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Akbulut's corks and h-cobordisms of smooth, simply connected 4-manifolds

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Theorem: Let M^5 be a smooth 5-dimensional h-cobordism between two simply connected, closed 4-manifolds, M_0 and M_1. Then there exists a sub-h-cobordism $A^5 \subset M^5$ between $A_0 \subset M_0$ and $A_1 \subset M_1$ with the properties:

(1): A_0 and hence A and A_1 are compact contractible manifolds, and

(2): $M - intA$ is a product h-cobordism, i.e. it is diffeomorphic to $(M_0 - intA_0) \times [0, 1]$.

This theorem first appeared in a preprint of Curtis & Hsiang in fall 1994. Soon after, much shorter proofs were found by Freedman & Stong [3], Matveyev [9], and Z. Bižaca. The following improvements were also shown:

Addenda: The h-cobordism A can be chosen so that,

(A): $M - A$ (and hence each $M_i - A_i$) is simply connected (Freedman & Stong) [3],

(B): A is diffeomorphic to B^5 (Bižaca, Kirby) (but not, of course, preserving the structure of the h-cobordism),

(C): $A_0 \times I$ and $A_1 \times I$ are diffeomorphic to B^5 [9],

(D): A_0 is diffeomorphic to A_1 by a diffeomorphism which, restricted to $\partial A_0 = \partial A_1$, is an involution [9].

Corollary: Any homotopy 4-sphere, Σ^4, can be constructed by cutting out a contractible 4-manifold, A_0 from S^4 and gluing it back in by an involution of ∂A_0.

Remark: Since there are many examples of non-trivial h-cobordisms (the first ones were discovered by Donaldson [4]), there are as many examples of non-trivial, rel boundary, h-cobordisms A. However these A are delicate objects; their non-triviality vanishes when a trivial h-cobordism is added. That is, if we add $A_0 \times I$ to A along $\partial A_0 \times I$, then it follows from the Addenda that we have an h-cobordism between S^4 on the bottom as well as S^4 on the top; thus the h-cobordism is the trivial $S^4 \times I$.

1. The first example of a non-trivial h-cobordism, A, on B^5 was found by Akbulut [1]. It is the prototype of the h-cobordisms A in the theorem, and it seems appropriate to call such h-cobordisms *Akbulut's corks*, for any exotic h-cobordism can be constructed from the product h-cobordism by pulling out a *cork* and putting it back in with a *twist*, (which preserves A_0 but not the structure of the h-cobordism).

Akbulut constructs a homology $S^2 \times S^2 - point$, called $A_{1/2}$, by adding two 2-handles to the symmetric link L of unknots drawn in Figure 1. Because this link L is symmetric, there

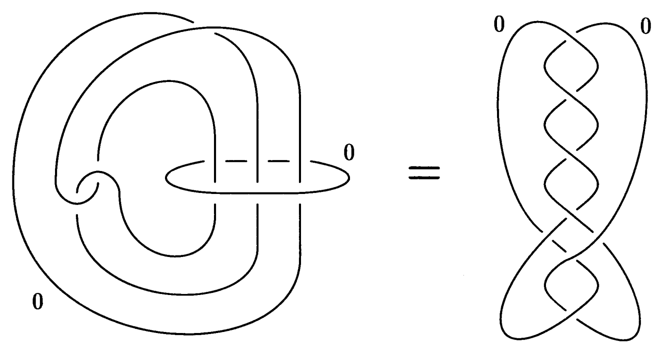

Figure 1.

is an involution $\tau : \partial A_{1/2} \to \partial A_{1/2}$, which extends to a diffeomorphism $\overline{\tau} : A_{1/2} \to A_{1/2}$ which switches the 2-handles (to extend τ over the 0-handle of $A_{1/2}$, one can either cone the involution on S^3 obtaining an involution which is not smooth at the cone point, or extend over the 0-handle using the fact that the involution on S^3 is diffeotopic to the identity).

Since the components of L are 0-framed unknots, one can trade either of the 2-handles (but not both for L is not the unlink) for a 1-handle, obtaining A_0 or A_1 respectively. (Recall that adding a 2-handle to a 0-framed unknot gives $S^2 \times B^2$, adding an orientable 1-handle to B^4 gives $B^3 \times S^1$, and both have boundary $S^2 \times S^1$; thus one may change the 4-manifold by trading 2-handles for 1-handles, or vice versa, and then adding all other handles to the $S^2 \times S^1$ boundary as before the trade.) The new 1-handles are denoted by the same unknot but with a dot on it, which means that any arcs going through the dotted circle actually go over a 1-handle. This can be seen by observing that a dotted circle means: remove the obvious, properly imbedded, 2-ball in the 0-handle leaving $S^1 \times B^3$, which is a 0-handle union a 1-handle. This operation does not change boundaries, so $\partial A_{1/2} = \partial A_0 = \partial A_1$.

Akbulut proves (using a long series of handle moves culminating in an application of Donaldson's invariants) that the identity map, $id : \partial A_0 \to \partial A_1$, does not extend to a diffeomorphism of A_0 to A_1.

The notation A_0, $A_{1/2}$, A_1, suggests there is an h-cobordism lurking about, and this is correct. The operation of trading 2-handles for 1-handles can also be done by adding 3-handles in the right way. Each component of L, K_0 or K_1, determines a 2-sphere, S_0 or S_1, composed of the core of the 2-handle and the obvious slice disk that K_i bounds in the 0-handle. Each 2-sphere has a trivial normal bundle (because of the 0-framing), and $S_0 \cap S_1$ is three points (algebraically one). To construct an h-cobordism A, start with $A_{1/2} \times [1/2 - \epsilon, 1/2 + \epsilon]$ and add a 3-handle to $S_0 \times (1/2 - \epsilon)$ and a 3-handle to $S_1 \times (1/2 + \epsilon)$. The new boundary on the bottom will be A_0 because $S_0 \times B^2$ has been removed from $A_{1/2} \times (1/2 - \epsilon)$, thus removing the 2-handle and the slice disk which has the effect of switching the 2-handle to a 1-handle. Similarly for A_1. The structure of the h-cobordism A is to add a 2-handle to A_0 (the 3-handle turned upside down) and the 3-handle to S_1. A must be non-trivial because it is a product over ∂A_0, describing $id : \partial A_0 \to \partial A_1$, and Akbulut showed this cannot extend to a diffeomorphism from A_0 to A_1.

There is a natural generalization of Akbulut's cork. Let the 0-handle in $A_{1/2}$ be replaced by a contractible 4-manifold $B_{1/2}$. Suppose D is a collection of $2n$ properly imbedded 2-balls, $D_{0,i} \cup D_{1,i}, i = 1 \ldots n$, in $B_{1/2}$, and suppose that $D_{0,i} \cap D_{0,j} = \emptyset = D_{1,i} \cup D_{1,j}$ for all $i, j \in 1, \ldots, n$, and that algebraically $D_{0,i} \cap D_{1,j} = \delta_{ij}$. Then we can form $A_{1/2}$ by adding 2-handles with 0-framings to each $\partial D_{0,i}$ and $\partial D_{1,i}$, $i = 1, \ldots, n$. This produces obvious 2-spheres with trivial normal bundles, $S_{0,i}$ and $S_{1,i}$, $i = 1, \ldots, n$. Then we form our h-cobordism A by adding n 3-handles below and n 3-handles above to the $S_{0,i}$'s and the $S_{1,i}$'s respectively.

We can conjecture that the product structure on the sides of A, namely $\partial A_0 \times I$, extends over A iff D is *concordant* in $B_{1/2} \times I$ to the ∂-connected sum of n copies of $B^2 \times 0 \cup 0 \times B^2$ in $B^4 \subset B_{1/2}$.

2. Here is a proof of the Theorem. The exposition is not particularly original, but gains by organizing all the steps into a whole rather than having them split into the two papers [3] and [9].

We begin with a Morse function $f : (M, M_0, M_1) \to (I, 0, 1)$ and its associated handlebody structure which adds k-handles, $0 \leq k \leq 5$, to M_0. We can cancel all 0-handles and 5-handles since M is connected. We can cancel all 1-handles and 4-handles (at the cost of new 2 and 3-handles) just as Smale did in the original proof of the higher dimensional h-cobordism theorem ([10] Lemma 6.15). Alternatively, we may have the h-cobordism provided by Wall (see [11], [5] Chapter 9) which begins with homotopy equivalent, simply connected closed, smooth 4-manifolds M_0 and M_1 and constructs M using only 2 and 3-handles. (This involves no loss of generality because any two h-cobordisms between M_0 and M_1 are diffeomorphic [7, 8].)

We can assume that $M_{1/2} = f^{-1}(1/2)$ has all the 2-handles below and all the 3-handles above. Note that $M_{1/2}$ is 1-connected which is always true of the upper boundary when 2-handles are attached to a {simply connected 4-manifold} $\times I$. Each 2-handle has an ascending 3-ball which meets $M_{1/2}$ in a smoothly imbedded 2-sphere; call these $S_{0,i}, i = 1, \ldots, n$. Similarly each 3-handle descends to meet $M_{1/2}$ in $S_{1,i}, i = 1, \ldots, n$. We can assume, perhaps after some handle slides, that the boundary map from 3-chains (generated by the 3-handles) to 2-chains (generated by 2-handles) is given by the identity matrix, or, equivalently, that algebraically $S_{0,i} \cap S_{1,j} = \delta_{ij}$.

3. Choose a base point $*$ in $M_{1/2}$ minus all the spheres $S_{k,i}$. Choose $2n$ arcs in general position which connect $*$ to basepoints $*_{k,i}$ in the $S_{k,i}$. A regular neighborhood of these arcs will be our 0-handle in a forthcoming handlebody structure on $M_{1/2}$. In each $S_{k,i}$, run disjoint arcs from $*_{k,i}$ to each point of intersection of $S_{k,i}$ with some $S_{k',j}$ (with $k \neq k'$). These arcs come in pairs, from $*_{k,i}$ to a point in $S_{k,i} \cap S_{k',j}$ to $*_{k',j}$, and a regular neighborhood forms a 1-handle attached to the 0-handle. Each $S_{k,i}$, minus the regular neighborhood of the tree of arcs in it, gives a 2-handle, $H_{k,i}$, which is added to the 0-handle and 1-handles. Figure 2 shows how the $H_{k,i}$'s behave with respect to the 1-handles; note that each $H_{k,i}$ is attached to an unknot and the $H_{0,i}$'s and the $H_{1,i}$'s are each attached to unlinks of n components. Note that we can assume that the $H_{0,i}$ do not go over any 1-handles, whereas the $H_{1,i}$ go over and back so that, if the one handles correspond to generators $x_1, x_2, \ldots x_r$ of the fundamental group $\pi_1(M_{1/2})$, then the $H_{1,i}$ give relators equal to a product of $x_i\overline{x}_i$'s and $\overline{x}_i x_i$'s, $i \in 1, \ldots, r$.

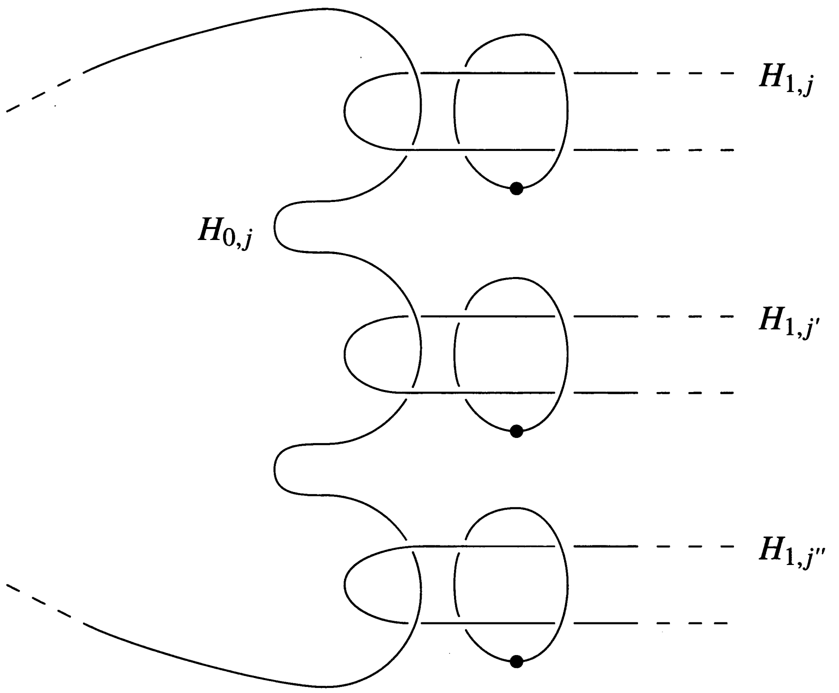

Figure 2.

So far we have chosen a 0-handle and some 1 and 2-handles in $M_{1/2}$. Extend this handlebody to a handlebody structure on all of $M_{1/2}$ (it may have more 1-handles (still indexed by $1 \ldots r$) as well as 2 and 3-handles, but extra 0- and 4-handles may be avoided). Since $M_{1/2}$ is 1-connected and the $H_{k,i}$ give trivial relators, it follows that the other 2-handles $H_l, l = 1, \ldots s$ must homotopically kill the 1-handles, where we assume that the attaching circle of each H_l has a base point $*_l$ which has been connected by an arc to $*$.

Note that when we slide H_l over H_m, along an arc λ joining $*_l$ to $*_m$ then we replace the relator r_l by the relator $r_l \lambda r_m^{\pm 1} \overline{\lambda}$; λ can be chosen to be trivial if necessary.

It follows from elementary combinatorial group theory that we can slide 2-handles over 2-handles so as to end up with the $H_l, l = 1, \ldots r$, exactly killing the generators $x_1, \ldots x_r$; that is, $H_i = x_i w_i$ where w_i cancels away to 1 using only the relations $x_j \overline{x}_j = 1 = \overline{x}_j x_j$. During this process, it may have been necessary to add cancelling pairs of 2- and 3-handles, so as to slide a new 2-handle over some H_l which is about to be altered by sliding over another handle; the new 2-handle preserves the relator r_l for later use in the sequence of Tietze moves which reduces the original presentation to the *trivial* one. These new 2-3 pairs may be necessary to avoid the difficulties inherent in the Andrews-Curtis Conjecture [2], [6] Problem 5.2.

Let $B_{1/2}$ be the contractible manifold formed by the 0-handle, all the 1-handles, and the 2-handles $H_l, l = 1, \ldots, r$. Let $A_{1/2}$ be $B_{1/2}$ union the 2-handles $H_{k,i}, k \in 0, 1, i \in 1, \ldots n$. Then A will be $A_{1/2}$ (thickened by crossing with $[1/2 - \epsilon, 1/2 + \epsilon]$) together with the 3-handles added below to the $S_{0,i}$'s and above to the $S_{1,i}$'s.

Clearly A is contractible (since $B_{1/2}$ is contractible and the 3-handles cancel the $H_{k,i}$). Since A contains the 2 and 3-handles of the h-cobordism M, it follows that $M - intA$ is a product h-cobordism. This finishes the proof of the Theorem.

4. Proof of the Addenda:

(B), (C) and (D) are easiest to prove so we start there.

A is diffeomorphic to $B_{1/2} \times I$ because the 3-handles added to $A_{1/2}$ geometrically cancel the 2-handles $H_{k,i}$, $k = 0, 1$, $i = 1, \ldots n$. Furthermore $B_{1/2} \times I$ is diffeomorphic to B^5 because homotopic circles in a 4-manifold are isotopic, so the attaching maps of the H_l's can be isotoped to geometrically cancel the 1-handles, leaving only the 0-handle of $B_{1/2} \times I$. This proves (B).

A_0 is contractible because A is, but we need to also know that each 1-handle of A_0 is homotopically cancelled by a 2-handle. A_0 is $A_{1/2}$ but with a dot on each attaching circle of the $H_{0,i}$'s. These dotted circles give n new generators, y_i, $i = 1, \ldots, n$, to the presentation for $\pi_1(B_{1/2})$, and the $H_{1,i}$, $i = 1, \ldots, n$, are n new relators, s_i, $i = 1, \ldots, n$. At this point we need to go back and make a careful choice of the arcs in each $S_{1,i}$ which join $*_{1,i}$ to the points of intersection of $S_{1,i}$ with the spheres $S_{0,j}$, $j = 1, \ldots, n$. We first run arcs from $*_{i,1}$ to all points of intersection with $S_{0,1}$, then with $S_{0,2}$, then $S_{0,3}$, and so on to $S_{0,n}$. This is easy to do because trees do not separate points in dimension 2. With this choice of arcs, it follows that the attaching circle of $H_{1,i}$ reads off the word $w_1 w_2 \ldots w_n$ where w_j is a word in the y_j and $\overline{y}_j$ with exponent sum zero if $j \neq i$ and exponent sum one if $j = i$.

Thus the 2-handles H_l, $l = 1, \ldots, r$, kill $x_1, \ldots, x_r$ and then the 2-handles $H_{1,i}$, $i = 1, \ldots, n$, kill the generators $y_1, \ldots, y_n$. Therefore, $A_0 \times I$ is diffeomorphic to B^5, because homotopy implies isotopy for 1-manifolds in 4-manifolds, so the 2-handles geometrically cancel the 1-handles since they do so homotopically. Similarly $A_1 \times I$ is B^5. This finishes the proof of Addenda (C).

5. To prove (D), we increase the size of the h-cobordism A. Choose a 4-ball B_0^4 in M_0 such that $B_0^4 \cap A_0 = \partial B_0^4 \cap \partial A_0 = B^3$. M is a product, $B^4 \times I$, over B_0^4.

Since A is B^5, it follows that $\partial A = A_0 \cup_\partial A_1 = S^4$. If we remove an open 4-ball, which intersects ∂A_i in a 3-ball, from ∂A, then the result, $(A_0 \cup_\partial A_1)_0$, can be identified with B_0^4. Similarly, using the fact that $A_0 \times I$ is B^5, we can identify $(A_0 \cup_\partial A_0)_1$ with B_1^4. Then the product h-cobordism $(B^4 \times I, B_0^4, B_1^4)$ can be identified with $(A_0 \times I \cup_\partial A^{-1}, (A_0 \cup_\partial A_1)_0, (A_0 \cup_\partial A_0)_1)$ where A^{-1} is A upside down and $A_0 \times I$ and A^{-1} are joined along $(\partial A_0 - intB^3) \times I$.

Now we enlarge the h-cobordism A by adding A^{-1} to it (see Figure 3). Clearly the complement is still a product, and clearly the top and bottom of $A \cup A^{-1}$, namely $(A_1 \cup_{B^3} A_0)_1$ and $(A_0 \cup_{B^3} A_1)_0$, are diffeomorphic by the obvious involution. This proves (D).

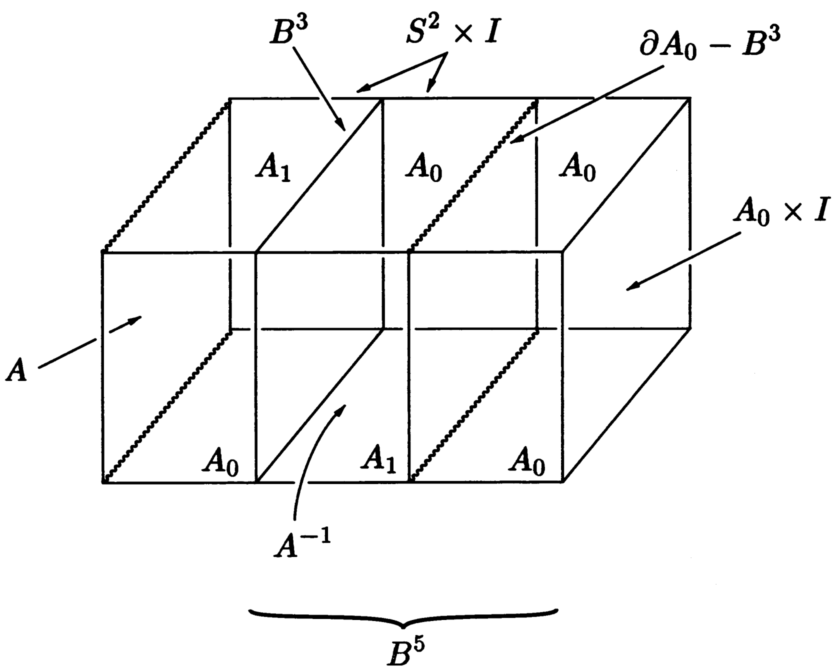

Figure 3.

6. To prove Addendum (A), that A can be chosen so that the complement $C = M - intA$ is simply connected, we must go back to the point in the argument in which $A_{1/2}$ was constructed with r 1-handles, r 2-handles H_l, $l = 1, \ldots, r$, and the $2n$ 2-handles $H_{k,i}, k \in 0, 1, i \in 1, \ldots n$. The complement of $A_{1/2}$ has zero first homology, but it may not be simply connected.

Let L_1 be a level set of $M_{1/2}$ after the 1-handles have been attached to the 0-handle, ($L_1 = \#rS^1 \times S^2$), and let L_3 ($= \#tS^1 \times S^2$) be a level set of $M_{1/2}$ just before the

3-handles are attached (equivalently, the boundary of the 4-handle union the 3-handles). Let $L_1 \cap L_3$ be denoted by Q^3; it can be thought of as L_1 minus the attaching circles of all the 2-handles, or L_3 minus the co-circles of the 2-handles.

Let $C_{1/2} = M_{1/2} - A_{1/2}$. Since $H_1(C_{1/2}) = 0$, it follows that $\pi_1(C_{1/2})$ is generated by commutators, so we can change it to zero if we have a method of sliding 2-handles that gives us new 2-handles which kill commutators, but does not affect $\pi_1(A_{1/2}) = 0$. Here is such a method:

All slides of 2-handles over other 2-handles must take place along arcs λ lying in Q (with endpoints at $*_\alpha$ and $*_\beta$, which are connected to $*$ for fundamental group computations). If H_α and H_β are 2-handles giving relations r_α and r_β in the generators $x_1 \dots x_r$ of $\pi_1(L_1)$, and if we slide H_α over H_β using the arc λ and then slide H_α back over H_β using the arc μ, then r_α is replaced by

$$r_\alpha \lambda r_\beta \lambda^{-1} \mu \bar{r}_\beta \mu^{-1} = r_\alpha [\bar{\mu}\lambda, r_\beta]^\mu.$$

(Note that if λ is homotopic to μ in $\pi_1(L_1)$, then r_α is unchanged.) The effect of these two slides on the generators of $\pi_1(L_3)$ is this: the co-circles of H_α and H_β provide relations r'_α and r'_β in the generators of $\pi_1(L_3)$. When the 2-handle dual to H_β slides over the 2-handle dual to H_α, and then back again, r'_β is replaced by

$$r'_\beta[\mu'\overline{\lambda}', r'_\alpha]^{\overline{\mu}'},$$

where λ' and μ' describe the homotopy classes of λ and μ in $\pi_1(L^3)$.

7. Proposition: It is possible to choose an arc λ which represents any two given elements in $\pi_1(L_1)$ and $\pi_1(L_3)$. That is, if $j_i : \pi_1(Q) \to \pi_1(L_i)$, $i = 1, 3$, then

$$\pi_1(Q) \xrightarrow{i_1 \oplus i_3} \pi_1(L_1) \oplus \pi_1(L_3)$$

is onto.

Proof: ∂Q is a collection of tori T_α; each contains loops $\gamma_{\alpha,1}$ and $\gamma_{\alpha,3}$ defined by $h_\alpha(S^1 \times \textit{point})$ and $h_\alpha(\textit{point} \times S^1)$ for the attaching map $h_\alpha : S^1 \times B^2 \to L_1$ for the 2-handle H_α. The $\{\gamma_{\alpha,1}\}$ normally generate $\pi_1(L_3)$ and represent 0 in $\pi_1(L_1)$, and similarly the $\{\gamma_{\alpha,3}\}$ normally generate $\pi_1(L_1)$ and represent 0 in $\pi_1(L_3)$. Thus one can represent $(g_1, g_3) \in \pi_1(L_1) \oplus \pi_1(L_3)$ by representing g_1 by a loop which is a product of conjugates of the $\{\gamma_{\alpha,3}\}$'s, and similarly g_3, and then composing the two loops.

8. Thus, by choosing λ and μ so that they are homotopic in L_1 but are arbitrary in L_3, we can slide H_α over H_β and back so as to replace r'_α with r'_α times any conjugate of the commutator of any element with r'_β, without changing r_α.

Recall that we have 2-handles H_l, $l = 1, \dots s$ in $M_{1/2}$ such that the H_l, $l = 1, \dots r$ belong to $A_{1/2}$ and give relators r_l killing $\pi_1(L_1)$; the cocores of the H_l, $l = 1, \dots s$ give relators r'_l, and the cocores of the $\{H_{k,i}\}$ give relators $r'_{k,i}$ which together must kill $\pi_1(L_3)$.

Since $H_1(C_{1/2}) = 0 = \pi_1(C_{1/2})/[\pi_1, \pi_1]$, it follows that the 2-handles in $C_{1/2}$, namely H_l, $l = r+1, \dots s$, give relators r'_l which, modulo $[\pi_1, \pi_1]$, kill $\pi_1(L_3)$. More precisely, the relators r'_l times a certain product of conjugates of commutators of arbitrary elements of

$\pi_1(L_3)$, i.e.

$$r'_l \prod_j [a_{l,j}, b_{l,j}]^{c_{l,j}}, \quad l = r+1, \dots s,$$

form a set of elements of $\pi_1(L_3)$ which normally generate it. If we had 2-handles $H_{l,j}$ whose cocores represented each of the $b_{l,j}$, then we could replace r'_l by $r'_l \prod_j [a_{l,j}, b_{l,j}]^{c_{l,j}}$ by sliding $H_{l,j}$ over H_l and back using arcs $\lambda_{l,j}$ and $\mu_{l,j}$ where $\mu'_{l,j}\overline{\lambda}'_{l,j} = a_{l,j}$ and $\overline{\mu}'_{l,j} = c_{l,j}$, and so that each arc is trivial in $\pi_1(L_1)$ so that the core of $H_{l,j}$ does not change its homotopy type. Having done this replacement, the cocores of the new H_l, $l = r+1, \dots s$, would kill $\pi_1(L_3)$.

So it suffices to find the 2-handles $H_{l,j}$. Suppose there are m of the $b_{l,j}$. Then we introduce m cancelling 1-,2-handle pairs into the handlebody structure on $M_{1/2}$ and include these pairs in $A_{1/2}$. Each $b_{l,j}$ is a product of conjugates of the relators r'_l, $l = 1, \dots s$, so if we slide the corresponding 2-handles H_l or $H_{k,i}$ over $H_{l,j}$, then the cocore of $H_{l,j}$ slides over the cocores and ends up representing $b_{l,j}$. Of course, the core of H_l still kills its original 1-handle, and sliding $H_{k,i}$ merely changes the isotopy class of the 2-sphere $S_{k,i}$. (This step is essentially nothing but the observation that one can always add a cancelling pair of 2-,3-handles where the 2-handle represents any desired word in the 1-handles.)

It may be useful to summarize here the whole construction. In $M_{1/2}$, choose a base point, $*$, hence a 0-handle, and then r 1-handles corresponding to each point of intersection between the ascending and descending 2-spheres. Each of these 2-spheres then provides a 2-handle $H_{k,i}$. Extend this handle structure to $M_{1/2}$. Slide 2-handles to get r 2-handles H_l which homotopically cancel the 1-handles (stablization by cancelling pairs of 2-, 3-handles to avoid Andrews-Curtis issues may have been necessary). Add some spare pairs of cancelling 1-,2-handles for later use. Inverting $M_{1/2}$ so that 1-handles become 3-handles, etc., we slide the spare 2-handles over the other 2-handles so that they will represent certain words, namely the $b_{l,j}$. Then we slide the 2-handles H'_l, $l = r+1 \dots s$, over the spare 2-handles and back so as to create relators which kill $\pi_1(L_3)$.

Now $B_{1/2}$ will consist of all of the 1-handles and all their homotopically cancelling 2-handles, so that $B_{1/2}$ is contractible. $A_{1/2}$ is $B_{1/2}$ union the $H_{k,i}$'s, as before. Finally $C_{1/2}$ is the 3-handles union the final version of the H_l, $l = 1, \dots s$. Both $A_{1/2}$ and $C_{1/2}$ are simply connected.

We use this new $A_{1/2}$ and proceed to prove Addenda (B), (C) and (D) as before (it is easy to check in proving (D) that the complement remains simply connected). This completes the proof of the Theorem and all its Addenda.

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