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ON AFFINE SELECTION AND SEPARATION OF VECTOR VALUED FUNCTIONS

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Abstract

Let K be a Choquet simplex, E a Fréchet space and $\Phi : K \to 2^E$ a map such that $\Phi(x)$ is closed and convex for each $x \in K$. A sufficient condition is given for Φ to have a continuous affine selection. This is employed to prove a separation theorem for vector valued semicontinuous functions from a Choquet simplex into an ordered Fréchet space.

1. Preliminaries

Let K be a convex subset of a topological vector space and E another topological vector space. Suppose that Φ is a map from K into 2^E, the family of all non-empty subsets of E. An affine continuous function f from K to E is called an affine continuous selection for Φ if $f(x) \in \Phi(x)$ for every $x \in K$. A map $\Phi : K \to 2^E$ is called affine if $\Phi(x)$ is a non-empty convex subset of E for every $x \in K$ and

$$\lambda \Phi(x_1) + (1-\lambda)\Phi(x_2) \subseteq \Phi(\lambda x_1 + (1-\lambda)x_2)$$

whenever $0 < \lambda < 1$ and $x_1, x_2 \in K$. A map $\Phi : K \to 2^E$ is called a lower semicontinuous carrier if for each open subset U of E $\{x \in K : \Phi(x) \cap U \neq \emptyset\}$ is open in K. The existence of continuous selections for lower semicontinuous carriers was intensively studied in a series of papers from which [6] and [7] are the closest to subject matter of this note.

The problem of finding affine continuous selections was studied in [5]. It was shown that the problem of finding an affine continuous selection has always a solution when E is a Fréchet space and the domain of Φ is a Choquet simplex. Recall that a compact convex subset K of a locally convex space is called a Choquet simplex if the cone with the origin as vertex and having K as its base induces a lattice ordering in the linear space spanned by K. The fundamental properties of simplexes may be found in [1].

A Selection Theorem

The following selection theorem of Lazar is the adaptation for Choquet simplexes of a selection theorem for lower semicontinuous carriers defined on topological spaces [6], [7].

Theorem [Lazar]. *Let E be a Fréchet space, K a Choquet simplex and $\Phi : K \to 2^E$ an affine lower semicontinuous carrier such that $\Phi(x)$ is closed for every $x \in K$. Then there exists an affine continuous selection for Φ i.e., an affine continuous function $f : K \to E$ with $f(x) \in \Phi(x)$ for each $x \in X$.*

Unfortunately, the intersection of two lower semicontinuous carriers need not be lower semicontinuous. However, Corollary 1.3 of [3] is sufficiently more general to answer our needs. When reformulated in our context it reads as follows.

Lemma 1. *Let X be a topological space, E a topological vector space and U be a non-empty open subset of E. If $\Phi_1, \Phi_2 : X \to 2^E$ are lower semicontinuous carriers with $\Phi_1(x) \cap (\Phi_2(x) + U) \neq \emptyset$ for each $x \in X$ then the map $\Psi : X \to 2^E$ defined by $\Psi(x) = \Phi_1(x) \cap (\Phi_2(x) + U)$ is a lower semicontinuous carrier.*

Proof. Let V be an open set in E. If $V = \emptyset$, then $\{x \in X : \Psi(x) \cap V \neq \emptyset\}$ is the empty set. Suppose $V \neq \emptyset$. Then $W = \{(a,b) \in E \times E : a - b \in U\}$ is an open set in $E \times E$. Let $\Phi_1 \times \Phi_2 : X \to 2^{E \times E}$ be defined by $(\Phi_1 \times \Phi_2)(x) = \Phi_1(x) \times \Phi_2(x)$. It is easy to see that $\Phi_1 \times \Phi_2$ is a lower semicontinuous carrier. Since $W \cap (V \times E)$ is an open set in $E \times E$ and

$$\{x \in X : \Psi(x) \cap V \neq \emptyset\} = \{x \in X : (\Phi_1 \times \Phi_2)(x) \cap W \cap (V \times E) \neq \emptyset\}$$

Ψ is lower semicontinuous. □

The basic device for showing that a map has an affine continuous selection is the following. It is an analogue of Proposition 1.2 in [2]. We denote the open ball of radius β about 0 in E by B_β.

Lemma 2. *Let K be a convex set, E a Fréchet space and $\Phi : K \to 2^E$ be a map. Assume that there exists a constant α such that for each $0 < \beta \in \mathbb{R}$ there exist affine continuous functions $f_\beta, g_\beta : K \to E$ such that for all $x \in K$*

$$f_\beta(x) \in \overline{\Phi(x) + B_\beta} \tag{1}$$

$$g_\beta(x) \in \overline{(\Phi(x) + B_{\beta/2}) \cap (f_\beta(x) + B_{\alpha\beta})} \tag{2}$$

Then there exists an affine continuous function $h : K \to E$ such that $h(x) \in \overline{\Phi(x)}$ for all $x \in K$.

Proof. It follows from (1) and (2) that we can construct by induction a sequence (f_n) of affine continuous functions from K into E such that

$$f_n(x) \in \overline{\Phi(x) + B_{\epsilon/2^n}} \subseteq \Phi(x) + B_{1/2^n}$$

and

$$f_{n+1}(x) \in \overline{f_n(x) + B_{\alpha\epsilon/2^n}} \subseteq f_n(x) + B_{\alpha/2^n} \quad \text{for all} \quad x \in X$$

where $0 < \epsilon < 1$. It follows that $d(f_n(x), f_{n+i}(x)) < \frac{\alpha}{2^{n-1}}$ for all $x \in X$ and $n, i = 1, 2, \ldots$ where d is the metric on E. Hence $\{f_n\}$ converges uniformly to some affine continuous function f. It is immediate that $f(x) \in \overline{\Phi(x)}$ for all $x \in X$. □

Proposition 1. *Let K be a Choquet simplex, E a Fréchet space and $\Phi : K \to 2^E$ be a map such that $\Phi(x)$ is closed and convex for each $x \in K$. Suppose also that for each $\epsilon > 0$ there exists an affine lower semicontinuous carrier $\Phi_\epsilon : K \to 2^E$ such that $\Phi(x) \subseteq \Phi_\epsilon(x) \subseteq \Phi(x) + B_\epsilon$ for each $x \in K$. Then there is an affine continuous function $f : K \to E$ with $f(x) \in \Phi(x)$ for each $x \in K$.*

Proof. Let us first note that if X and Y are topological spaces and if $\Phi : X \to 2^Y$ is a lower semicontinuous carrier, then for each open set U in Y

$$\{x \in X : \Phi(x) \cap U \neq \phi\} = \{x \in X : \overline{\Phi(x)} \cap U \neq \phi\}$$

so that $\Psi : X \to 2^Y$, defined by $\Psi(x) = \overline{\Phi(x)}$ is also a lower semicontinuous carrier. Therefore the map $x \to \overline{\Phi_\epsilon(x)}$ is an affine lower semicontinuous carrier. By Lazar's selection theorem for each $0 < \epsilon$ there is an affine continuous function $f_\epsilon : K \to 2^E$ such that

$$f_\epsilon(x) \in \overline{\Phi_\epsilon(x)} \subseteq \overline{\Phi(x) + B_\epsilon} \quad \text{for} \quad x \in K.$$

We define $\Psi_\epsilon : K \to 2^E$ as

$$\Psi_\epsilon(x) = \left(\Phi_{\frac{\epsilon}{4}}(x) + B_{\frac{\epsilon}{4}}\right) \cap \left(f_\epsilon(x) + B_\epsilon\right).$$

Since

$$f_\epsilon(x) \in \overline{\Phi_\epsilon(x)} \subseteq \overline{\Phi(x) + B_\epsilon} \subseteq \overline{\Phi_{\frac{\epsilon}{4}} + B_\epsilon},$$

each neighbourhood, in particular, $f_\epsilon(x) + B_{\frac{\epsilon}{4}}$, of $f_\epsilon(x)$ intersects $\Phi_{\frac{\epsilon}{4}}(x) + B_\epsilon$. As $B_{\frac{\epsilon}{4}}$ and B_ϵ are absolutely convex, this yields

$$\left(f_\epsilon(x) + B_\epsilon\right) \cap \left(\Phi_{\frac{\epsilon}{4}}(x) + B_{\frac{\epsilon}{4}}\right) \neq \emptyset.$$

Consequently, lower semicontinuity of the map $x \to f_\epsilon(x) + B_\epsilon$ implies that of Ψ_ϵ by Lemma 1. Ψ_ϵ is also affine as each of $x \to \Phi_{\frac{\epsilon}{4}}(x) + B_{\frac{\epsilon}{4}}$ and $x \to f_\epsilon(x) + B_\epsilon$ is affine. Since $\overline{\Psi_\epsilon(x)}$ is a closed convex set for each x, we conclude that the map $x \to \overline{\Psi_\epsilon(x)}$ is an affine lower semicontinuous carrier. Thus there exists an affine continuous function $g_\epsilon : K \to 2^E$ with

$$\begin{aligned} g_\epsilon(x) \in \overline{\Psi_\epsilon(x)} &\subseteq \overline{\left(\Phi_{\frac{\epsilon}{4}}(x) + B_{\frac{\epsilon}{4}}\right) \cap (f_\epsilon(x) + B_\epsilon)} \\ &\subseteq \overline{\left(\Phi(x) + B_{\frac{\epsilon}{4}} + B_{\frac{\epsilon}{4}}\right) \cap (f_\epsilon(x) + B_\epsilon)} \\ &\subseteq \overline{\left(\Phi(x) + B_{\frac{\epsilon}{2}}\right) \cap (f_\epsilon(x) + B_\epsilon)}. \end{aligned}$$

Hence by Lemma 2 there exists an affine continuous function $f : K \to E$ such that $f(x) \in \overline{\Phi(x)} = \Phi(x)$ for all $x \in K$. □

Separation of Vector Valued Functions

The cone of positive elements in an ordered vector space E will be denoted by E^+. E^+ will assumed to be closed whenever E is an ordered topological vector space.

Vector valued semicontinuous functions can be defined by suitable modifications of the definitions of real valued semicontinous function [4].

Definition. *If K is a topological space and E an ordered topological vector space, a function $f : K \to E$ is called upper semicontinuous if $f^{-1}(U - E^+)$ is open for each open set U in E. f is called lower semicontinuous if $f^{-1}(U + E^+)$ is open for each open set U in E.*

Since, for any open subset U of $\mathbb{R}$, the sets $U + \mathbb{R}^+$ and $U - \mathbb{R}^+$ are infinite open intervals, it is clear that for real valued functions the above definitions coincide with the classical ones.

f is an upper semicontinuous function if and only if $-f$ is lower semicontinuous and vice-versa. Clearly, also, if U is open then so are the sets $U + E^+$ and $U - E^+$, so that continuous functions are both upper and lower semicontinuous. This proves the necessity part of the following proposition.

Proposition 2. *If X is a topological space and E an locally solid ordered topological vector space then a function $f : X \to E$ is continuous if and only if it is both upper and lower semicontinuous.*

Proof. Any open subset U of E can be written as a union of order convex open sets. Therefore it suffices to show $f^{-1}(U)$ is open for each order convex open subset U. But, for an order convex $U, f(x) \in U$ if and only if there are $\alpha, \beta \in U$ with $\alpha \leq f(x) \leq \beta$, so that $f^{-1}(U) = f^{-1}(U + E^+) \cap f^{-1}(U - E^+)$ which is certainly open. □

Lemma 3. *If X is a topological space and E an ordered topological vector space then*

(i) $f : X \to E$ is lower semicontinuous if and only if the map $F : X \to 2^E$ defined by $F(x) = f(x) - E^+$ is a lower semicontinuous carrier.

(ii) $g : X \to E$ is upper semicontinuous if and only if $G : X \to 2^E$ defined by the map $G(x) = g(x) + E^+$ is a lower semicontinuous carrier.

Proof. We prove only (i), the proof of (ii) being similar. For an arbitrary subset U of E and for $x_0 \in X$ the following string of equivalences is obvious:

$$\begin{aligned} & x_0 \in \{x \in X : U \cap (f(x) - E^+) \neq \emptyset\} \Leftrightarrow \\ & \quad \exists\, z \in E^+ \quad \text{with} \quad f(x_0) - z \in U \\ & \Leftrightarrow \exists\, z \in E^+ \quad \text{with} \quad f(x_0) \in z + U \subseteq U + E^+ \\ & \Leftrightarrow x_0 \in f^{-1}(U + E^+). \end{aligned}$$

Now, assuming that U is open, the claim follows from the definitions. □

Let K be a compact convex subset of a locally convex Hausdorff space. If E is a partially ordered vector space then a function $f : K \to E$ is called convex if

$$f(\lambda x_1 + (1-\lambda)x_2) \leq \lambda f(x_1) + (1-\lambda) f(x_2) \quad \text{for all} \quad \lambda \in (0,1)$$

and x_2, x_2 in K. A function $g : K \to E$ is called concave if $-g$ is convex. A function $f : K \to E$ is called affine if it is convex and concave. The space of all affine continuous functions from K into E will be denoted by $A(K,E)$. Clearly, if E is an ordered vector space then $A(K,E)$ endowed with the ordering $a \geq 0$ if and only if $a(x) \geq 0$ for all $x \in K$, is an ordered vector space. For more details on vector valued affine continuous functions we refer the reader to [2].

A theorem of Edwards [1] states that if K is a Choquet simplex and $f, -g : K \to \mathbb{R}$ are upper semicontinuous convex functions then there exists an affine continuous function $h : K \to \mathbb{R}$ with the property $f \leq h \leq g$.

The following result is a vector valued version of this theorem.

Proposition 3. *Let K be a Choquet simplex, E a locally solid ordered Fréchet space with Riesz separation property. If $f, -g$ are upper semicontinuous convex functions with $f(x) \leq g(x)$ for all $x \in K$. Then there exists an affine continuous function $h : K \to E$ with $f \leq h \leq g$.*

Proof. For $\epsilon > 0$, let B_ϵ be the open ball centred at 0 with radius $\epsilon > 0$. The topology of E admits a neighbourhood base at 0 consisting of absolutely convex and solid sets [8]. We choose an absolutely convex, order convex and positively generated neighbourhood V_ϵ of 0 such that $V_\epsilon + V_\epsilon + V_\epsilon \subseteq B_\epsilon$.

By Lemma 3 the set valued maps $x \rightsquigarrow f(x)+E^+$ and $x \rightsquigarrow g(x)-E^+$ are lower semicontinuous carriers and their values are convex sets. The map $x \rightsquigarrow g(x)-E^+ + V_\epsilon$ is a lower semicontinuous carrier as

$$\{x : (g(x)-E^+ + V_\epsilon) \cap U \neq \emptyset\} = g^{-1}(U - V_\epsilon + E^+)$$

for each open set U in E. Applying Lemma 1 to the maps $x \rightsquigarrow g(x)-E^+ + V_\epsilon$ and $x \rightsquigarrow f(x)+E^+$ we obtain the lower semicontinuity of the map

$$\Phi_\epsilon(x) = (f(x)+E^+ + V_\epsilon) \cap (g(x)-E^+ + V_\epsilon).$$

The values of Φ_ϵ are convex sets. It is straightforward to see that Φ_ϵ is an affine map. Let $\Phi : K \to 2^E$ be the map defined by

$$\Phi(x) = (f(x)+E^+) \cap (g(x)-E^+).$$

The values of Φ are closed convex subsets of E and we have $\Phi(x) \subseteq \Phi_\epsilon(x)$ for each $x \in K$. Next we show $\Phi_\epsilon(x) \subseteq \Phi(x) + B_\epsilon$ for all $x \in K$. Let $y \epsilon \Phi_\epsilon(x)$. Then $v_1 \leq y - f(x)$ and $y - g(x) \leq v_2$ for some v_1, v_2 in V_ϵ. As V_ϵ is positively generated, there exist v_1^+, v_1^- in $V_\epsilon \cap E^+$ with $v_1 = v_1^+ - v_1^-$. Similarly, $v_2 = v_2^+ - v_2^-$ for some $v_2^+, v_2^- \epsilon V_\epsilon \cap E^+$. Let $v = v_1^+ + v_2^+$. Then $v \in B_\epsilon$ and we have $v_1, y - g(x) \leq y - f(x), v$. By Riesz separation property, we can find w with $v_1, y-g(x) \leq w \leq y - f(x), v$. It is routine to show that $w \in B_\epsilon$ and $y - w \in (f(x)+E^+) \cap (g(x)-E^+)$. Thus $y \in \Phi(x) + B_\epsilon$ and we have

$$\Phi(x) \subseteq \Phi_\epsilon(x) \subseteq \Phi(x) + B_\epsilon \quad \text{for all} \quad x \in K.$$

By Proposition 1, there exists an affine continuous function $h : K \to E$ with $h(x) \in \Phi(x)$ for all $x \in K$. It is easy to see that $f \leq h \leq g$ as claimed. □

Let K, f and g be as in the proposition. Let E be a Fréchet lattice. Then there exists an affine continuous function $h : K \to E$ such that $f \leq h \leq g$.

Let K be a compact convex set and F a convex subset of K. F is called a face of K if $x_1, x_2 \in F$ whenever $\lambda x_1 + (1-\lambda)x_2 \in F$ for some $\lambda \in (0,1)$ and $x_1, x_2 \in K$.

Corollary. *Let K and E be as in the proposition and F be a closed face of K. Let $f, -g : K \to E$ be upper semicontinuous convex function with $f \leq g$ on K. Let $h : F \to E$ be an affine continuous function with $f|_F \leq h \leq g|_F$. Then there exists $\hat{h} \in A(K,E)$ with $\hat{h}|_F = h$ and $f \leq \hat{h} \leq g$ on K.*

Proof. We define $f_1(x) = f(x)$ if $x \in K \setminus F$, $f_1(x) = h(x)$ if $x \in F$ and $g_1(x) = g(x)$ if $x \in K \setminus F$, $g_1(x) = h(x)$ otherwise. Then $f_1, -g_1$ are upper semicontinuous convex functions on K.

We prove only that g_1 is lower semicontinuous and note that f_1 is upper semicontinuous if and only if $-f_1$ is lower semicontinuous. Lower semicontinuity of $(-f_1)$ can be proved similarly. To this end let U be an open set in E then

$$g_1^{-1}(U+E^+) = h^{-1}(U+E^+) \cup (g^{-1}(U+E^+) \cap (K \setminus F)).$$

Since $h \leq g|_F$, we have $h^{-1}(U+E^+) \subseteq g^{-1}(U+E^+) \cap F$. The set $h^{-1}(U+E^+)$ is relatively open in F so there is an open set $W \subset K$ with $h^{-1}(U+E^+) = W \cap F$. Noting that $W \cup (K \setminus F)$ is also open, it follows that

$$g_1^{-1}(U+E^+) = (W \cup (K \setminus F)) \cap g^{-1}(U+E^+)$$

is open as claimed. Noting that $f_1(x) \leq g_1(x)$ for $x \in K$ we have, by Proposition 3, $\hat{h} \in A(K,E)$ with $f_1 \leq \hat{h} \leq g_1$ on K. $\hat{h}$ is the required extension of h. □

Corollary. *Let K be a compact convex set and E a Fréchet lattice. K is a Choquet simplex if and only if $A(K,E)$ has the Riesz separation property.*

Proof. Let K be a simplex. Suppose $f_1, f_2 \leq g_1, g_2$ in $A(K,E)$. $C(K,E)$ the space of continuous functions from K into E is a vector lattice with pointwise lattice operations. Hence $f_1 \vee f_2$ and $g_1 \wedge g_2$ are in $C(K,E)$. As $f_1 \vee f_2$ is convex and $g_1 \wedge g_2$ is concave, there exists a continuous affine map h with $f_1 \vee f_2 \leq h \leq g_1 \wedge g_2$. Clearly, h does the required separation.

Let $e \in E^+$ be fixed and choose $0 \leq F \in E'$ with $F(e) = 1$. The map $f \to f \otimes e : f \otimes e(k) = f(k)e$ is a positive injection of $A(K)$ into $A(K,E)$. If $A(K,E)$ has the separation property, $f_1 \otimes e, f_2 \otimes e \leq h \leq g_1 \otimes e, g_2 \otimes e$ for some $h \in A(K,E)$. Applying F to the last string of inequalities we have $f_1, f_2 \leq F(h) \leq g_1, g_2$ in $A(K)$. Thus $A(K)$ has the Riesz separation property. Therefore K is a simplex by II.3.11 in [1] □

The rest of the applications of the proposition will depend on the following construction.

Lemma 4. *Let F be a closed face of the compact convex set K and E be an ordered topological vector space. Suppose $f_1 : K \to E$, $f_2 : F \to E$ be lower semicontinuous concave functions and that $f_2 \leq f_1|_F$. If we define the function f as follows*

$$f(x) = \begin{cases} f_1(x), & x \in K \setminus F, \\ f_2(x), & x \in F. \end{cases}$$

Then f is lower semicontinuous and concave.

Proof. The lower semicontinuity of f follows from that of f_1 and f_2 together with the identity

$$\{x \in K : f(x) \in U + E^+\} = \{x \in K : f_1(x) \in U + E^+\} \cup \{x \in F : f_2(x) \in U + E^+\}$$

for each open set U in E. To show f is concave we have to establish that $\lambda f(x_1) + (1-\lambda) f(x_2) \leq f(\lambda x_1 + (1-\lambda)x_2)$ for any choice of $\lambda \in (0,1)$ and $x_1, x_2 \in K$. The only non-trivial case arises when $x_1 \in X \setminus F$ and $x_2 \in F$ (or vice versa). In this case $\lambda x_1 + (1-\lambda)x_2$ cannot be in F as F is a face and so

$$\begin{aligned} f(\lambda x_1 + (1-\lambda)x_2) &= f_1(\lambda x_1 + (1-\lambda)x_2) \geq \lambda f_1(x_1) + (1-\lambda)f_1(x_2) \\ &\geq \lambda f_1(x) + (1-\lambda)f_2(x_2) = \lambda f(x_1) + (1-\lambda)f(x_2). \end{aligned}$$

Hence f is concave and the lemma is proved. □

Corollary. *Let K be a Choquet simplex, E be a Fréchet lattice and F be a closed face of K. If $a_1, a_2, b \in A(K,E)$ with $a_i \leq b$ and $a_1|_F \leq a_2|_F$, then there exists an element $c \in A(K,E)$ for which $a_i \leq c \leq b$ and $c|_F = a_2|_F$.*

Proof. We define $g : K \to E$ by

$$g(x) = \begin{cases} b(x), & x \in K \setminus F, \\ a_2(x), & x \in F. \end{cases}$$

Let $f(x) = a_1(x) \vee a_2(x)$. Then $-f$ and g are lower semicontinuous concave functions. We have $f \leq g$ and $f|_F = g|_F = a_2|_F$. By the proposition there exists $c \in A(K,E)$ with $f \leq c \leq g$ and this c plainly meets the requirements of the corollary. □

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Corollary. *Let K and E be as in the proposition and F, G be disjoint closed faces of K. Then for any $e \in E^+$ there exists $f \in A(K,E)$ such that $f|_F = 0$, $f|_G = e$ and $0 \leq f(x) \leq e$ for each $x \in K$.*

Proof. We define $u, v : K \to E$ as follows;

$$u(x) = \begin{cases} 0, & x \in K \setminus G, \\ e, & x \in G, \end{cases} \quad v(x) = \begin{cases} e, & x \in K \setminus F, \\ 0, & x \in F. \end{cases}$$

$-u, v$ are lower semicontinuous concave functions with $u \leq v$. The proposition furnishes f in $A(K,E)$ with $u \leq f \leq v$, and this f has the desired properties. □

We will need the following notion. The faces F_1, F_2 of the convex set K are said to be complementary if $F_1 \cap F_2 = \emptyset$ and $\operatorname{conv}(F_1 \cup F_2) = K$. We will say that $f : K \to E$ is order bounded if $f(K)$ is an order bounded subset of E.

Corollary. *Let K and E be as in the proposition. Suppose that F_1 and F_2 are complementary closed faces of K, and $f_i \in A(F_i, E), (i = 1,2)$ be order bounded. Then there exists a unique $f \in A(K,E)$ such that $f|_{F_i} = f_i$, $(i = 1,2)$.*

Proof. Let $e \in E^+$ be such that $f_1(x), f_2(x) \le e$ for all $x \in K$. We define $u, v : K \to E$ as follows

$$\begin{array}{ll} -u(x) = v(x) = e, & x \in K \setminus (F_1 \cup F_2), \\ u(x) = v(x) = f_1(x), & x \in F_1, \\ u(x) = v(x) = f_2(x), & x \in F_2. \end{array}$$

$-u$ and v are lower semicontinuous concave functions and $u \le v$. By the proposition there exists $f \in A(K, E)$ with $u \le f \le v$. Clearly f meets the requirements of the corollary. By Milman's theorem $\partial K \subseteq F_1 \cup F_2$. The uniqueness follows from the fact that a continuous affine function is completely determined by its values on ∂K. □

References

[1] E.M. Alfsen, *Compact Convex Sets and Boundary Integrals*, Ergebnisse der Mathematik und ihrer Grenzgebiete **57**, Springer Verlag, Berlin - Heidelberg - New York 1971.

[2] L. Asimov, H. Atkinson, Dominated extension of continuous affine functions with range an ordered Banach spaces, *Quart. J. Math. Oxford* (2) **23** (1972) 383-389.

[3] Z. Ercan, Riesz spaces of Riesz space valued functions, Ph.D. Thesis, Queen's University of Belfast, 1993.

[4] Z. Ercan, A. Wickstead, Banach lattices of Banach lattice valued maps, Preprint.

[5] A.J. Lazar, Spaces of affine continuous functions, *Trans. Amer. Math. Soc.* **134** (1968) 503-525.

[6] E.M. Michael, Selected selection theorems, *Amer. Math. Montly* **63** (1956) 233-238.

[7] E.M. Michael, Continuous Selections I, *Israel J. Math.* **2** (1964) 201-204.

[8] K. Ng, Y. Wong; *Partially Ordered Topological Vector Spaces*, Clarendon Press, Oxford (1971).

VEKTÖR DEĞERLİ FONKSİYONLAR İÇİN AYIRMA VE AFİN SEÇİM ÜZERİNE

Özet

K Choquet simpleksi E, Frechet uzayı ve ϕ, her $x \in K$ için $\phi(x)$'in kapalı konveks küme olduğu, $K \to 2^E$ işlevi ise ϕ için sürekli, afin seçim yapılanabilmesi için yeterli bir koşul verilmektedir. Bunun yardımı ile bir Choquet simpleksi üzerinde tanımlı ve sıralamalı Frechet uzayında değer alan yarı sürekli fonksiyonlar için sürekli afin fonksiyonlar ile ayırma yapılmış ve sonuçları elde edilmiştir.

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