

SPECIAL ISSUE

VOLUME | ISSUE | YEAR

46 | 5 | 2022

ISSN 1300-0098 • E-ISSN 1303-6149

TURKISH JOURNAL OF  
**MATHEMATICS**

<http://journals.tubitak.gov.tr/math/>

**Trends in Combinatorial Ring Theory and Applications**

**Guest Editors:**

Prof. Dr. Peter V. DANCHEV

Prof. Dr. Antonio GIAMBRUNO

Prof. Dr. Plamen E. KOSHLUKOV

Prof. Dr. Louis H. ROWEN



Published by the Scientific and  
Technological Research Council of Turkey

**TÜBİTAK**

# TURKISH JOURNAL OF MATHEMATICS

Editor-in-Chief

Adnan **TERCAN**, Hacettepe University, Turkey

Assistant Editor-in-Chief

Aydın **GEZER**, Atatürk University, Turkey

**VOLUME 46 • ISSUE 5 • 2022**

ISSN 1300-0098 • E-ISSN 1303-6149

## SCOPE

- The Turkish Journal of Mathematics is published electronically 6 times a year by the Scientific and Technological Research Council of Turkey (TÜBİTAK).
- Accepts English-language original research manuscripts in the field of mathematics. Review articles are only considered for publication in exceptional circumstances.
- Contribution is open to researchers of all nationalities.

## EDITORIAL BOARD

- Alexander DEGTAREV, Bilkent University, Turkey
- Ali Bülent EKİN, Ankara University, Turkey
- Ali Sinan SERTÖZ, Bilkent University, Turkey
- Anar DOSİ, Middle East Technical University, Turkey
- Arne WINTERHOF, Johann Radon Institute, Austria
- Bayram ŞAHİN, Ege University, Turkey
- Christine BESSENRODT, Leibniz University of Hannover, Germany
- Cornelia-Livia BEJAN, University of "Al. I. Cuza", Romania
- Eduard EMEYANOV, Middle East Technical University, Turkey
- Emrah KILIÇ, TOBB University of Economics & Technology, Turkey
- Enrique ZUAZUA, Friedrich-Alexander Universität, Germany
- Francis BONAHON, University of Southern California, U.S.A.
- Heybetkulu MUSTAFAYEV, Van Yüzüncü Yıl University, Turkey
- Hüseyin DEMİR, Ondokuz Mayıs University, Turkey
- İlham A. ALIEV, Akdeniz University, Turkey
- Jacek DZIOK, University of Rzeszow, Poland
- Janusz SOKOL, University of Rzeszow, Poland
- Kazım İlhan İKEDA, Boğaziçi University, Turkey
- Louis H. ROWEN, Bar-Ilan University, Israel
- Marat AKHMET, Middle East Technical University, Turkey
- Mario KRNIC, University of Zagreb, Croatia
- Mark LAWSON, Heriot-Watt University, United Kingdom
- Martin BOHNER, Missouri University of Science and Technology, U.S.A.
- Nicolae POPA, Institute of Mathematics of the Romanian Academy, Romania
- Pedro A. Guil ASENSIO, University of Murcia, Spain
- Peter DANCHEV, Bulgarian Academy of Sciences, Bulgaria
- Salvador ROMAGUERA, València Polytechnic University, Spain
- Seher Melike AYDOĞAN, Istanbul Technical University, Turkey
- Shyuichi IZUMIYA, Hokkaido University, Japan
- Snezhana HRISTOVA, University of Plovdiv, Bulgaria
- Tamar DATUASHVILI, Ivane Javakhishvili Tbilisi State University, Georgia
- Toma ALBU, Institute of Mathematics of the Romanian Academy, Romania
- Varga K. KALANTAROV, Koç University, Turkey
- W. A. ZUNIGA-GALINDO, University of Texas Rio Grande Valley, USA, Centro de Investigación y de Estudios Avanzados del I. P. N, Mexico
- William WEISS, University of Toronto at Mississauga, Canada
- Zekeriya ARVASI, Osmangazi University, Turkey

## ABSTRACTED

- Science Citation Index Expanded
- Scopus
- Academic Search Complete
- Zentralblatt MATH
- MathSciNet
- Current Index to Statistics

# CONTENTS

Preface: Trends in Combinatorial Ring Theory and Applications .....	i
<b>Peter DANCHEV, Antonio GIAMBRUNO, Plamen KOSHLUKOV, Louis ROWEN</b>	
Tensor products of graded-simple $s$ -modules .....	1619
<b>Yuri BAHTURIN, Abdallah SHIHADDEH</b>	
Capelli identities on algebras with involution or graded involution .....	1644
<b>Francesca Saviella BENANTI, Angela VALENTI</b>	
$GL_n$ -invariant functions on $M_n(\mathcal{G})$ .....	1667
<b>Allan BERELE</b>	
Symmetric polynomials in free associative algebras .....	1674
<b>Silvia BOUMOVA, Vesselin DRENSKY, Deyan DZHUNDREKOV, Martin KASSABOV</b>	
Jordan maps and zero Lie product determined algebras .....	1691
<b>Matej BREŠAR</b>	
On the expansion of the multiplicity-free plethysms $p_2[s_{(a,b)}]$ and $p_2[s_{(1^r, 2^t)}]$ .....	1699
<b>Luisa CARINI</b>	
On Nowicki's conjecture: a survey and a new result .....	1709
<b>Lucio CENTRONE, Andre DUSHIMIRIMANA, Şehmus FINDIK</b>	
Henselian discrete valued stable fields .....	1735
<b>Ivan CHIPCHAKOV</b>	
Bound for the cocharacters of the identities of irreducible representations of $\mathfrak{sl}_2(\mathbb{C})$ .....	1749
<b>Mátyás DOMOKOS</b>	
Invariants of symplectic and orthogonal groups acting on $GL(n, \mathbb{C})$ -modules .....	1759
<b>Vesselin DRENSKY, Elitza HRISTOVA</b>	
On the Mesyan conjecture .....	1794
<b>Pedro Souza FAGUNDES, Thiago Castilho de MELLO, Pedro Henrique da Silva dos SANTOS</b>	
Symmetric polynomials in the free metabelian associative algebra of rank 2 .....	1809
<b>Şehmus FINDIK</b>	
$\mathbb{Z}$ -graded identities on the infinite-dimensional Grassmann algebra and arithmetic tools: revisited ..	1814
<b>Claudemir FIDELIS</b>	
Nonlinear evolution equations related to Kac-Moody algebras $A_r^{(1)}$ : spectral aspects .....	1828
<b>Vladimir S. GERDJIKOV</b>	
Computing forgotten topological index of zero-divisor graphs of commutative rings .....	1845
<b>Arif GÜRISOY, Necla KIRCALI GÜRISOY, Alper ÜLKER</b>	

A variant of Rosset's approach to the Amitsur-Levitzki theorem and some $\mathbb{Z}_2$ -graded identities of $M_n(E)$ .....	1864
<b>Szilvia HOMOLYA, Jenő SZIGETI</b>	
Polynomial identities in matrix algebras with pseudoinvolution .....	1871
<b>Antonio IOPPOLO</b>	
Depth and Stanley depth of the quotient rings of edge ideals of some lobster trees and unicyclic graphs .....	1886
<b>Adnan IQBAL, Muhammad ISHAQ</b>	
On almost $s$ -weakly regular rings .....	1897
<b>Kanchan JANGRA, Dinesh UDAR</b>	
A note on the $\mathcal{A}$ -generators of the polynomial algebra of six variables and applications .....	1911
<b>Tin NGUYEN KHAC</b>	
Spectral theory of B-Weyl elements and the generalized Weyl's theorem in primitive $C^*$ -algebra ...	1927
<b>Yingying KONG, Yanxun REN, Lining JIANG</b>	
Structure of annihilators of powers .....	1945
<b>Jongwook BAECK, Nam Kyun KIM, Tai Keun KWAK, Yang LEE</b>	
On central polynomials and codimension growth .....	1965
<b>Fabrizio MARTINO</b>	
Codimensions of algebras with additional structures .....	1975
<b>Daniela LA MATTINA</b>	
On the restricted graded Jacobson radical of rings of Morita context .....	1985
<b>Puguh Wahyu PRASETYO, Hidetoshi MARUBAYASHI, Indah Emilia WIJAYANTI</b>	
On $(a, d)$ -edge local antimagic coloring number of graphs .....	1994
<b>Rajkumar SUNDARAMOORTHY, Nalliah MOVIRI CHETTIAR</b>	
A new approach to word standardization and some of its applications .....	2003
<b>Wesam TALAB</b>	
On $m_j$ -clean ring and strongly $m_j$ -clean ring .....	2015
<b>Gülşen ULUCAK, Arda KÖR</b>	
Nilpotent varieties and metabelian varieties .....	2023
<b>Angela VALENTI, Sergey MISHCHENKO</b>	
On $S$ -comultiplication modules .....	2034
<b>Eda YILDIZ, Ünsal TEKİR, Suat KOÇ</b>	

## Trends in Combinatorial Ring Theory and Applications

### Preface

The present volume contains the proceedings of the International Conference “*Trends in Combinatorial Ring Theory*” which was held partly online via zoom and partly at the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences in Sofia, Bulgaria, from September 20 to September 24, 2021. The purpose of the conference was to present the current state of art in the area of Ring Theory, its applications, and related fields. The conference and this volume are dedicated to the 70-th anniversary of Professor Vesselin Drensky. In his scientific career, Vesselin Drensky made major contributions to the fields of *Combinatorial and Computational Ring Theory*, *Algebras with Polynomial Identities*, *Commutative and Noncommutative Invariant Theory*, *Representation Theory of Groups*, and many other related fields in algebra. Some of his more important contributions include the development of the interplay of the representations of the symmetric and general linear groups in the context of PI algebras, counterexamples to the Specht problem for nonassociative algebras, a new proof of Nowicki’s conjecture, computation of the Hilbert series of the algebras of invariants of the special linear group, constructions of minimal bases of the polynomial identities of the full matrix algebra of order 2, and of the simple Lie algebra of dimension 3, description of the multilinear structure of the corresponding relatively free algebras, the proof of the strong Anick conjecture, the solution of a problem posed by Claudio Procesi concerning the identities of matrix algebras in positive characteristic, the construction of central polynomials of low degrees for matrix algebras, and many others.

Most of Professor Drensky’s collaborators, colleagues, and students took part in the conference. It was organized by Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Institute of Mathematics, Statistics and Scientific Computing, State University of Campinas, Brazil, and Memorial University of Newfoundland, Canada. The main topics of the conference included Combinatorial and Computational Ring Theory, Group Theory, Theory of PI-algebras, Commutative and Noncommutative Invariant Theory, Lie and Jordan Algebras and Superalgebras, Matrix Algebras and many other topics. The meeting brought together mathematicians from 16 countries. The program consisted of 26 talks which were 40 min long and 37 talks which were 25 min long. Many of the participants in the conference contributed also to the present volume.

The organizers of the conference and the editors of the present volume are really very grateful to all participants in the conference and to all contributors to the present volume, to the anonymous referees for their careful and patient work, to the Editorial Staff and to the Editor-in-Chief, Professor Adnan Tercan, of the Turkish Journal of Mathematics, and to the Bulgarian National Science Fund for providing financial support for the conference.

Organizers: *Ivan CHIPCHAKOV*, *Peter DANCHEV*, *Elitza HRISTOVA* and *Plamen KOSHLUKOV*  
Guest Editors: *Peter DANCHEV*, *Antonio GIAMBRUNO*, *Plamen KOSHLUKOV* and *Louis ROWEN*  
June 07, 2022, Sofia, Campinas, St. John’s