Turkish Journal of Mathematics

http://journals.tubitak.gov.tr/math/

Research Article

Turk J Math
(2022) 46: 1113 – 1118
© TÜBİTAK
doi:10.3906/mat-2201-80

On the paper "Generalized hyperideals in locally associative left almost semihypergroups"

Niovi KEHAYOPULU*
TBAE, The Scientific and Technological Research Council of Turkey, Turkey

Received: 16.01.2022 • **Accepted/Published Online:** 23.02.2022 • **Final Version:** 11.03.2022

Abstract: This note is written to show that the definition of the $\mathcal{LA}$-semihypergroup by V. Amjad, K. Hila and F. Yousafzai "Generalized hyperideals in locally associative left almost semihypergroups, New York J. Math. 2014" should be corrected and that it is not enough to replace the multiplication "$\cdot$" of an $\mathcal{LA}$-semigroup by the hyperoperation "$\circ$" to pass from an $\mathcal{LA}$-semigroup to an $\mathcal{LA}$-semihypergroup. The two examples of the paper based on the definition of the $\mathcal{LA}$-semihypergroup are wrong that is a further indication that this definition needs correction. According to the last section of the paper, the paper generalizes the results of an $\mathcal{LA}$-semigroup by M. Akram, N. Yaqoob and M. Khan "On (m,n)-ideals of left almost semigroups, Appl. Math. Sci. (Ruse) 2013" while the paper duplicates, without citation, the section 4 of the paper by W. Khan, F. Yousafzai, W. Guo and M. Khan "On (m,n)-ideals of left almost semigroups, J. Semigroup Theory Appl. 2014" with the usual change of "$\cdot$" to "$\circ$".

Key words: Left almost semihypergroup, Abel-Grassmann's groupoid

According to the introduction of [2], "a left almost semigroup ($\mathcal{LA}$-semigroup) is a groupoid S whose elements satisfy the following left invertive law $(ab)c = (cb)a$ for all $a,b,c \in S$" ("left invertive" according to the bibliography —should be possibly changed to "left invertible", "left inverted" or "left inverse"). The concept "$\mathcal{LA}$-semigroup" gives the impression that this is a semigroup having the $\mathcal{LA}$-property. The fact that each commutative groupoid is a semigroup is no excuse for being called "almost semigroup". Many structures could be called almost semigroups if we adopt this definition. This structure being a groupoid and not a semigroup, the term $\mathcal{LA}$-semigroup should be replaced by $\mathcal{AG}$-groupoid also called Abel-Grassmann's groupoid (see, for example, [3, 12, 13, 17, 18]). Some authors use the concept "nonassociative ordered semigroup" which is certainly wrong as an ordered semigroup cannot be nonassociative. Phrases like "this is a structure midway between a groupoid and a commutative semigroup" make the very simple definition incomprehensible.

Again by the introduction, an $\mathcal{LA}$-semigroup is a generalization of a semigroup and it has vast applications in semigroups as well as in other branches of mathematics [2, p. 1064, l. 12–14]. That means that every semigroup is an $\mathcal{LA}$-semigroup. How is it possible? In fact, a semigroup is a generalization of a commutative $\mathcal{LA}$-semigroup.

The Definition of the $\mathcal{LA}$-semigroup in [2]: A map $\circ : H \times H \to \mathcal{P}^*(H)$ is called hyperoperation or join operation on the set H, where H is a nonempty set and $\mathcal{P}^*(H) = \mathcal{P}(H)\backslash\emptyset$ denotes the set of all nonempty subsets of H. A hypergroupoid is a set H together with a (binary) hyperoperation. Let A and B be two

*Correspondence: niovi.kehayopulu-tbae@tubitak.gov.tr
2010 *AMS Mathematics Subject Classification:* 20N99

nonempty subsets of H, then we denote $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$, $a \circ A = \{a\} \circ A$ and $a \circ B = \{a\} \circ B$. A hypergroupoid $(H, \circ)$ is called an $\mathcal{LA}$-semihypergroup if $(x \circ y) \circ z = (z \circ y) \circ x$ for all $x, y, z \in H$.

We cannot write $(x \circ y) \circ z$ as $x \circ y$ is a subset of H, z an element of H and $\circ$ is an "operation" between two elements of H called hyperoperation as it assigns to each couple of elements a, b of H a nonempty subset (instead of an element) of H. It could also be called operation. Even if we identify the z by the singleton $\{z\}$, we cannot write $(x \circ y) \circ \{z\}$ as $x \circ y$ and $\{z\}$ are sets while $\circ$ operation between elements. We will justify what we say using the two examples in [2]. In addition, we cannot write $A \circ B$, as $\circ$ is an "operation" between elements. We cannot use the same symbol both for elements and sets (different subjects); if we do that, a great confusion arises (see also [8]). To use a single symbol to represent different operations could possibly be a fruitful idea when implementing algorithms in a computer using a table of multiplication, but definitely not suitable for a research article on pure mathematics.

We will use the terms $\mathcal{AG}$-groupoid, $\mathcal{AG}$-hypergroupoid instead of the $\mathcal{LA}$-semigroup, $\mathcal{LA}$-semihypergroup given in [2].

This is the Example 4 in [2]: The set $H = \{a, b, c, d, e\}$ with the hyperoperation $\circ$ given by Table 1, is an $\mathcal{AG}$-hypergroupoid.

Table 1. The hyperoperation of Example 4 in [2].

$\circ$	a	b	c	d	e
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a,e\}$	$\{a,e\}$	$\{a,c\}$	$\{a,e\}$
c	$\{a\}$	$\{a,e\}$	$\{a,e\}$	$\{a,b\}$	$\{a,e\}$
d	$\{a\}$	$\{b\}$	$\{c\}$	$\{d\}$	$\{e\}$
e	$\{a\}$	$\{a,e\}$	$\{a,e\}$	$\{a,e\}$	$\{a,e\}$

We wrote $\{a\}$ in the table instead of the a in [2]. According to the definition of the $\mathcal{AG}$-hypergroupoid in [2], to show that this is an $\mathcal{AG}$-hypergroupoid we have to show that $(x \circ y) \circ z = (z \circ y) \circ x$ for every $x, y, z \in H$. This being so, the $(a \circ b) \circ d$, for example, should be equal to $(d \circ b) \circ a$. We have $(a \circ b) \circ d = \{a\} \circ d$ and $(d \circ b) \circ a = \{b\} \circ a$, while both the $\{a\} \circ d$ and the $\{b\} \circ a$ are without meaning.

Let us give now the correct definition to see the difference.

Definition 1 *Let H be a nonempty set and $\circ$ an hyperoperation on H; that is a mapping of $H \times H$ into the set $\mathcal{P}^*(H)$ of nonempty subsets of H. For two nonempty subsets A and B of H we denote by $*$ the operation on $\mathcal{P}^*(H)$ defined by*

$$* : \mathcal{P}^*(H) \times \mathcal{P}^*(H) \to \mathcal{P}^*(H) \mid (A, B) \to A * B := \bigcup_{a \in A,\, b \in B} a \circ b.$$

Then H is called $\mathcal{AG}$-hypergroupoid if

$$(a \circ b) * \{c\} = (c \circ b) * \{a\} \text{ for all } a, b, c \in H.$$

When is convenient and no confusion is possible, we identify the singleton $\{x\}$ with its element and write, for short, $(a \circ b) * c = (c \circ b) * a$.

As one can easily see, for any a, b we have $\{a\} * \{b\} = a \circ b$ (see, for example [5–7]).

Using this definition, let us check if the Example 4 in [2] (given by Table 1 above) is correct. We have

$$(d \circ d) * \{b\} = \{d\} * \{b\} = d \circ b = \{b\},$$

$$(b \circ d) * \{d\} = \{a, c\} * \{d\} = \bigcup_{x \in \{a,\, c\}} x \circ d = (a \circ d) \cup (c \circ d) = \{a\} \cup \{a, b\} = \{a, b\} \text{ and}$$

$$(d \circ d) * \{b\} \neq (b \circ d) * \{d\}.$$

So the Example 4 in [2] is wrong.

This is the Example 5 in [2]: The set $H = \{a, b, c, d\}$ defined by Table 2 is an $\mathcal{AG}$-hypergroupoid.

Table 2. The hyperoperation of Example 5 in [2].

$\circ$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a, b, c, d\}$	$\{a, b, c\}$	$\{a, b, c\}$
c	$\{a\}$	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$
d	$\{a\}$	$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$

The Example 5 in [2] is also wrong as

$$(c \circ d) * \{b\} = \{a, b, c\} * \{b\} = \bigcup_{x \in \{a,b,c\}} x \circ b = (a \circ b) \cup (b \circ b) \cup (c \circ b) = \{a, b, c, d\},$$

$$(b \circ d) * \{c\} = \{a, b, c\} * \{c\} = \bigcup_{x \in \{a,b,c\}} x \circ c = (a \circ c) \cup (b \circ c) \cup (c \circ c) = \{a, b, c\} \text{ but}$$

$$(c \circ d) * \{b\} \neq (b \circ d) * \{c\}.$$

Also,

$$(b \circ c) * \{d\} = \{a, b, c\} * \{d\} = (a \circ d) \cup (b \circ d) \cup (c \circ d) = \{a, b, c\} \text{ while}$$

$$(d \circ c) * \{b\} = \{a, b, c\} * \{b\} = (a \circ b) \cup (b \circ b) \cup (c \circ b) = \{a, b, c, d\}.$$

It is very difficult (not to say impossible) to check the examples of $\mathcal{AG}$-hypergroupoids given by a table of multiplication by hand. As we see, the examples in [2] are not correct that means nobody has checked (or able to check) if they are correct or not during the refereeing process; so it should be mentioned in the papers the way they have been constructed. On the other hand, such examples can be obtained in a very easy way from corresponding examples on $\mathcal{AG}$-groupoids which is a much simpler structure but still an explanation is needed to check if they are correct.

According to the last section, the results of the paper generalize the result by M. Akram, N. Yaqoob and M. Khan [1] and by Q. Mushtaq and S.M. Yusuf [16]. The results in [1] are different; regarding [16] the only we have is the review MR0596763 by M. Friedberg in MathSciNet and the review Zbl 0445.20033 by P. Pondelicek in Zentralblatt —does not appear to be related to [2].

In what follows, the aim is to show that is not enough to pass from an $\mathcal{AG}$-groupoid to an $\mathcal{AG}$-hypergroupoid by replacing the multiplication "$\cdot$" of the $\mathcal{AG}$-groupoid by the hyperoperation "$\circ$" of the $\mathcal{AG}$-hypergroupoid.

Strange symbols like

$$[(H \circ R) \circ (H \circ R^{m+n-2})] \circ [(H \circ L) \circ (H \circ L^{m+n-2})],$$

$$[\{(a^{nm} \circ H^{m}) \circ a^{mn}\} \circ H] \circ \{(a^{mn} \circ H^{n}) \circ a^{nn}\}$$

have been used throughout the paper (see, for example p. 1068, l.6; p. 1071, l.10). Unless a clear definition of the (m, n)-hyperideal and the (m, n)-regularity, some necessary lemmas, and an explanation how they can be applied to the theorems of the paper, these symbols have no sense. But even in case of a “trivial extension” in which the contexts between the two structures are different only some necessary lemmas and an explanation about the rest of the paper is necessary (see, for example, [4, 9, 10]).

The paper by V. Amjad, K. Hila and F. Yousafzai [2] is an exact copy (word by word) of section 4 of the paper by W. Khan, F. Yousafzai, W. Guo and M. Khan [14], not cited in [2], with the only difference that the multiplication “$\cdot$” in [14] has been replaced by “$\circ$”.
In fact,

Lemma 1 in [2] is the Lemma 7 in [14];

Theorem 1 in [2] is the Theorem 6 in [14];

Theorem 2 in [2] is the Theorem 7 in [14];

Theorem 3 in [2] is the Theorem 8 in [14];

Theorem 4 in [2] is the Theorem 9 in [14];

Corollary 1 in [2] is the Corollary 5 in [14];

Theorem 5 in [2] is the Theorem 10 in [14];

Lemma 3 in [2] is the Lemma 9 in [14];

Corollary 2 in [2] is the Corollary 6 in [14];

Theorem 6 in [2] is the Theorem 11 in [14];

Theorem 7 in [2] is the Theorem 12 in [14];

Theorem 8 in [2] is the Theorem 13 in [14].

Finally, 29 papers have been cited in References while the paper is based only on [14] not cited in References.

On the other hand, the paper by W. Khan, F. Yousafzai and M. Khan [15] is an exact copy (word by word) of the paper by W. Khan, F. Yousafzai, W. Guo and M. Khan [14] with the only difference that 17 papers have been cited in References in [15] instead of 13 in [14] and a little change in Introduction. The results in [14] should be checked as well.

The following theorem is useful for applications.

Theorem 2 *Let H is an $\mathcal{AG}$-hypergroupoid and A, B, C nonempty subset of H. Then we have*

$$(A * B) * C = (C * B) * A.$$

Proof Let $x \in (A * B) * C$. Then $x \in u \circ c$ for some $u \in A * B$, $c \in C$. Since $u \in A * B$, we have $u \in a \circ b$ for some $a \in A$, $b \in B$. Then we have

$$x \in u \circ c = \{u\} * \{c\} \subseteq (a \circ b) * \{c\} = (c \circ b) * \{a\} \subseteq (C * B) * A.$$

Thus we have $(A*B)*C \subseteq (C*B)*A$. If $x \in (C*B)*A$, then $x \in u \circ a$ for some $u \in C*B$, then $u \in c \circ b$ for some $c \in C$, $b \in B$ and $x \in \{u\}*\{a\} \subseteq (c \circ b)*\{a\} = (a \circ b)*\{c\} \subseteq (A*B)*C$ and so $(C*B)*A \subseteq (A*B)*C$ and equality holds. □

Note All the results on hypersemigroups or $\mathcal{AG}$-hypersemigroups obtained using only sets are corollaries of corresponding results on le-semigroups (: lattice ordered semigroups having a greatest element e with respect to the order) (see also [11]).

With my best thanks to the anonymous referee for his/her time to read the paper carefully.

References

[1] Akram M, Yaqoob N, Khan M. On (m,n)-ideals in LA-semigroups. Applied Mathematical Sciences (Ruse) 2013; 7 (41-44): 2187-2191. doi: 10.12988/ams.2013.13195

[2] Amjad V, Hila K, Yousafzai F. Generalized hyperideals in locally associative left almost semihypergroups. New York Journal of Mathematics 2014; 20: 1063-1076. doi: 10.1080/00927872.2010.521932

[3] Dudek WA, Gigon RS. Completely inverse AG^{**}-groupoids. Semigroup Forum 2013; 87 (1): 201-229. doi: 10.1007/s00233-013-9465-z

[4] Kehayopulu N. On ordered Γ-semigroups. Scientiae Mathematicae Japonicae 2010; 71 (2): 179-185.

[5] Kehayopulu N. On hypersemigroups. Pure Mathematics Applications (PU.M.A.) 2015; 25 (2): 151-156. doi: 10.1515/puma-2015-0015

[6] Kehayopulu N. How we pass from semigroups to hypersemigroups. Lobachevskii Journal Mathematics 2018; 39 (1): 121-128. doi: 10.1134/S199508021801016X

[7] Kehayopulu N. Lattice ordered semigroups and hypersemigroups. Turkish Journal Mathematics 2019: 43 (5): 2592-2601. doi: 10.3906/mat-1907-86

[8] Kehayopulu N. Erratum to: "Application of (m,n)-Γ-hyperideals in characterization of LA-Γ-semihypergroups" by Abul Basar. Discussiones Mathematicae - General Algebra and Applications 2020; 40 (1): 135-138. doi: 10.7151/dmgaa.1329

[9] Kehayopulu N. From $\vee e$-semigroups to hypersemigroups. Discussiones Mathematicae - General Algebra and Applications. 2021; 41 (1): 113-126. doi: 10.7151/dmgaa.1353

[10] Kehayopulu N. On ordered Γ-hypersemigroups, minimal bi-ideals, and minimal left ideals. Turkish Journal of Mathematics 2021; 45 (2): 909-918. doi:10.3906/mat-2011-35

[11] Kehayopulu N. Bi-interior ideal elements in $\wedge e$-semigroups. European Journal of Pure and Applied Mathematics 2021; 14 (1): 43-52. doi: 10.29020/nybg.ejpam.v14i1.3931

[12] Khan M, Saima A. On semilattice decomposition of an Abel-Grassmann's groupoid. Acta Mathematica Sinica (English Series) 2012; 28 (7): 1461-1468. doi: 10.1007/s10114-012-0014-6

[13] Khan M, Saima A. An analogy of Clifford decomposition theorem for Abel-Grassmann groupoids. Algebra Colloquium 2014; 21 (2): 347-353. doi: 10.1142/S1005386714000303

[14] Khan W, Yousafzai F, Guo W, Khan M. On (m,n)-ideals of left almost semigroups. Journal of Semigroup Theory and Applications 2014; 1: 1-20.

[15] Khan W, Yousafzai F, Khan M. On generalized ideals of left almost semigroups. European Journal Pure Applied Mathematics 2016; 9 (3): 277-291.

[16] Mushtaq Q, Yusuf SM. On locally associative LA-semigroups. The Journal of Natural Sciences and Mathematics. 1979; 19 (1): 57-62.

[17] Protić PV, Stevanović N. AG-test and some general properties of Abel-Grassmann's groupoids. Pure Mathematics Applications 1995; 6 (4): 371-383.

[18] Qaiser M, Madad K. Semilattice decomposition of locally associative AG**-groupoids. Algebra Colloquium 2009; 16 (1): 17-22. doi: 10.1142/S1005386709000030