le \limsup_{n\to\infty} \left(\sum_{v=1}^{\infty} \left| \sum_{i=v}^{\infty} v^{1/k} \Omega_{iv} u_{ni} \right|^{k^*} \right)^{1/k^*}.$$

Also, if $\limsup_{n\to\infty} \sum_{v=1}^{\infty} \left| \sum_{i=v}^{\infty} v^{1/k} \Omega_{iv} u_{ni} \right|^{k^*} = 0$*, then it* U *is a compact operator.*

Corollary 4.6 *Let* $1 < k < \infty$ *and* $X \in \{c_0, c, l_\infty\}$*. Then, if* $U \in \left(|C_{\lambda,\mu}|_k, X\right)$*, then*

$$\|L_U\| = \sup_n \left(\sum_{v=1}^{\infty} \left| \sum_{i=v}^{\infty} v^{1/k} \Omega_{iv} u_{ni} \right|^{k^*} \right)^{1/k^*},$$

and $U \in (|C_{\lambda,\mu}|, \Lambda)$*, then*

$$\|L_U\| = \sup_{n,v} \left| \sum_{i=v}^{\infty} \Omega_{iv} u_{ni} \right|.$$

References

[1] Başarır M, Kara EE. On some difference sequence spaces of weighted means and compact operators, Annals of Functional Analysis. 2011; 2 : 114-129.

[2] Djolovic I, Malkowsky E. Matrix transformations and compact operators on some new mth-order difference sequences. Applied Mathematics and Computation 2008; 198 (2): 700-714.

[3] Gökçe F, Sarıgöl MA. Some matrix and compact operators of the absolute Fibonacci series spaces. Kragujevac Journal of Mathematics. 2020; 44 (2): 273-286.

[4] Gökçe F, Sarıgöl MA. Series spaces derived from absolute Fibonacci summability and matrix transformations. Bollettino dell'Unione Matematica Italiana 2020; 13: 29-38.

[5] Gökçe F, Sarıgöl MA. On absolute Euler spaces and related matrix operators.Proceedings of the National Academy of Sciences, India. Section A. Physical Sciences 2020; 90 (5): 769-775.

[6] Gökçe F, Sarıgöl MA. Generalization of the absolute Cesàro space and some matrix transformations. Numerical Functional Analysis and Optimization 2019; 40:1039-1052.

[7] Gökçe F, Sarıgöl MA. Generalization of the space $l(p)$ derived by absolute Euler summability and matrix operators. Journal of Inequalities and Applications 2018; 2018: 133.

[8] Hazar Güleç GC. Compact matrix operators on absolute Cesaro spaces. Numerical Functional Analysis and Optimization 2020; 41 (1) :1-15.

[9] Hazar Güleç GC. Characterization of some classes of compact and matrix operators on the sequence spaces of Cesàro means. Operator and Matrices. 2019; 13 (3) : 809-822.

[10] Hazar Güleç G.C, Sarıgöl MA. On absolute Nörlund spaces and matrix operators. Acta Mathematica Sinica, English Series 2018; 34 (5): 812-826.

[11] Ilkhan Kara M, Kara EE. Matrix transformations and compact operators on Catalan sequence spaces. Journal of Mathematical Analysis and Applications. 2021; 498 (1): 124925.

[12] Ilkhan M. A new conservative matrix derived by Catalan numbers and its matrix domain in the spaces c and c_0. Linear and Multilinear Algebra. 2020; 68 (2): 417-434.

[13] Ilkhan M, Hazar Güleç GC. A study on absolute Euler totient series space and certain matrix transformations, Mugla Journal of Science and Technology. 2020; 6 (1): 112-119.

[14] Ilkhan M. Matrix Domain of a Regular Matrix Derived by Euler Totient Function in the Spaces c_0 and c. Mediterranean Journal of Mathematics 2020; 17 (1) : 1-21.

[15] Jarrah AM, Malkowsky E. Ordinary absolute and strong summability and matrix transformations. Filomat 2003; 17 : 59-78.

[16] Kara EE, Başarır M, On compact operators and some Euler $B^{(m)}$ difference sequence spaces. Journal of Mathematical Analysis and Applications. 2011; 379 : 499–511.

[17] Malkowsky E. Compact matrix operators between some $BK-$ spaces, in: M. Mursaleen (Ed.), Modern Methods of Analysis and Its Applications. Anamaya Publ., New Delhi, 2010; 86-120.

[18] Malkowsky E, Rakocevic V. On matrix domains of triangles. Applied Mathematics and Computation 2007; 189 : 1146-1163.

[19] Malkowsky E, Rakocevic V. An introduction into the theory of sequence space and measures of noncompactness. Zbornik Radova. (Beograd) 2000; 9: 143-234.

[20] Mohapatra RN., Sarıgöl M.A. On matrix operators on the series space $|\bar{N}_p^{\theta}|_k$. Ukrains'kyi Matematychnyi Zhurnal 2017; 69 (11) : 1524-1533.

[21] Mursaleen M, Noman AK. Compactness of matrix operators on some new difference sequence spaces.Linear Algebra and its Applications 2012; 436 : 41-52.

[22] Mursaleen M, Noman AK. Compactness by the Hausdorff measure of noncompactness. Nonlinear Analysis: Theory, Methods & Applications. 2010; 73 (8) : 2541-2557.

[23] Sarıgöl MA, Agarwal R. Banach spaces of absolutely k-summable series. Georgian Mathematical Journal 2021.

[24] Sarıgöl MA. Matrix transformations on fields of absolute weighted mean summability. Studia Scientiarum Mathematicarum Hungarica 2011; 48 (3) : 331-341.

[25] Sarıgöl MA. On the local properties of factored Fourier series. Applied Mathematics and Computation. 2010; 216 : 3386-3390.

[26] Stieglitz M, Tietz H. Matrix transformationen von Folgenraumen Eine Ergebnisübersicht. Mathematische Zeitschrift 1977; 154 : 1-16.

[27] Sulaiman WT. On summability factors of infinite series. Proceedings of the American Mathematical Society 1992; 115 : 313- 317.

[28] Wilansky A. Summability Through Functional Analysis. Mathematics Studies 85. North Holland , Amsterdam, 1984.