

A NOTE ON DOUBLE SEQUENCES OF FUZZY NUMBERS

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Abstract

In this paper we introduce double convergent sequences of fuzzy numbers and we also study the space of double convergent sequences of fuzzy numbers.

1. Introduction

The concept of fuzzy sets was first introduced by Zadeh [4] and subsequently several authors including Zadeh have discussed various aspects of the theory and applications of fuzzy sets. Convexity in fuzzy linear spaces over any valued field was introduced and discussed by Nanda [2].

In this paper we introduce double convergent sequences of fuzzy numbers and we also study the space of double convergent sequences of fuzzy numbers.

2. Let D denote the set of all closed bounded intervals $A = [\underline{A}, \bar{A}]$ on the real line R where \underline{A} and \bar{A} denote the end points of A . For $A, B \in D$ we define,

$$A \leq B \text{ iff } \underline{A} \leq \underline{B} \text{ and } \bar{A} \leq \bar{B},$$
$$d(A, B) = \max(|\underline{A} - \underline{B}|, |\bar{A} - \bar{B}|).$$

It is not hard to see that d defines a metric on D and $d(A, B)$ is called the distance between the intervals A and B . Also, it is easy to see that \leq defined above is a partial order relation in D .

A fuzzy number is a fuzzy subset of the real line R which is bounded convex and normal. Let $L(R)$ denote the set of all fuzzy numbers which are upper semicontinuous and have compact support. In other words if $X \in L(R)$ then for any $\alpha \in [0, 1]$, X^α is compact set in R , where,

$$X^\alpha = \begin{cases} t : X(t) \geq \alpha & \text{if } \alpha \in (0, 1], \\ t : X(t) > 0 & \text{if } \alpha = 0. \end{cases}$$

Define a map $\bar{d} : L(R) \times L(R) \rightarrow R$ by the rule

$$\bar{d}(X, Y) = \sup_{0 \leq \alpha \leq 1} d(X^\alpha, Y^\alpha).$$

It is straightforward to see that d is a metric in $L(R)$. For $X, Y \in L(R)$ define,

$$X \leq Y \text{ iff } X^\alpha \leq Y^\alpha \text{ for any } \alpha \in [0, 1].$$

A subset E of $L(R)$ is said to be bounded above if there exists a fuzzy number C , called an upper bounded of E , such that $X \leq C$ for every $X \in E$. C is called the least upper bound (l.u.b. or sup) of E . If C is an upper bound and is the smallest of all upper bounds. A lower bound and the greatest lower bound (g.l.b or inf) are defined similarly. E is said to be bounded if it is both bounded above and bounded below.

3. Main Results

We now introduce double sequences of fuzzy numbers.

Definition 1. A double sequence $X = (X_{mn})$ of fuzzy numbers is a function X from $N \times N$ (N is the set of all positive integers) into $L(R)$. The fuzzy number X_{mn} denotes the value of the function at a point $(m, n) \in N \times N$ and is called the (m, n) - term of the double sequence.

Definition 2. A double sequence $X = (X_{mn})$ of fuzzy numbers is said to be convergent in Pringsheim's sense if there exists a fuzzy number X_0 such that X_{mn} converges to X_0 as both m and n tend to ∞ , independently of one another:

$$\lim_{m, n} X_{mn} = X_0 \tag{3.1}$$

It is almost trivial that $X = (X_{mn})$ converges in Pringsheim's sense if and only if for every $\epsilon > 0$ there exists an integer $N = N(\epsilon)$ such that $\bar{d}(X_{jk}, X_{mn}) \leq \epsilon$ whenever $\min(j, k, m, n) \geq N$.

The crucial difference between the convergence of single sequences of fuzzy numbers and the convergence in Pringsheim's sense of double sequences of fuzzy numbers is that the latter does not imply the boundedness of the terms of the double sequence of fuzzy numbers in question.

Let C_d denote the set of all double convergent sequences of fuzzy numbers.

$X = (X_{mn})$ is said to be Cauchy sequence if for every $\epsilon > 0$ there exists $i_0 \in N$ such that

$$\bar{d}(X_{mn}^i, X_{mn}^j) \leq \epsilon \quad \text{if } \min(i, j) \geq i_0$$

It can be easily seen that $L(R)$ is a complete metric space with the metric \bar{d} . We have the following result.

Theorem 1. C_d is complete metric space with the metric defined by,

$$\rho(X, Y) = \sup_{m, n} \bar{d}(X_{mn}, Y_{mn})$$

where $X = (X_{mn})$ and $Y = (Y_{mn})$ are double convergent sequences of fuzzy numbers.

Proof. It is straightforward to see that ρ is a metric on C_d . To show that C_d is complete in this metric, let (X_{mn}^i) be a Cauchy sequence in C_d . Then for fixed m, n , (X_{mn}^i) is a Cauchy sequence in $L(R)$. But $(L(R), d)$ is complete. Hence $\lim_i X_{mn}^i = X_{mn}$ for each m, n . Put $X = (X_{mn})$. We shall now show that $\lim_i X^i = X$ and $X \in C_d$. Since (X^i) is a Cauchy sequence in C_d , given $\epsilon > 0$ there exists $i_0 \in N$ such that,

$$\rho(X^i, X^j) \leq \epsilon \quad \text{if } \min(i, j) \geq i_0 \quad (3.2)$$

Consequently the finite limits $\lim_i X_{mn}^i = X_{mn}$ exists for all m and n . Letting $j \rightarrow \infty$ in (3.2) yields,

$$\bar{d}(X_{mn}^i, X_{mn}) \leq \epsilon \quad \text{if } i \geq i_0. \quad (3.3)$$

for all m and n . Setting $X = (X_{mn})$, (3.3) shows that

$$\lim_i \rho(X^i, X) = 0 \quad (3.4)$$

we still have to verify that $X \in C_d$. By (2.3),

$$\begin{aligned} \bar{d}(X_{qk}, X_{mn}) &\leq \bar{d}(X_{qk}, X_{qk}^i) + \bar{d}(X_{qk}^i, X_{mn}^i) + \bar{d}(X_{mn}^i, X_{mn}) \\ &\leq 2\epsilon + \bar{d}(X_{qk}^i, X_{mn}^i) \end{aligned} \quad (3.5)$$

where $i \geq i_0$ is fixed. Even simpler inequalities hold for difference $\bar{d}(X_{qn}, X_{mn})$ and $\bar{d}(X_{mk}, X_{mn})$. Since $X^i \in C_d$, we can conclude $X \in C_d$ as well and this proves the completeness of C_d . □

SAVAŞ

References

- [1] Matloka, M. Sequences of Fuzzy numbers, Busefal 28 (1986), 28-37.
- [2] Nanda, S. Fuzzy Linear Spaces Over Valued fields, Fuzzy Sets and Systems, 42 (1991), 351-354.
- [3] Sava E. and Nuray F., Statistical convergence of sequence of fuzzy numbers, Math. Slovaca, 44(1994) (to appear)
- [4] Zadeh, L.A. Fuzzy Sets, Inform. and Control 8 (1965), 338-353.

FUZZY SAYILARIN ÇİFT DİZİLERİ ÜZERİNE BİR NOT

Özet

Bu çalışmada, fuzzy sayıların çift dizilerini verdik ve ayrıca fuzzy sayıların yakınsak çift diziler uzayını çalıştık.

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