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Novel correlation coefficients for interval-valued Fermatean hesitant fuzzy sets with pattern recognition application

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Abstract: A combination of interval-valued Fermatean fuzzy sets with Fermatean hesitant fuzzy elements in the form of interval values is known as an interval-valued Fermatean hesitant fuzzy set. Since Fermatean hesitant fuzzy sets are effective instruments for representing more complex, ambiguous, and hazy information, interval-valued Fermatean hesitant fuzzy sets are expansions of these sets. This investigation will concentrate on four different types of correlation coefficients for Fermatean hesitant fuzzy sets and expand them to include correlation coefficients and weighted correlation coefficients for interval-valued Fermatean hesitant fuzzy sets. Finally, the numerical examples demonstrate the viability and usefulness of the suggested methodologies in decision-making under many criteria.

Key words: Correlation, correlation coefficient, Fermatean hesitant fuzzy set, informational energy, interval-valued Fermatean hesitant fuzzy set

1. Introduction

1.1. Correlation coefficients

The correlation coefficient (KK) is a precise metric used in correlation studies to express the strength of the linear relationship between two variables. By measuring the distance of each data point from the variable mean, the formula indicates how well the link between the variables can be fit to an example line drawn across the data for two variables. A correlation is a sign of cochange. Using this statistical method, we may measure the magnitude and direction of the link between two or more variables. The KK is denoted by r. The KK can have a value between -1 and $+1$, according to mathematics. The sign of this coefficient indicates the direction of the link between the two variables, and its numerical value indicates the strength of the correlation. The reciprocal contact is taken into consideration while interpreting the connection. Correlation is a reciprocal relationship rather than a cause-and-effect relationship that can be used to explain the difference or resemblance between two variables. Due to correlation, the values of the related variables may be maximized if the affecting factors can be controlled or the relationship between the variables can be estimated by looking at the value of one variable.

A KK is a bivariate statistic if it depicts the connection between just two variables and a multivariate statistic if there are more than two variables. There are, thus, many diverse fields of study, from engineering to physics, from medicine to economics. In statistics, finding a KK between any two parameters or variables is quite common. Pearson's KK has been used in statistics research on data analysis and classification, pattern

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2010 *AMS Mathematics Subject Classification:* 03E72,91A06, 62C86 recognition, clustering, medical diagnosis, and decision-making. It is shown that conventional correlation cannot handle data with ambiguous problems. The fuzzy logic approach attempts to quantify human perception and cognitive uncertainties. Scientists are used to employing binary logic to analyze data. Human logic is ambiguous and complicated; therefore, using binary logic to study human mental processes causes some distortion. Fuzzy logic is built on the base of human cognitive processes. Fuzzy logic, for instance, is defined as the "modeling of thinking and decision systems that enable people to make consistent and correct judgments in the presence of extensive and inaccurate information." The fuzzy type KKs have been expanded based on mathematical statistics, statistical KK, and the fuzzy logic method. The KK generated for fuzzy data shows both the strength of the relationship between fuzzy sets as well as whether fuzzy sets are positively or negatively related.

1.2. Uncertainty

Numerous academic fields, including psychology, philosophy, cognitive science, and artificial intelligence, are interested in how humans reason and make decisions in the face of everyday situations. Usually, numerous mathematical and statistical models are used to try to characterize these processes. The issue of decision-making surfaces in this process. Decision-making (DM) is the process of choosing one or more of the available options for behavior when a person or institution is trying to accomplish a certain objective. According to research, while many daily judgments may be made instinctively, complicated and important choices require more than this. Multicriteria decision making (MCDM) is a group of analytical techniques that assesses the benefits and drawbacks of options based on a variety of criteria. To pick one or more alternatives from a group of alternatives with varying qualities by competing criteria or to rank these alternatives, MCDM approaches are employed to help the DM process. In other words, decision-makers use MCDM approaches to rate options with various features by comparing them to a variety of criteria. MCDM is a collection of techniques that are regularly applied at all levels and in all spheres of life.

A key idea in decision-making (DM) difficulties is uncertainty. Unpredictable events characterize uncertainty. Routine choices cannot be discussed under ambiguous circumstances. It is important to consider both the advantages and disadvantages of potential outcomes under unclear circumstances. It is crucial to do a thorough analysis of the environmental influences at this stage. Benefiting from prior experiences and decisions is not always successful when there is ambiguity, even while ultimate judgments are not in doubt. Thanks to Zadeh's notion of "fuzzy sets" (FS) [76], linguistic terms that we unintentionally employ regularly have become "computable". Fuzzy logic enabled the grading system to expand the realm of classical mathematics, which was previously restricted to certainty. This notion resulted in a paradigm shift that spread throughout the world as a result of its successful implementations in everyday situations. An element with a distinctive function is either an element of a set in the traditional sense of the word or it is not. The membership function (MF), which assigns each item a degree of membership in the range $[0,1]$, in the FS notion, determines whether or not an apple is a member of a set.

The degree to which an element belongs to a set in the FS A is $\rho(A)$, and the degree to which it does not belong is $1-\rho(A)$. As a result, one is equal to the total of the degrees of belonging and nonbelonging. This circumstance, however, falls short of adequately explaining the ambiguity in several issues. The intuitionistic fuzzy set (IFS) theory, which is an extension of the FS theory, was developed by Atanassov [4] as a result. In IFS theory, the nonmembership degree (ND) is specified in addition to the membership degree (MD), whereas FS theory is modeled to only reveal the membership degree (MD) defined in the range $[0,1]$. According to IFS theory, MD and ND are both in the $[0,1]$ region. Yager [71] proposed Pythagorean fuzzy sets (PFS) and in certain circumstances developed them as an extension of IFSs because IFSs cannot adequately convey uncertainty. PFSs employ the notion that the sum of the squares of MD and ND is less than or equal to 1 for circumstances when decision-making is impossible when MD and ND are added together. In the literature, there is a lot of study on FS and its many expansions ([1, 15, 16, 20, 21, 28, 29, 36, 50, 74]).

1.3. Motivation

Tools like aggregation operators and information measurements are frequently used in decision-making difficulties. Another method for determining the best option is to utilize the KKs measure of the level of dependence between two sets. Using KKs, one may assess how strongly two variables are associated. Since the information is frequently vague, ambiguous, and incomplete in many situations, several academics have developed the KKs in fuzzy contexts. In addition to supplying the correlation for fuzzy information in accordance with conventional statistics, Chiang and Lin [10] provided a technique for KK of FSs. According to the conventional understanding of "KKs," the KK of fuzzy information has been investigated in [44] using a mathematical programming approximation. According to [4], intuitionistic fuzzy set (IFS) results were more complete and exact based on the findings from the FS theory. Both membership degree (MD) and nonmembership degree (ND) are taken into consideration by the IFS theory, and it requires that their sum be one or less than one. There are several applications for the IFS-derived KKs, including DM, cluster analysis, image processing, and pattern recognition ([42, 64–66, 69]). Thanks to Pythagorean fuzzy set (PFS) ([29, 34, 49, 50, 70, 71, 73, 77]), which were developed to address an IFS issue, several DM problems involving Pythagorean fuzzy information have been published in the literature.

Senapati and Yager [53] were the first to propose the Fermat fuzzy set (FFS). The MD and ND in the FFS achieve the property $0leqmA3 + nA3leq1$. When identifying uncertainties, the FFS, a novel idea in the literature, performs better than the IFS and PFS. As an illustration, consider $0.9+0.6 > 1$, $0.92+0.62 > 1$, and $0.93+0.63 > 1$. Some FFS characteristics, score, and accuracy functions are provided in [53]. Additionally, the TOPSIS approach, which is widely used to solve MCDM issues, has been employed to solve FFS. Additionally, Senapati and Yager [53] used the TOPSIS method, which is frequently used in MCDM issues, to solve FFS difficulties. Senapati and Yager [54] continued this work by investigating a number of additional operations including arithmetic mean operations over FFSs in addition to using the FF weighted product model to address MCDM issues. New aggregation operations that are FFS-related are described and their associated attributes are studied in [55]. Shahzadi and Akram [56] created the new aggregated operators and provided a new decision support algorithm for the FFSS. Garg et al. [26] defined new FFS type aggregated operators defined by t-norm and t-conorm. Donghai et al. [13] suggested the concept of FF linguistic term sets. Operations, score, and accuracy functions belonging to these sets are given. In [14], a new similarity measure related to FF linguistic term sets is constructed. The new measurement is a combination of Euclidean distance measure and cosine similarity measure. Kirisci [30] defined FF soft sets and gave the measure of entropy based on FF soft sets. In [31], a new hesitant fuzzy set called the fermatean hesitant fuzzy set is given and some of its properties are investigated. Kirisci and Simsek [33] offer aggregation operations to extend FFHSs to interval-valued Fermatean hesitant fuzzy sets (IVFHFS) and to improve MCGDM methods in IVFHF environments. In [32], the ELECTRE I method is defined with Fermatean fuzzy sets according to the group DM process in which more than one individual interacts at the same time. In [**?**], various FF reference relations (consistent, incomplete, consistent incomplete, acceptable incomplete) are defined. An additive consistency based on a priority vector is given. In addition, a model is presented to obtain missing decisions in incomplete FF preference relations.

Torra and Narukawa [61] and Torra [62] provide HFSs, which authorize an object's MD to a set of potential values. Particularly useful in GDM difficulties, HFSs have generated strong solutions for these issues. Because of these qualities, HFSs have gained prominence in the literature. Following that, several interval-valued hesitant fuzzy sets were investigated [2, 3, 5, 7, 9, 12, 27, 40, 41, 48, 51]. KKs for HFSs have been studied in [68] and [8]. In these investigations, the hesitant fuzzy element (HFE), which has fewer element cardinalities, has had its possible MDs enlarged by the addition of some extreme values. If the extreme value is distant from the other possible membership values, the extended HFE will be very different from the original HFE. Additionally, it is impossible to determine the KKs between two HFS if one HFS's hesitant MDs are always zero. With respect to the mean of an HFE, Liao et al. [43] established a unique KK formula and expanded the KKs' range to the interval $[-1;1]$. The KK between an HFS and other HFSs will still not be calculated when an HFS is characterized by a constant function. The improvement of the counterintuition of the current KKs of HFSs in [8, 43, 68] was the main goal of Sun et al.'s [59] study. Its key contribution is to better the weighted KKs; however, the general case is not improved. Meng and Chen [45] suggested the KKs of HFSs based on IVF measures, and they [46] extended the technique to study the situation. The term is too complicated, though. The KKs of dual HFSs were also explored in [63] and [75]. The KKs of HF soft sets were discussed by Das et al. [11]. Although there are many KKs for HFSs, these notions might lead to a lot of irrational claims.

The originality of this work can be stated as follows. There have been various extensions of the classical KKs such as fuzzy KK, IF KK, and PF KK. These extensions have improved the performance of the KKs. FFSs can handle problems with ambiguity and incomplete information more efficiently than that of IFSs and PFSs Figure. In this study, the interval-valued Fermatean fuzzy KKs were developed considering the hesitant KKs, intuitionistic fuzzy KKs, Pythagorean fuzzy KKs, and Fermatean fuzzy KKs studies. Since the $MD^3 + ND^3 \leq 1$ requirement is satisfied for an object in the use of FFSs, there will be the possibility to cover more elements than IFSs and PFSs. Examples of pattern recognition and medical decision-making regarding the new KKs are given. KKs based on different fuzzy sets given in previous studies were compared with newly proposed KKs.

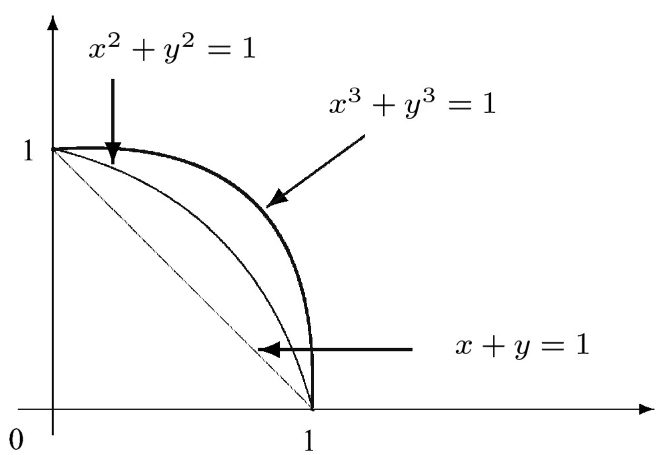

Figure 1. Comparison of space of FMGs, PMGs. and IFGs [53].

In this study, for IVFHFSs, KKs and weighted KKs were defined, and some characteristics were examined. The correlation coefficient for PFSs is a particular instance of the correlation coefficient for FFSs. As a result, IVFHFS's discovery of correlation coefficients provides more alternatives for handling uncertainty. The suggested correlation coefficient is consequently more general than the current ones, making it more appropriate for solving problems in the real world. Illustrative examples such as pattern recognition and medical decision-making to show the practicality and efficiency of the suggested KKs are given.

The organization of this work's structure is as follows. Fundamental ideas and findings from HFSs, FHFSs, and IVFHFSs are given in Section 2. The informational energies, correlations, and KKs and weighted KKs of IVFHFSs are defined in Section 3. Using the weighted KKs recommended in IVFHF environments, numerical examples are given in Section 4 demonstrating the effectiveness of the proposed method. A comparison of the new method with previous methods is made in Section 5 and this section is concluded with a subsection explaining the advantages of the new method.

2. Preliminaries

It will be regarded as the $\mathfrak{X} = \{x_1, x_2, \cdots, x_n\}$ initial universe throughout the work.

The intuitionistic fuzzy set(IFS) in $\mathfrak{X}$ is defined:

$$N = \{(x, \zeta_N(x), \eta_N(x)) | x \in \mathfrak{X}\}. \tag{2.1}$$

In this definition, $\zeta_N(x), \eta_N(x) : \mathfrak{X} \to [0,1]$ is said to be MD and ND, with $\zeta_N(x) + \eta_N(x) \leq 1$.

The Pythagorean fuzzy set (PFS) is characterized as:

$$N = \{(x, \zeta_N(x), \eta_N(x)) | x \in \mathfrak{X}\}, \tag{2.2}$$

if $\zeta_N(x), \eta_N(x) : \mathfrak{X} \to [0,1]$ are MD and ND of element of the $x \in \mathfrak{X}$, with $\zeta_N^2(x) + \eta_N^2(x) \leq 1$.

Fermatean fuzzy set (FFS) is given as:

$$N = \{(x, \zeta_N(x), \eta_N(x)) | x \in \mathfrak{X}\}, \tag{2.3}$$

if $\zeta_F(x), \eta_F(x) : \mathfrak{X} \to [0,1]$ are MD and ND of element of the $x \in \mathfrak{X}$, with $\zeta_N^3(x) + \eta_N^3(x) \leq 1$.

The set

$$\Gamma = \{(u, \tau_\Gamma(u)) : u \in U\} \tag{2.4}$$

is called hesitant fuzzy set, where $\tau_\Gamma(u)$ indicates the set of some values in unit interval, that is probable MD of $u \in U$ to Γ [67].

The set

$$F_T = \{(u, \zeta_{F_T}(u), \eta_{F_T}(u)) : u \in U\} \tag{2.5}$$

is called a Fermatean hesitant fuzzy set (FHFS) [31], where

(i.) For each element $u \in U$, $\zeta_{F_T}(u)$, $\eta_{F_T}(u)$ are functions from U to $[0,1]$, demonstrating a likely MD and ND of element $u \in U$ in F_T respectively,

(ii.) $\forall \quad t_{F_T}(u) \in \zeta_{F_T}(u)$, $\exists \quad t'_{F_T}(u) \in \eta_{F_T}(u)$, such that $0 \leq t^3_{F_T}(u) + t'^3_{F_T}(u) \leq 1$, (iii.) $\forall \quad t^{'}_{F_T}(u) \in \eta_{F_T}(u)$, $\exists \quad t_{F_T}(u) \in \zeta_{F_T}(u)$, such that $0 \leq t^3_{F_T}(u) + t^{'3}_{F_T}(u) \leq 1$.

From this stage on, the set of all elements belonging to FHFSs will be denoted by $\mathfrak{FH}(U)$. If U has only $(u, \zeta_{F_T}(u), \eta_{F_T}(u))$, then it is said to be a fermatean hesitant fuzzy number (FHFN) and is represented by $\widetilde{t} = \{\zeta_{\widetilde{t}}, \eta_{\widetilde{t}}\}$.

$$\mathcal{F} = \{(u, h_{\mathcal{F}}(u) : u \in U\}, \tag{2.6}$$

is called an interval-valued Fermatean hesitant ($\mathbb{IVFH}$) $\mathcal{F}$ on U, where

$$h_{\mathcal{F}}(u) = \left\{(\widehat{\zeta}_{\mathcal{F}}(u), \widehat{\eta}_{\mathcal{F}}(u)) : \widehat{\zeta}_{\mathcal{F}}(u) = [\zeta^-_{\mathcal{F}}, \zeta^+_{\mathcal{F}}] \in D[0,1], \widehat{\eta}_{\mathcal{F}}(u) = [\eta^-_{\mathcal{F}}, \eta^+_{\mathcal{F}}] \in D[0,1], (\zeta^+_{\mathcal{F}})^3 + (\eta^+_{\mathcal{F}})^3 \leq 1\right\},$$

where $\widehat{\zeta}_{\mathcal{F}}(u)$ is the possible Fermatean membership interval and $\widehat{\eta}_{\mathcal{F}}(u)$ is the possible Fermatean nonmembership intervals of $\mathcal{F}$. Throughout this article, Υ will show the set of all $\mathbb{IVFH}$s.

Principle of recognition is defined as: In discourse universe $\mathfrak{X}$, let it be assumed that there are m patterns defined by FFS $\mathfrak{N}_k$ $(k = 1, 2, \cdots, m)$. Again, let us suppose that there is a model to be identified with FFS $\mathfrak{P}$ in $\mathfrak{X}$.

The relationship index degree between FFSs $\mathfrak{N}_k$ and $\mathfrak{P}$ is described as:

$$R(\mathfrak{N}_{k0}, \mathfrak{P}) = \max_{1 \leq k \leq m} \{R(\mathfrak{N}_k, \mathfrak{P})\}.$$

In this case, it is decided that sample $\mathfrak{P}$ belongs to $\mathfrak{N}_{k0}$.

The set

$$IE_F(N) = \sum_{i=1}^{n} \left[\zeta^6_N(x_i) + \eta^6_N(x_i) + \theta^6_N(x_i)\right] \tag{2.7}$$

is called informational energies of Fermatean fuzzy set N [35].

For FFSs N and M, correlation and KK are defined as [35]:

$$C_F(N, M) = \sum_{i=1}^{n} \left[\zeta^3_N(x_i).\zeta^3_M(x_i) + \eta^3_N(x_i).\eta^3_M(x_i) + \theta^3_N(x_i).\theta^3_M(x_i)\right] \tag{2.8}$$

$$\mathfrak{C}_F(N, M) = \frac{C_F(N, M)}{\sqrt{IE_F(N).IE_F(M)}} \tag{2.9}$$

$$= \frac{\sum_{i=1}^{n} \left[\zeta^3_N(x_i).\zeta^3_M(x_i) + \eta^3_N(x_i).\eta^3_M(x_i) + \theta^3_N(x_i).\theta^3_M(x_i)\right]}{\sqrt{\sum_{i=1}^{n} [\zeta^6_N(x_i) + \eta^6_N(x_i) + \theta^6_N(x_i)]}.\sqrt{\sum_{i=1}^{n} [\zeta^6_M(x_i) + \eta^6_M(x_i) + \theta^6_M(x_i)]}} \tag{2.10}$$

$$\mathfrak{D}_F(N, M) = \frac{C_I(N, M)}{\max\{IE(N).IE(M)\}} \tag{2.11}$$

$$= \frac{\sum_{i=1}^{n} \left[\zeta_N(x_i).\zeta_M(x_i) + \eta_N(x_i).\eta_M(x_i) + \theta^3_N(x_i).\theta^3_M(x_i)\right]}{\max\left[\sum_{i=1}^{n} [\zeta^6_N(x_i) + \eta^6_N(x_i) + \theta^6_N(x_i)].\sum_{i=1}^{n} [\zeta^6_{MB}(x_i) + \eta^6_M(x_i) + \theta^6_M(x_i)]\right]}. \tag{2.12}$$

3. Correlation coefficients

The set

$$\mathcal{F} = \{(u, h_{\mathcal{F}}(u) : u \in U\}, \tag{3.1}$$

is called an $\mathbb{IVFH}$ $\mathcal{F}$ on U, where

$$h_{\mathcal{F}}(u) = \left\{(\widehat{\zeta}_{\mathcal{F}}(u), \widehat{\eta}_{\mathcal{F}}(u)) : \widehat{\zeta}_{\mathcal{F}}(u) = [\zeta^-_{\mathcal{F}}, \zeta^+_{\mathcal{F}}] \in D[0,1], \widehat{\eta}_{\mathcal{F}}(u) = [\eta^-_{\mathcal{F}}, \eta^+_{\mathcal{F}}] \in D[0,1], (\zeta^+_{\mathcal{F}})^3 + (\eta^+_{\mathcal{F}})^3 \leq 1\right\},$$

where $\widehat{\zeta}_{\mathcal{F}}(u)$ is the possible Fermatean membership interval and $\widehat{\eta}_{\mathcal{F}}(u)$ is the possible Fermatean nonmembership intervals of $\mathcal{F}$. Throughout this article, Υ will show the set of all $\mathbb{IVFH}$s.

The indeterminacy degree of u to $\mathcal{F}$ is described as $\theta_{\mathcal{F}}(u) = \sqrt[3]{1 - (\zeta^3_{\mathcal{F}}(u) + \eta^3_{\mathcal{F}}(u))}$, for any FFS $\mathcal{F}$ and $u \in U$.

Let $\mathcal{F} = \{u_i, \zeta_{\mathcal{F}}(u_i, \eta_{\mathcal{F}}(u_i)|u_i \in U\}$ be a IVFHFS, where $\zeta_{\mathcal{F}}(u_i), \eta_{\mathcal{F}}(u_i) \in [0,1]$ and $\zeta^3_{\mathcal{F}}(u_i + \eta^3_{\mathcal{F}}(u_i)|u_i \leq 1$ for each $u_i \in U$, we define the informational energy of the FFS $\mathcal{F}$ as

$$IE(\mathcal{F}) = \sum_{i=1}^{n} \left[\left(\zeta^-_{\mathcal{F}}(u_i)\right)^6 + \left(\zeta^+_{\mathcal{F}}(u_i)\right)^6 + \left(\eta^-_{\mathcal{F}}(u_i)\right)^6 + \left(\zeta^+_{\mathcal{F}}(u_i)\right)^6 + \left(\theta^-_{\mathcal{F}}(u_i)\right)^6 + \left(\theta^+_{\mathcal{F}}(u_i)\right)^6\right]. \tag{3.2}$$

Suppose that two IVFHFS's $\mathcal{F} = \{u_i, \zeta_{\mathcal{F}}(u_i, \eta_{\mathcal{F}}(u_i)|u_i \in U\}$ and $\mathcal{G} = \{u_i, \zeta_{\mathcal{G}}(u_i, \eta_{\mathcal{G}}(u_i)|u_i \in U\}$, where $\zeta_{\mathcal{F}}(u_i, \eta_{\mathcal{F}}(u_i), \zeta_{\mathcal{F}}(u_i, \eta_{\mathcal{F}}(u_i) \in [0,1]$ for each $u_i \in U$. Hence, the correlation of the FFVs $\mathcal{F}$, $\mathcal{G}$ is defined:

$$C(\mathcal{F}, \mathcal{G}) = \sum_{i=1}^{n} \Bigg[(\zeta^-_{\mathcal{F}}(u_i))^3(\zeta^-_{\mathcal{G}}(u_i))^3 + (\eta^-_{\mathcal{F}}(u_i))^3(\eta^-_{\mathcal{G}}(u_i))^3 + (\theta^-_{\mathcal{F}}(u_i))^3(\theta^-_{\mathcal{G}}(u_i))^3 + \tag{3.3}$$

$$(\zeta^+_{\mathcal{F}}(u_i))^3(\zeta^+_{\mathcal{G}}(u_i))^3 + (\eta^+_{\mathcal{F}}(u_i))^3(\eta^+_{\mathcal{G}}(u_i))^3 + (\theta^+_{\mathcal{F}}u_i))^3(\theta^+_{\mathcal{G}}(u_i))^3\Bigg].$$

For the correlation of FFSs, the conditions

(1) $C(\mathcal{F}, \mathcal{F}) = IE(\mathcal{F})$

(2) $C(\mathcal{F}, \mathcal{G}) = C(\mathcal{G}, \mathcal{F})$

are hold.

Definition 3.1 *Choose two FFSs $\mathcal{F}$ and $\mathcal{G}$ on X. Then the KK between $\mathcal{F}, \mathcal{G}$ is defined by*

$$\mathfrak{C}(\mathcal{F}, \mathcal{G}) = \frac{C(\mathcal{F}, \mathcal{G})}{[IE(\mathcal{F}).IE(\mathcal{G})]^{1/2}} \tag{3.4}$$

$$= \frac{\sum_{i=1}^{n}(\zeta^-_{\mathcal{F}}(u_i))^3(\zeta^-_{\mathcal{G}}(u_i))^3 + (\eta^-_{\mathcal{F}}(u_i))^3(\eta^-_{\mathcal{G}}(u_i))^3 + (\theta^-_{\mathcal{F}}(u_i))^3(\theta^-_{\mathcal{G}}(u_i))^3}{\sqrt{\sum_{i=1}^{n}\left[\left(\zeta^-_{\mathcal{F}}(u_i)\right)^6 + \left(\zeta^+_{\mathcal{F}}(u_i)\right)^6 + \left(\eta^-_{\mathcal{F}}(u_i)\right)^6 + \left(\zeta^+_{\mathcal{F}}(u_i)\right)^6 + \left(\theta^-_{\mathcal{F}}(u_i)\right)^6 + \left(\theta^+_{\mathcal{F}}(u_i)\right)^6\right]}}$$

$$\frac{+(\zeta^+_{\mathcal{F}}(u_i))^3(\zeta^+_{\mathcal{G}}(u_i))^3 + (\eta^+_{\mathcal{F}}(ux_i))^3(\eta^+_{\mathcal{G}}(u_i))^3 + (\theta^+_{\mathcal{F}}(u_i))^3(\theta^+_{\mathcal{G}}(u_i))^3}{\times\sqrt{\sum_{i=1}^{n}\left[\left(\zeta^-_{\mathcal{G}}(u_i)\right)^6 + \left(\zeta^+_{\mathcal{G}}(u_i)\right)^6 + \left(\eta^-_{\mathcal{G}}(u_i)\right)^6 + \left(\zeta^+_{\mathcal{G}}(u_i)\right)^6 + \left(\theta^-_{\mathcal{G}}(u_i)\right)^6 + \left(\theta^+_{\mathcal{G}}(u_i)\right)^6\right]}}$$

Theorem 3.2 *For any two FFSs $\mathcal{F},\mathcal{G}$ in X, the KK of FFSs satisfies the following conditions:*

(P1) $\mathfrak{C}(\mathcal{F},\mathcal{G})=\mathfrak{C}(\mathcal{G},\mathcal{F})$,

(P2) If $\mathcal{F}=\mathcal{G}$, *then* $\mathfrak{C}(\mathcal{F},\mathcal{G})=1$,

(P3) $0\leq\mathfrak{C}(\mathcal{F},\mathcal{G})\leq 1$.

Proof We only proved condition (P2). Obviously, $\mathfrak{C}(\mathcal{F},\mathcal{G})\geq 0$.

$$\begin{aligned}
C(\mathcal{F},\mathcal{G}) &= \sum_{i=1}^{n}\Bigg[(\zeta_{\mathcal{F}}^{-}(u_i))^3(\zeta_{\mathcal{G}}^{-}(u_i))^3+(\eta_{\mathcal{F}}^{-}(u_i))^3(\eta_{\mathcal{G}}^{-}(u_i))^3+(\theta_{\mathcal{F}}^{-}(u_i))^3(\theta_{\mathcal{G}}^{-}(u_i))^3\\
&+ (\zeta_{\mathcal{F}}^{+}(u_i))^3(\zeta_{\mathcal{G}}^{+}(u_i))^3+(\eta_{\mathcal{F}}^{+}(u_i))^3(\eta_{\mathcal{G}}^{+}(u_i))^3+(\theta_{\mathcal{F}}^{+}(u_i))^3(\theta_{\mathcal{G}}^{+}(u_i))^3\Bigg]\\
&= \Bigg[(\zeta_{\mathcal{F}}^{-}(u_1))^3(\zeta_{\mathcal{G}}^{-}(u_1))^3+(\eta_{\mathcal{F}}^{-}(u_1))^3(\eta_{\mathcal{G}}^{-}(u_1))^3+(\theta_{\mathcal{F}}^{-}(u_1))^3(\theta_{\mathcal{G}}^{-}(u_1))^3\\
&+ (\zeta_{\mathcal{F}}^{+}(u_1))^3(\zeta_{\mathcal{G}}^{+}(u_1))^3+(\eta_{\mathcal{F}}^{+}(u_1))^3(\eta_{\mathcal{G}}^{+}(u_1))^3+(\theta_{\mathcal{F}}^{+}(u_1))^3(\theta_{\mathcal{G}}^{+}(u_1))^3\Bigg]\\
&= \Bigg[(\zeta_{\mathcal{F}}^{-}(u_2))^3(\zeta_{\mathcal{G}}^{-}(u_2))^3+(\eta_{\mathcal{F}}^{-}(u_2))^3(\eta_{\mathcal{G}}^{-}(u_2))^3+(\theta_{\mathcal{F}}^{-}(u_2))^3(\theta_{\mathcal{G}}^{-}(u_2))^3\\
&+ (\zeta_{\mathcal{F}}^{+}(u_2))^3(\zeta_{\mathcal{G}}^{+}(u_2))^3+(\eta_{\mathcal{F}}^{+}(u_2))^3(\eta_{\mathcal{G}}^{+}(u_2))^3+(\theta_{\mathcal{F}}^{+}(u_2))^3(\theta_{\mathcal{G}}^{+}(u_2))^3\Bigg]\\
&+ \cdots+=\Bigg[(\zeta_{\mathcal{F}}^{-}(u_n))^3(\zeta_{\mathcal{G}}^{-}(u_n))^3+(\eta_{\mathcal{F}}^{-}(u_n))^3(\eta_{\mathcal{G}}^{-}(u_n))^3+(\theta_{\mathcal{F}}^{-}(u_n))^3(\theta_{\mathcal{G}}^{-}(u_n))^3\\
&+ (\zeta_{\mathcal{F}}^{+}(u_n))^3(\zeta_{\mathcal{G}}^{+}(u_n))^3+(\eta_{\mathcal{F}}^{+}(u_n))^3(\eta_{\mathcal{G}}^{+}(u_n))^3+(\theta_{\mathcal{F}}^{+}(u_n))^3(\theta_{\mathcal{G}}^{+}(u_n))^3\Bigg].
\end{aligned}$$

Using the Cauchy–Schwarz inequality, for $(\zeta_1+\cdots+\zeta_n)\in R^n$ and $(\eta_1+\cdots+\eta_n)\in R^n$,

$$(\zeta_1\eta_1+\zeta_2\eta_2+\cdots+\zeta_n\eta_n)^2\leq\left(\zeta_1^2+\cdots+\zeta_n^2\right).\left(\eta_1^2+\cdots+\eta_n^2\right).$$

Then,

$$\begin{aligned}
\Big[C(\mathcal{F},\mathcal{G})\Big]^2 &\leq \Big[\big((\zeta^-_{\mathcal{F}}(u_1))^6+(\eta^-_{\mathcal{F}}(u_1))^6+(\theta^-_{\mathcal{F}}(u_1))^6\big)+\big((\zeta^+_{\mathcal{F}}(u_1))^6+(\eta^+_{\mathcal{F}}(u_1))^6+(\theta^+_{\mathcal{F}}(u_1))^6\big)\\
&+ \big((\zeta^-_{\mathcal{F}}(u_2))^6+(\eta^-_{\mathcal{F}}(u_2))^6+(\theta^-_{\mathcal{F}}(u_2))^6\big)+\big((\zeta^+_{\mathcal{F}}(u_2))^6+(\eta^+_{\mathcal{F}}(u_2))^6+(\theta^+_{\mathcal{F}}(u_2))^6\big)+\cdots\\
&+ \big((\zeta^-_{\mathcal{F}}(u_n))^6+(\eta^-_{\mathcal{F}}(u_n))^6+(\theta^-_{\mathcal{F}}(u_n))^6\big)+\big((\zeta^+_{\mathcal{F}}(u_n))^6+(\eta^+_{\mathcal{F}}(u_n))^6+(\theta^+_{\mathcal{F}}(u_n))^6\big)\Big]\\
&\times \Big[\big((\zeta^-_{\mathcal{G}}(u_1))^6+(\eta^-_{\mathcal{G}}(u_1))^6+(\theta^-_{\mathcal{G}}(u_1))^6\big)+\big((\zeta^+_{\mathcal{G}}(u_1))^6+(\eta^+_{\mathcal{G}}(u_1))^6+(\theta^+_{\mathcal{G}}(u_1))^6\big)\\
&+ \big((\zeta^-_{\mathcal{G}}(u_2))^6+(\eta^-_{\mathcal{G}}(u_2))^6+(\theta^-_{\mathcal{G}}(u_2))^6\big)+\big((\zeta^+_{\mathcal{G}}(u_2))^6+(\eta^+_{\mathcal{G}}(u_2))^6+(\theta^+_{\mathcal{G}}(u_2))^6\big)+\cdots\\
&+ \big((\zeta^-_{\mathcal{G}}(u_n))^6+(\eta^-_{\mathcal{G}}(u_n))^6+(\theta^-_{\mathcal{G}}(u_n))^6\big)+\big((\zeta^+_{\mathcal{G}}(u_n))^6+(\eta^+_{\mathcal{G}}(u_n))^6+(\theta^+_{\mathcal{G}}(u_n))^6\big)\Big]\\
&= \sum_{i=1}^{n}\big[\big((\zeta^-_{\mathcal{F}}(u_i))^6+(\eta^-_{\mathcal{F}}(u_i))^6+(\theta^-_{\mathcal{F}}(u_i))^6\big)+\big((\zeta^+_{\mathcal{F}}(u_i))^6+(\eta^+_{\mathcal{F}}(u_i))^6+(\theta^+_{\mathcal{F}}(u_i))^6\big)\big]\\
&\times \sum_{i=1}^{n}\Bigg[\big((\zeta^-_{\mathcal{G}}(u_i))^6+(\eta^-_{\mathcal{G}}(u_i))^6+(\theta^-_{\mathcal{G}}(u_i))^6\big)+\big((\zeta^+_{\mathcal{G}}(u_i))^6+(\eta^+_{\mathcal{G}}(u_i))^6+(\theta^+_{\mathcal{G}}(u_i))^6\big)\Bigg]\\
&= IE(\mathcal{F}).IE(\mathcal{G}).
\end{aligned}$$

Therefore, $[C(\mathcal{F},\mathcal{G})]^2 \leq IE(\mathcal{F}).IE(\mathcal{G})$ and $\mathfrak{C}(\mathcal{F},\mathcal{G}) \leq 1$. □

Example 3.3 *Take three FFSs* $\mathcal{F}=\{([0.6,0.8],[0.5,0.7])\}$, $\mathcal{G}=\{([0.3,0.7],[0.5,0.6]),([0.4,0.7],[0.5,0.7])\}$ *in* $\mathfrak{X}$. *By Equation 3.2, the informational energies of* $\mathcal{F}$, $\mathcal{G}$:

$$IE(\mathcal{F})=(0.6^6+0.8^6+0.87^6)+(0.5^6+0.7^6+0.53^6)=0.9$$

$$IE(\mathcal{G})=(0.3^6+0.7^6+0.95^6)+(0.5^6+0.6^6+0.76^6)+(0.4^6+0.7^6+0.93^6)+(0.5^6+0.7^6+0.68^6)=2.74$$

By using Equation 3.4, the correlation between the FFSs $\mathcal{F},\mathcal{G}$ *is written as:*

$$\begin{aligned}
C(\mathcal{F},\mathcal{G}) &= \Big[(\zeta^-_{\mathcal{F}})^3(\zeta^-_{\mathcal{G}_1})^3+(\eta^-_{\mathcal{F}})^3(\eta^-_{\mathcal{G}_1})^3+(\theta^-_{\mathcal{F}})^3(\theta^-_{\mathcal{G}_1})^3+(\zeta^+_{\mathcal{F}})^3(\zeta^+_{\mathcal{G}_1})^3+(\eta^+_{\mathcal{F}})^3(\eta^+_{\mathcal{G}_1})^3+(\theta^+_{\mathcal{F}})^3(\theta^+_{\mathcal{G}_1})^3+\\
&\quad (\zeta^-_{\mathcal{F}})^3(\zeta^-_{\mathcal{G}_2})^3+(\eta^-_{\mathcal{F}})^3(\eta^-_{\mathcal{G}_2})^3+(\theta^-_{\mathcal{F}})^3(\theta^-_{\mathcal{G}_2})^3+(\zeta^+_{\mathcal{F}})^3(\zeta^+_{\mathcal{G}_2})^3+(\eta^+_{\mathcal{F}})^3(\eta^+_{\mathcal{G}_2})^3+(\theta^+_{\mathcal{F}})^3(\theta^+_{\mathcal{G}_2})^3\Big]\\
&= 0.6^3 0.3^3+0.8^3 0.7^3+0.87^3 0.95^3+0.5^3 0.5^3+0.7^3 0.6^3+0.537^3 0.76^3\\
&+ 0.6^3 0.4^3+0.8^3 0.7^3+0.87^3 0.93^3+0.5^3 0.5^3+0.7^3 0.7^3+0.53^3 0.68^3=1.81
\end{aligned}$$

Hence, the KK *between the FFSs* $\mathcal{F},\mathcal{G}$ *is given by:*

$$\mathfrak{C}(\mathcal{F},\mathcal{G})=\frac{C(\mathcal{F},\mathcal{G})}{[IE(\mathcal{F}).IE(\mathcal{G})]^{1/2}}=\frac{1.81}{[(0.9).(2.74)]^{1/2}}=0.734$$

Definition 3.4 *For $\mathcal{F}$ and $\mathcal{G}$, the definition of KK as:*

$$\mathfrak{D}(\mathcal{F},\mathcal{G}) = \frac{C(\mathcal{F},\mathcal{G})}{\max\left[IE(\mathcal{F}).IE(\mathcal{G})\right]} \tag{3.5}$$
$$= \frac{\sum_{i=1}^{n}(\zeta_{\mathcal{F}}^{-}(u_i))^3(\zeta_{\mathcal{G}}^{-}(u_i))^3+(\eta_{\mathcal{F}}^{-}(u_i))^3(\eta_{\mathcal{G}}^{-}(u_i))^3+(\theta_{\mathcal{F}}^{-}(u_i))^3(\theta_{\mathcal{G}}^{-}(u_i))^3}{\max\left\{\sum_{i=1}^{n}\left[\left(\zeta_{\mathcal{F}}^{-}(u_i)\right)^6+\left(\zeta_{\mathcal{F}}^{+}(u_i)\right)^6+\left(\eta_{\mathcal{F}}^{-}(u_i)\right)^6+\left(\zeta_{\mathcal{F}}^{+}(u_i)\right)^6+\left(\theta_{\mathcal{F}}^{-}(u_i)\right)^6+\left(\theta_{\mathcal{F}}^{+}(u_i)\right)^6\right],\right.}$$
$$\frac{+(\zeta_{\mathcal{F}}^{+}(u_i))^3(\zeta_{\mathcal{G}}^{+}(u_i))^3+(\eta_{\mathcal{F}}^{+}(u_i))^3(\eta_{\mathcal{G}}^{+}(u_i))^3+(\theta_{\mathcal{F}}^{+}(u_i))^3(\theta_{\mathcal{G}}^{+}(u_i))^3}{\left.\sum_{i=1}^{n}\left[\left(\zeta_{\mathcal{G}}^{-}(u_i)\right)^6+\left(\zeta_{\mathcal{G}}^{+}(u_i)\right)^6+\left(\eta_{\mathcal{G}}^{-}(u_i)\right)^6+\left(\zeta_{\mathcal{G}}^{+}(u_i)\right)^6+\left(\theta_{\mathcal{G}}^{-}(u_i)\right)^6+\left(\theta_{\mathcal{G}}^{+}(u_i)\right)^6\right]\right\}}$$

Theorem 3.5 *For any two FFSs $\mathcal{F},\mathcal{G}$ $\mathfrak{D}(\mathcal{F},\mathcal{G})$ satisfies the following conditions:*

(P1) $\mathfrak{D}(\mathcal{F},\mathcal{G})=\mathfrak{D}(\mathcal{G},\mathcal{F})$,

(P2) $\mathcal{F}=\mathcal{G}$ *iff* $\mathfrak{D}(\mathcal{F},\mathcal{G})=1$,

(P3) $0\leq\mathfrak{D}(\mathcal{F},\mathcal{G})\leq 1$.

Proof We only proved condition (P2). It is clear that $\mathfrak{D}(\mathcal{F},\mathcal{G})\geq 0$. Since from Theorem 3.2, $\left[C(\mathcal{F},\mathcal{G})\right]^2\leq IE(\mathcal{F}).IE(\mathcal{G})$. Therefore, $C(\mathcal{F},\mathcal{G})\leq\max[IE(\mathcal{F}).IE(\mathcal{G})]$; thus, $\mathfrak{D}(\mathcal{F},\mathcal{G})\leq 1$. □

It is possible to define these KKs in a different way. Weights will be used for these new definitions because, in numerous real-life practices, the distinct sets can have diverse weights. Therefore, weight ρ_i of every element $x_i\in\mathfrak{X}$ must be considered in new definitions. The KKs to be defined by weights will be called weighted KKs. For these definitions, choose the weight vector as ρ with the condition $\sum_{i=1}^{n}\rho_i=1$ for $\rho_i\geq 1$. Therefore, $C_\rho(\mathcal{F},\mathcal{G})$, $\mathfrak{C}_\rho(\mathcal{F},\mathcal{G})$ are defined as follows:

$$\mathfrak{C}_\rho(\mathcal{F},\mathcal{G}) = \frac{C_\rho(\mathcal{F},\mathcal{G})}{\left[IE_\rho(\mathcal{F}).IE_\rho(\mathcal{G})\right]^{1/2}} \tag{3.6}$$
$$= \frac{\sum_{i=1}^{n}\rho_i(\zeta_{\mathcal{F}}^{-}(u_i))^3(\zeta_{\mathcal{G}}^{-}(u_i))^3+(\eta_{\mathcal{F}}^{-}(u_i))^3(\eta_{\mathcal{G}}^{-}(u_i))^3+(\theta_{\mathcal{F}}^{-}(u_i))^3(\theta_{\mathcal{G}}^{-}(u_i))^3}{\sqrt{\sum_{i=1}^{n}\rho_i\left[\left(\zeta_{\mathcal{F}}^{-}(u_i)\right)^6+\left(\zeta_{\mathcal{F}}^{+}(u_i)\right)^6+\left(\eta_{\mathcal{F}}^{-}(u_i)\right)^6+\left(\zeta_{\mathcal{F}}^{+}(u_i)\right)^6+\left(\theta_{\mathcal{F}}^{-}(u_i)\right)^6+\left(\theta_{\mathcal{F}}^{+}(u_i)\right)^6\right]}}$$
$$\frac{+(\zeta_{\mathcal{F}}^{+}(u_i))^3(\zeta_{\mathcal{G}}^{+}(u_i))^3+(\eta_{\mathcal{F}}^{+}(u_i))^3(\eta_{\mathcal{G}}^{+}(u_i))^3+(\theta_{\mathcal{F}}^{+}(x_i))^3(\theta_{\mathcal{G}}^{+}(u_i))^3}{\sqrt{\sum_{i=1}^{n}\left[\left(\zeta_{\mathcal{G}}^{-}(u_i)\right)^6+\left(\zeta_{\mathcal{G}}^{+}(u_i)\right)^6+\left(\eta_{\mathcal{G}}^{-}(u_i)\right)^6+\left(\zeta_{\mathcal{G}}^{+}(u_i)\right)^6+\left(\theta_{\mathcal{G}}^{-}(u_i)\right)^6+\left(\theta_{\mathcal{G}}^{+}(u_i)\right)^6\right]}}$$

$$\mathfrak{D}_\rho(\mathcal{F},\mathcal{G}) = \frac{C_\rho(\mathcal{F},\mathcal{G})}{\max\left[IE_\rho(\mathcal{F}).IE_\rho(\mathcal{G})\right]} \tag{3.7}$$

$$= \frac{\sum_{i=1}^n \rho_i(\zeta_{\mathcal{F}}^-(u_i))^3(\zeta_{\mathcal{G}}^-(u_i))^3 + (\eta_{\mathcal{F}}^-(u_i))^3(\eta_{\mathcal{G}}^-(u_i))^3 + (\theta_{\mathcal{F}}^-(u_i))^3(\theta_{\mathcal{G}}^-(u_i))^3}{\max \sum_{i=1}^n \rho_i \left(\zeta_{\mathcal{F}}^-(u_i)\right)^6 + \left(\zeta_{\mathcal{F}}^+(u_i)\right)^6 + \left(\eta_{\mathcal{F}}^-(u_i)\right)^6 + \left(\zeta_{\mathcal{F}}^+(u_i)\right)^6 + \left(\theta_{\mathcal{F}}^-(u_i)\right)^6 + \left(\theta_{\mathcal{F}}^+(u_i)\right)^6}$$

$$\frac{+(\zeta_{\mathcal{F}}^+(u_i))^3(\zeta_{\mathcal{G}}^+(u_i))^3 + (\eta_{\mathcal{F}}^+(u_i))^3(\eta_{\mathcal{G}}^+(u_i))^3 + (\theta_{\mathcal{F}}^+(u_i))^3(\theta_{\mathcal{G}}^+(u_i))^3}{,\sum_{i=1}^n \rho_i \left[\left(\zeta_{\mathcal{G}}^-(u_i)\right)^6 + \left(\zeta_{\mathcal{G}}^+(u_i)\right)^6 + \left(\eta_{\mathcal{G}}^-(u_i)\right)^6 + \left(\zeta_{\mathcal{G}}^+(u_i)\right)^6 + \left(\theta_{\mathcal{G}}^-(u_i)\right)^6 + \left(\theta_{\mathcal{G}}^+(u_i)\right)^6\right]}$$

The following theorems are proved as similar to Theorems 3.2 and 3.5:

Theorem 3.6 *Take a weight vector of x_i as ρ and it is considered to satisfy the $\sum_{i=1}^n \varrho_i = 1$ condition for $\rho_i \geq 1$. The weighted KK between the FFSs $\mathfrak{N},\mathfrak{M}$ defined by Equation 3.6 satisfies:*

(P1) $\mathfrak{C}_\rho(\mathcal{F},\mathcal{G}) = \mathfrak{C}_\rho(\mathcal{F},\mathcal{G})$,

(P2) $0 \leq \mathfrak{C}_\rho(\mathcal{F},\mathcal{G}) \leq 1$;

(P3) $\mathfrak{C}_\rho(\mathcal{F},\mathcal{G}) = 1$ *iff* $\mathcal{F} = \mathcal{G}$

Theorem 3.7 *The weighted KK between the FFSs $\mathcal{F},\mathcal{G}$ defined by Equation 3.7 satisfies:*

(P1) $\mathfrak{D}_\rho(\mathcal{F},\mathcal{G}) = \mathfrak{D}_\rho(\mathcal{F},\mathcal{G})$,

(P2) $0 \leq \mathfrak{D}_\rho(\mathcal{F},\mathcal{G}) \leq 1$;

(P3) $\mathfrak{D}_\rho(\mathcal{F},\mathcal{G}) = 1$ *iff* $\mathcal{F} = \mathcal{G}$

4. Numerical examples

4.1. Pattern recognition application

In this subsection, an example for the multicriteria decision-making problem of alternatives, from the field of pattern recognition is used as the demonstration of the application of the proposed decision-making method, as well as the effectiveness of the proposed method.

Choose three known patterns PR_1, PR_2, PR_3. For the finite universe $U = \{u_1, u_2, u_3\}$ as

$$PR_1 = \{(u_1, [1.0, 0.0], [0.9, 0.2]), (u_2, [0.8, 0.0], [0.7, 0.1]), (u_3, [0.7, 0.4], [0.6, 0.5])\}$$
$$PR_2 = \{(u_1, [0.8, 0.1], [0.9, 0.1]), (u_2, [1.0, 0.0], [0.9, 0.1]), (u_3, [0.8, 0.3], [0.7, 0.4])\}$$
$$PR_3 = \{(u_1, [0.6, 0.2], [0.7, 0.3]), (u_2, [0.8, 0.2], [0.7, 0.2]), (u_3, [1.0, 0.0], [0.9, 0.1])\}.$$

Consider an unknown pattern $UP \in FFS(U)$ that will be recognized, where

$$UP = \{(u_1, [0.5, 0.3], [0.4, 0.3]), (u_2, [0.6, 0.2], [0.6, 0.3]), (u_3, [0.8, 0.1], [0.8, 0.2])\}.$$

The target of this problem is to classify the pattern UP in one of the classes PR_1, PR_2, PR_3. For it, proposed correlation coefficient index, $\mathfrak{C},\mathfrak{D}$, have been computed from UP to PR_i $(i = 1, 2, 3)$ and are given as follows:

$$\mathfrak{C}(PR_1, UP) = 0.098, \quad \mathfrak{C}(PR_2, UP) = 0.084, \quad \mathfrak{C}(PR_3, UP) = 0.1167,$$
$$\mathfrak{D}(PR_1, UP) = 0.2652, \quad \mathfrak{D}(PR_2, UP) = 0.1195, \quad \mathfrak{D}(PR_3, UP) = 0,4582.$$

Thus, from these two proposed correlation coefficient indexes, we conclude that the pattern UP belongs to the pattern PR_3.

On the other hand, if we assume that weights of u_1, u_2, u_3 are $0.5, 0.3, 0.2$, respectively, then we utilize the correlation coefficient $\mathfrak{C}_\rho$, $\mathfrak{D}_\rho$ for obtaining the most suitable pattern as:

$$\mathfrak{C}_\rho(PR_1, UP) = 0.175, \quad \mathfrak{C}_\rho(PR_2, UP) = 0.162, \quad \mathfrak{C}_\rho(PR_3, UP) = 0.29,$$
$$\mathfrak{D}_\rho(PR_1, UP) = 0.2, \quad \mathfrak{D}_\rho(PR_2, UP) = 0.056, \quad \mathfrak{D}_\rho(PR_3, UP) = 0,327.$$

Thus, ranking order of the three patterns PR_1, PR_2, PR_3 and hence PR_3 is the most desirable pattern to be classified with UP.

4.2. Multicriteria decision-making

This subsection will apply the correlation coefficients for interval-valued Pythagorean hesitant fuzzy sets (IVPHFSs) to MCDM problems and clustering analysis.

Let $L = \{L_i : i = 1, 2, \cdots, m\}$ be a finite set of alternatives and $K = \{K_j : j = 1, 2, \cdots, n\}$ be a set of criteria, ω be a weight vector of the criteria ($\omega_j \in [0,1], \sum_{j=1}^{n} \omega_j = 1$). Suppose $M = (h_{\mathcal{F}_{\rangle|}})_{m\times n}$ is an interval-valued Fermatean hesitant fuzzy decision matirx (IVFHFDM) where $h_{\mathcal{F}_{\rangle|}} = \{([\zeta_{ij}^-, \zeta_{ij}^+], [\eta_{ij}^-, \eta_{ij}^+]) : (\zeta_{ij}^+)^3 + (\eta_{ij}^+)^3 \leq 1\}$ ($i = 1, 2, \cdots, m; j = 1, 2, \cdots, n$) is an INFHFE given by decision makers to evaluate the alternative L_i with respect to the criteria K_j.

The concrete algorithm is listed as follows:

Step 1: Input $M = (h_{\mathcal{F}_{\rangle|}})_{m\times n}$, $\omega = (\omega_1, \cdots, \omega_n)^T$ and $\lambda \in [0,1]$.

Step 2: Make M the revised IVFHFDM $M' = (h'_{\mathcal{F}_{\rangle|}})_{m\times n}$.

Step 3: Compute the weighted correlation coefficients between each $\mathcal{F}_i$ and $\mathcal{F}_i^*$.

Step 4: Get the priority of the alternatives L_i by ranking the above correlation coefficients.

Example 4.1 *A scientific committee should evaluate four possible R&D projects L_i ($i = 1, 2, 3, 4$) according to the three criteria K_j ($j = 1, 2, 3$). Here C_1, C_2 are both benefit criteria and C_3 is a cost criterion. Take the weight vector of criteria as $\omega = (0.3, 0.5, 0.3)^T$. The decision matrix is given by the experts as Table 1.*

Considering that $|h_{\mathcal{F}_{\infty|}}| \neq |h_{\mathcal{F}_{\in|}}|$ for any $j \in \{1, 2, 3\}$, we revised the matrix M and M' in Table 2. Furthermore, compute the four correlation coefficients between these alternatives and the ideal alternatives in Table 3. The result shows the R&D project L_3 are always the most optimal, although the four ranking results (Table 4) are little different.

4.3. Medical decision-making

Example 4.2 *A clinic wants to buy a panoramic X-ray machine. In the research conducted for this purchase process, a selection will be made from four different brands ($\widehat{\mathcal{F}} = \{\widehat{\mathcal{F}_1}, \widehat{\mathcal{F}_2}, \widehat{\mathcal{F}_3}, \widehat{\mathcal{F}_4}\}$). When making this choice, three different criteria are considered: C_1: Robustness and permanence, C_2: service support, C_3: price*

Table 1. IVFHFDM M.

	C_1	C_2	C_3
$\mathcal{F}_1$	$\{([0.6, 0.8], [0.4, 0.5]),$ $([0.5, 0.8], [0.1, 0.3]),$	$\{([0.5, 0.7], [0.1, 0.3]),$ $([0.4, 0.6], [0.2, 0.5])\}$	$\{([0.5, 0.7], [0.3, 0.4])\}$
$\mathcal{F}_2$	$\{([0.4, 0.7], [0.2, 0.3])\}$	$\{([0.5, 0.6], [0.3, 0.4])\}$ $\{([0.3, 0.5], [0.2, 0.3])\}$ $\{([0.2, 0.7], [0.2, 0.4])\}$ $\{([0.3, 0.5], [0.2, 0.7])\}$	$\{([0.3, 0.7], [0.2, 0.4]),$ $([0.2, 0.5], [0.3, 0.4])\}$ $([0.2, 0.6], [0.2, 0.3])\}$
$\mathcal{F}_3$	$\{([0.3, 0.5], [0.1, 0.2])\}$ $\{([0.5, 0.7], [0.3, 0.4])\}$	$\{([0.6, 0.8], [0.2, 0.3]),$ $([0.6, 0.9], [0.1, 0.2])\}$	$\{([0.3, 0.4], [0.5, 0.6]),$ $\{([0.3, 0.4], [0.6, 0.7])\}$ $\{([0.4, 0.5], [0.4, 0.6])\}$
$\mathcal{F}_4$	$\{([0.5, 0.6], [0.2, 0.4])\}$	$\{([0.4, 0.5], [0.3, 0.4]),$ $([0.3, 0.5], [0.3, 0.4])\}$	$\{([0.5, 0.8], [0.3, 0.4]),$

Table 2. Revised IVFHFDM M'.

	C_1	C_2	C_3
$\mathcal{F}_1$	$\{([0.5, 0.8], [0.1, 0.3]),$ $([0.6, 0.8], [0.4, 0.5]),$	$\{([0.5, 0.7], [0.1, 0.3]),$ $([0.4, 0.6], [0.2, 0.5])\}$ $([0.4, 0.6], [0.2, 0.5])\}$ $([0.4, 0.6], [0.2, 0.5])\}$	$\{([0.5, 0.7], [0.3, 0.4])\}$ $\{([0.5, 0.7], [0.3, 0.4])\}$ $\{([0.5, 0.7], [0.3, 0.4])\}$
$\mathcal{F}_2$	$\{([0.4, 0.7], [0.2, 0.3])\}$ $\{([0.4, 0.7], [0.2, 0.3])\}$	$\{([0.5, 0.6], [0.3, 0.4])\}$ $\{([0.2, 0.7], [0.2, 0.4])\}$ $\{([0.3, 0.5], [0.2, 0.3])\}$ $\{([0.3, 0.5], [0.2, 0.7])\}$	$\{([0.3, 0.7], [0.2, 0.4])\},$ $\{([0.2, 0.6], [0.2, 0.3])\}$ $\{([0.2, 0.5], [0.3, 0.4])\}$
$\mathcal{F}_3$	$\{([0.5, 0.7], [0.3, 0.4])\}$ $\{([0.3, 0.5], [0.1, 0.2])\}$	$\{([0.6, 0.8], [0.2, 0.3])\},$ $\{([0.6, 0.8], [0.2, 0.3])\},$ $\{([0.6, 0.9], [0.1, 0.2])\}$ $\{([0.6, 0.9], [0.1, 0.2])\}$	$\{([0.4, 0.5], [0.4, 0.6])\},$ $\{([0.3, 0.4], [0.5, 0.6])\}$ $([0.3, 0.4], [0.6, 0.7])\}$
$\mathcal{F}_4$	$\{([0.5, 0.6], [0.2, 0.4])\}$ $\{([0.5, 0.6], [0.2, 0.4])\}$	$\{([0.4, 0.5], [0.3, 0.4])\},$ $([0.4, 0.5], [0.3, 0.4])\}$ $([0.3, 0.5], [0.3, 0.4])\}$ $([0.3, 0.5], [0.3, 0.4])\}$	$\{([0.5, 0.8], [0.3, 0.4])\},$ $\{([0.5, 0.8], [0.3, 0.4])\}$ $\{([0.5, 0.8], [0.3, 0.4])\}$

Table 3. Correlations Coefficients of IVFHFSs.

	$(\mathcal{F}_1, \mathcal{F}^*)$	$(\mathcal{F}_2, \mathcal{F}^*)$	$(\mathcal{F}_3, \mathcal{F}^*)$	$(\mathcal{F}_4, \mathcal{F}^*)$
$\mathfrak{C}$	0.3144	0.207	0.4737	0.2116
$\mathfrak{D}$	0.2225	0.1864	0.3974	0.1721
$\mathfrak{C}_\rho$	0.5937	0.4851	0.6803	0.498
$\mathfrak{D}_\rho$	0.4765	0.4469	0.5154	0.4319

Table 4. Ranking.

$\mathfrak{C}(\mathcal{F}_3, \mathcal{F}^*) > \mathfrak{C}(\mathcal{F}_1, \mathcal{F}^*) > \mathfrak{C}(\mathcal{F}_4, \mathcal{F}^*) > \mathfrak{C}(\mathcal{F}_2, \mathcal{F}^*)$
$\mathfrak{D}(\mathcal{F}_3, \mathcal{F}^*) > \mathfrak{D}(\mathcal{F}_1, \mathcal{F}^*) > \mathfrak{D}(\mathcal{F}_2, \mathcal{F}^*) > \mathfrak{D}(\mathcal{F}_4, \mathcal{F}^*)$
$\mathfrak{C}_\rho(\mathcal{F}_3, \mathcal{F}^*) > \mathfrak{C}_\rho(\mathcal{F}_1, \mathcal{F}^*) > \mathfrak{C}_\rho(\mathcal{F}_4, \mathcal{F}^*) > \mathfrak{C}_\rho(\mathcal{F}_2, \mathcal{F}^*)$
$\mathfrak{D}_\rho(\mathcal{F}_3, \mathcal{F}^*) > \mathfrak{D}_\rho(\mathcal{F}_1, \mathcal{F}^*) > \mathfrak{D}_\rho(\mathcal{F}_2, \mathcal{F}^*) > \mathfrak{D}_\rho(\mathcal{F}_4, \mathcal{F}^*)$

performance. C_1 and C_2 are benefit criteria, C_3 are cost criteria. Let the weight vector of criteria be $\omega = (0.28, 0.47, 0.25)^T$.

Table 5. IVFHF M.

	C_1	C_2	C_3
$\widehat{\mathcal{F}_1}$	$\{\langle[0.58, 0.68], [0.41, 0.52]\rangle,$ $\langle[0.57, 0.76], [0.18, 0.33]\rangle\}$	$\{\langle[0.46, 0.59], [0.15, 0.35]\rangle,$ $\langle[0.37, 0.58], [0.23, 0.54]\rangle\}$	$\{\langle[0.56, 0.72], [0.18, 0.37]\rangle\}$
$\widehat{\mathcal{F}_2}$	$\{\langle[0.46, 0.67], [0.27, 0.41]\rangle\}$	$\{\langle[0.54, 0.62], [0.35, 0.42]\rangle,$ $\langle[0.38, 0.51], [0.20, 0.33]\rangle$ $\langle[0.25, 0.65], [0.24, 0.40]\rangle,$ $\langle[0.35, 0.48], [0.21, 0.68]\rangle\}$	$\{\langle[0.39, 0.51], [0.22, 0.35]\rangle,$ $\langle[0.21, 0.53], [0.37, 0.42]\rangle$ $\langle[0.25, 0.58], [0.14, 0.36]\rangle\}$
$\widehat{\mathcal{F}_3}$	$\{\langle[0.33, 0.45], [0.16, 0.22]\rangle,$ $\langle[0.58, 0.68], [0.26, 0.37]\rangle\}$	$\{\langle[0.60, 0.80], [0.14, 0.22]\rangle,$ $\langle[0.66, 0.72], [0.24, 0.29]\rangle\}$	$\{\langle[0.33, 0.44], [0.37, 0.59]\rangle,$ $\langle[0.38, 0.44], [0.55, 0.75]\rangle$ $\langle[0.43, 0.58], [0.34, 0.67]\rangle\}$
$\widehat{\mathcal{F}_4}$	$\{\langle[0.55, 0.65], [0.22, 0.44]\rangle\}$	$\{\langle[0.46, 0.51], [0.37, 0.42]\rangle,$ $\langle[0.28, 0.48], [0.31, 0.42]\rangle\}$	$\{\langle[0.48, 0.77], [0.23, 0.37]\rangle\}$

IVFHF values are given in Table 5. Table 6 shows the revised IVFHF decision matrix, since $|h_{\widehat{\mathcal{F}_{1j}}}| \neq |h_{\widehat{\mathcal{F}_{2j}}}|$ ($j = 1, 2, 3$). Table 7 gives the correlations and informational energies of the four IVFHFSs. The values in Table 8 are the calculated four KKs between these alternatives and the ideal alternatives.

As can be seen from these results, there is not much difference between the ranking results, and it is seen that the $\widehat{\mathcal{F}_3}$ panoramic X-ray machine is suitable for all criteria.

5. Discussion

5.1. Comparison

The KK formulas given below were defined by Chen et al. [8] and Garg [78], respectively. For the values given below, we will calculate the correlation coefficients using the formulas 5.1 and 5.2.

In order to comprehend the examples, it is helpful to keep in mind the following details: The correlation coefficient is a measure of how strongly and in what direction the independent variables are related. The range of values for this coefficient is -1 to $+1$. A direct linear relationship is shown by a positive number, and an inverse linear relationship is indicated by a negative value. If the correlation coefficient is 0, the aforementioned variables do not have a linear connection. The closer the coefficient is to $+1$ or -1, the stronger the linearity of the relationship.

Table 6. Revised IVFHF M'.

	C_1	C_2	C_3
$\widehat{\mathcal{F}_1}$	$\{\langle[0.57,0.76],[0.18,0.33]\rangle, \langle[0.58,0.68],[0.41,0.52]\rangle\}$	$\{\langle[0.46,0.59],[0.15,0.35]\rangle, \langle[0.46,0.59],[0.15,0.35]\rangle\}$ $\{\langle[0.37,0.58],[0.23,0.54]\rangle, \langle[0.37,0.58],[0.23,0.54]\rangle\}$	$\{\langle[0.56,0.72],[0.18,0.37]\rangle, \langle[0.56,0.72],[0.18,0.37]\rangle\}$ $\{\langle[0.56,0.72],[0.18,0.37]\rangle\}$
$\widehat{\mathcal{F}_2}$	$\{\langle[0.46,0.67],[0.27,0.41]\rangle, \langle[0.46,0.67],[0.27,0.41]\rangle\}$	$\{\langle[0.54,0.62],[0.35,0.42]\rangle, \langle[0.38,0.51],[0.20,0.33]\rangle$ $\langle[0.2,0.7],[0.2,0.4]\rangle, \langle[0.35,0.48],[0.21,0.68]\rangle\}$	$\{\langle[0.39,0.51],[0.22,0.35]\rangle, \langle[0.25,0.58],[0.14,0.36]\rangle$ $\langle[0.21,0.53],[0.37,0.42]\rangle\}$
$\widehat{\mathcal{F}_3}$	$\{\langle[0.58,0.68],[0.26,0.37]\rangle, \langle[0.33,0.45],[0.16,0.22]\rangle\}$	$\{\langle[0.60,0.80],[0.14,0.22]\rangle, \langle[0.60,0.80],[0.14,0.22]\rangle\}$ $\langle[0.66,0.72],[0.24,0.29]\rangle, \langle[0.38,0.44],[0.55,0.75]\rangle\}$	$\{\langle[0.43,0.58],[0.34,0.67]\rangle, \langle[0.3,0.4],[0.4,0.6]\rangle$ $\langle[0.38,0.44],[0.55,0.75]\rangle\}$
$\widehat{\mathcal{F}_4}$	$\{\langle[0.55,0.65],[0.22,0.44]\rangle, \langle[0.55,0.65],[0.22,0.44]\rangle\}$	$\{\langle[0.46,0.51],[0.37,0.42]\rangle, \langle[0.46,0.51],[0.37,0.42]\rangle\}$ $\{\langle[0.28,0.48],[0.31,0.42]\rangle, \langle[0.28,0.48],[0.31,0.42]\rangle\}$	$\{\langle[0.48,0.77],[0.23,0.37]\rangle, \langle[0.48,0.77],[0.23,0.37]\rangle$ $\{\langle[0.48,0.77],[0.23,0.37]\rangle\}$

Table 7. Correlations of M'.

	C	D	C_ρ	D_ρ
$\widehat{\mathcal{F}_1}$	0.108	0.2623	0.3941	0.2616
$\widehat{\mathcal{F}_2}$	0.0963	0.2037	0.3628	0.1962
$\widehat{\mathcal{F}_3}$	0.1485	0.2809	0.4039	0.2508
$\widehat{\mathcal{F}_4}$	0.0842	0.1875	0.3612	0.2084

Table 8. Correlation coefficients of IVFHFSs.

	$(\widehat{\mathcal{F}_1},\widehat{\mathcal{F}^*})$	$(\widehat{\mathcal{F}_2},\widehat{\mathcal{F}^*})$	$(\widehat{\mathcal{F}_3},\widehat{\mathcal{F}^*})$	$(\widehat{\mathcal{F}_4},\widehat{\mathcal{F}^*})$
$\mathfrak{C}$	0.4033	0.2816	0.5413	0.2896
$\mathfrak{D}_\rho$	0.2861	0.2440	0.3837	0.2121
$\mathfrak{C}_\rho$	0.6514	0.5702	0.7302	0.5728
$\mathfrak{D}_\rho$	0.5029	0.4576	0.5734	0.4562

$$\mathfrak{C}(F,G)_{Chen_1} = \frac{\sum_{i=1}^{n}\left[\frac{1}{li}\sum_{j=1}^{li}\rho_{F\sigma(j)}(u_i)\rho_{G\sigma(j)}(u_i)\right]}{\left[\sum_{i=1}^{n}\left(\frac{1}{l_{Fi}}\sum_{j=1}^{l_{Fi}}\rho^2_{F\sigma(j)}(u_i)\right)\right]^{1/2}\left[\sum_{i=1}^{n}\left(\frac{1}{l_{(Gi)}}\sum_{j=1}^{l_{(Gi)}}\rho^2_{G\sigma(j)}(u_i)\right)\right]^{1/2}} \tag{5.1}$$

Consider the two HFSs $F_1 = \{(u,[0.0])\}$, $F_2 = \{(u,[0.2,0.3])\}$ for $U = \{u\}$. Using the formula 5.1, since the informational energy of F is equal to 0, then the correlation coefficient between F_1 and F_2 cannot compute.

For $h_{\mathcal{F}}(u) = \{\rho_{\mathcal{F}}(u)\}$, take the HFS $\mathcal{F} = \{(u, h_{\mathcal{F}}(u)) : u \in U\}$. Now, we can generalize $h_{\mathcal{F}}(u)$ to $\mathcal{F} = \{(\rho_{\mathcal{F}}(u), \sqrt{1-\rho_{\mathcal{F}}^2(u)})\}$. Therefore, $\mathcal{F}$ is extended to Pythagorean hesitant fuzzy set (PHFS). Zheng et al. [78] developed the LCME method to solve the problems caused by the different cardinati of the PHFEs. According to this method, the KK of the PHFS is as follows:

$$\mathfrak{C}(F,G)_{Chen_2} = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{3S_i}\sum_{j=1}^{S_i}\left[\rho_{F\sigma(j)}^2\rho_{G\sigma(j)}^2 + \eta_{F\sigma(j)}^2\eta_{G\sigma(j)}^2 + \theta_{F\sigma(j)}^2\theta_{G\sigma(j)}^2\right]\right)}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\rho_{Fj}^4(u_i) + \eta_{Fj}^4(u_i) + \theta_{Fj}^4(u_i)\right)}\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\rho_{Gj}^4(u_i) + \eta_{Gj}^4(u_i) + \theta_{Gj}^4(u_i)\right)}} \tag{5.2}$$

Consider the two PHFSs $F_1 = \{(u, [0.0])\}$, $F_2 = \{(u, [0.2, 0.3])\}$ for $U = \{u\}$. Then, we can obtain $h_{F_1}(u) = \{(0,1)\}$, $h_{F_2}(u) = \{(0.2, \sqrt{0.96}), (0.3, \sqrt{0.91})\}$. Using the formula 5.2, $\mathfrak{C}(F,G)_{Chen_2} = 0.935$.

If we use similar thinking for FHFS and calculate with the same values, then $h_{F_1}(u) = \{(0,1)\}$, $h_{F_2}(u) = \{(0.2, \sqrt{0.997}), (0.3, \sqrt{0.991})\}$, where $\mathcal{F} = \{(\rho_{\mathcal{F}}(u), \sqrt[3]{1-\rho_{\mathcal{F}}^3(u)})\}$. Hence, we can obtain $\mathfrak{C}(F,G) = 0.9969$.

5.2. Advantages of the new approach

The following merits have been taken into account from the proposed correlation coefficient:

As mentioned above, the FFS is one of the generalizations of the classic set, FS, IFS, and PFS. As PFS is one of the most successful extensions, which is characterized by the degrees of membership and nonmembership satisfaction of the particular alternative concerning the criteria, such that their sum of the square is equal to or less than 1. However, there may be a situation where the decision-maker may provide the degree of membership and nonmembership of a particular attribute in such a way that their sum of squares is greater than 1. Therefore, this situation is not properly handled in the PFS. To overcome this shortcoming, FFS theory is one of the more general and can handle not only incomplete information but also indeterminate information and inconsistent information, which exist commonly in real situations. Therefore, the Fermatean fuzzy information decision-making is more suitable for real scientific and engineering applications.

Furthermore, it has been observed from the existing studies that various researchers proposed an algorithm by using a correlation coefficient for PFSs. As mentioned above, some situations cannot be represented by PFSs, so their corresponding algorithm may not give appropriate results.

The correlation coefficient for PFSs is a special case of the correlation coefficient of FFSs. Accordingly, obtaining correlation coefficients according to IVFHFS gives wider opportunities to deal with uncertainty. Therefore, the proposed correlation coefficient is more generalized and suitable to solve real-life problems more accurately than the existing ones.

6. Conclusion

In this paper, a correlation coefficient for IVFHFSs has been proposed. The short-coming of the existing operators have also been highlighted in this paper. Based on that, the present paper have extended the theory of correlation coefficient from PFS to the FFSs in which the constraint condition of sum of membership and nonmembership degrees be less than one has been relaxed. Numerical example have been given that demonstrate that the proposed correlation coefficient can easily handle the situation where the existing correlation coefficient in PFS environment fails. The proposed correlation coefficient in FFS has been developed by taking the degree of membership, nonmembership, and their degree of hesitation between them. Also to deal with the situations where the elements in a set are correlative, a weighted correlation coefficient has been defined. To demonstrate the efficiency of the proposed coefficients, numerical examples of pattern recognition and multicriteria decision-making have been taken. From the experimental studies, it has been concluded that the proposed correlation coefficient in the FFS environment can suitably handle the real-life decision-making problem with their targets. In future studies, it will be possible to create new concepts with the methods proposed in this article and ideas from studies such as [37–39, 47, 57].

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