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On symplectic fillings of 3–manifolds

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Dedicated to Robion C. Kirby on the occasion of his 60th birthday

1. Introduction

1.1. Contact structures

Let Y be a closed 3–manifold. A field of 2–planes $\xi \subset TY$ is a *contact structure* if it is the kernel of a smooth 1–form θ on Y such that $\theta \wedge d\theta \neq 0$ at every point of Y. Notice that since ξ is oriented by the restriction of $d\theta$ the manifold Y is necessarily orientable. Moreover, an orientation on Y induces a coorientation on ξ and vice–versa. When Y has a prescribed orientation, ξ is said to be *positive* (*negative*, respectively), if the orientation on Y induced by ξ coincides with (is the opposite of, respectively) the given one. In this paper we shall only consider oriented 3–manifolds. Therefore, from now on by the expression "3–manifold" we shall always mean "oriented 3–manifold", and all contact structures will be implicitly assumed to be positive (for an introduction to contact structures and a guide to the literature we refer the reader to [Be, El3, Gi]).

By the work of Martinet and Lutz [Ma] we know that every closed, oriented 3–manifold Y admits a positive contact structure. Eliashberg defined a special class of contact structures, which he called *overtwisted*, and proved that in any homotopy class of cooriented 2–plane fields on a 3–manifold there exists a positive overtwisted contact structure, which is unique up to isotopy [El1]. This showed that the really interesting contact structures are the non–overtwisted ones, which Eliashberg called *tight*. For such contact structures, the questions of existence and uniqueness in a given homotopy class are known to have a negative answer, in general.

1.2. Symplectic fillings

All the tight contact structures known at present are fillable in one sense or another, i.e., loosely speaking, they are a 3–dimensional phenomenon induced by a 4–dimensional one. A *4–manifold with contact boundary* is a pair (X, ξ), where X is a connected, oriented smooth 4–manifold with boundary and ξ is a contact structure on ∂X (positive with respect to the boundary orientation). A *compatible symplectic form* on (X, ξ) is a symplectic form ω on X such that $\omega|_\xi > 0$ at every point of ∂X. A contact 3–manifold (Y, ζ) is called *symplectically fillable* if there exists a 4–manifold with contact boundary (X, ξ) carrying a compatible symplectic form ω and an orientation–preserving diffeomorphism $\phi\colon Y \to \partial X$ such that $\phi_*(\zeta) = \xi$. The triple (X, ξ, ω) is said to be a *symplectic filling* of Y. More generally, (Y, ζ) is called *symplectically semi–fillable* if the diffeomorphism ϕ sends Y onto a connected component of ∂X. In this case (X, ξ, ω) is called a *symplectic semi–filling* of Y. If (Y, ζ) is symplectically semi–fillable, then ζ is tight by a theorem of Eliashberg and Gromov (see [El2, La]).

1.3. Basic questions

The following fundamental question about the fillability of contact 3–manifolds (cf. [El3], question 8.2.1, and [Ki], question 4.142) remained unanswered for some time:

Question 1.1. *Does every oriented* 3*–manifold admit a fillable contact structure?*

Eliashberg's Legendrian surgery construction [El1, Go] provides a rich source of contact 3–manifolds which are filled by Stein surfaces (a special kind of 4–manifolds with contact boundary carrying exact compatible symplectic forms). Symplectically fillable contact structures are not necessarily fillable by Stein surfaces. For example, the 3–torus $S^1 \times S^1 \times S^1$ carries infinitely many isomorphism classes of symplectically fillable contact structures, but Eliashberg showed [El4] that only one of them can be filled by a Stein surface.

Gompf studied systematically the fillability of Seifert 3–manifolds using Eliashberg's construction. This led him to formulate the following:

Conjecture 1.2 ([Go])**.** *The Poincaré homology sphere, oriented as the boundary of the positive E_8 plumbing, does not admit positive contact structures which are fillable by a Stein surface.*

Another basic question one may ask concerns the uniqueness of symplectic fillings. One may loosely formulate the uniqueness question as follows (cf. question 10.2 in [El2]):

Question 1.3. *To what extent does a* 3*–manifold determine its symplectic fillings?*

A 3–manifold may have several symplectic fillings. Via Legendrian surgery one can construct, for instance, non–diffeomorphic (even after blow–up) symplectic fillings of certain 3–manifolds. On the other hand, S^3 is known to have just one symplectic filling up to blow–ups and diffeomorphisms [El2], and the same is true for the lens spaces $L(n, 1)$ when $n \neq 4$ (when $n = 4$ there are two possibilities) [McD].

1.4. Recent developments

Some progress in the understanding of contact structures has recently come from studying the spaces of solutions to the Seiberg–Witten equations. One of the outcomes of [LM] was a proof of the existence, for every natural number n, of integral homology 3–spheres carrying more than n homotopic, non–isomorphic tight contact structures. Generalizing to a non–compact setting the results of [Ta1, Ta2], Kronheimer and Mrowka [KM] introduced monopole invariants for smooth 4–manifolds with contact boundary, and used them to strengthen the results of [LM] as well as to prove new results, as for example that on every oriented 3–manifold there is only a finite number of homotopy classes of symplectically semi–fillable contact structures. In the paper [Li] we applied the results of [KM] to establish the following:

Theorem 1.4 ([Li], Theorem 1.4)**.** *Let (X, ξ) be a 4–manifold with contact boundary equipped with a compatible symplectic form. Suppose that a connected component of the boundary of X admits a metric with positive scalar curvature. Then, the boundary of X is connected and $b_2^+(X) = 0$.*

Notice how, as a consequence of this theorem, a symplectic semi–filling of a 3–manifold carrying positive scalar curvature metrics is necessarily a symplectic filling.

The following corollary of theorem 1.4 proves, in particular, Gompf's conjecture and provides a negative answer to question 1.1.

Corollary 1.5 ([Li], Corollary 1.5)**.** *Let Y denote the Poincaré homology sphere oriented as the boundary of the positive E_8 plumbing. Then, Y has no symplectically semi–fillable contact structures. Moreover, $Y \# -Y$ is not symplectically semi–fillable with any choice of orientation.*

2. Statement of results

In this paper we prove new results concerning questions 1.1 and 1.3. In fact, we produce new examples of oriented irreducible 3–manifolds which are not symplectically semi–fillable, and a new uniqueness result for the intersection forms of the symplectic fillings of certain rational homology 3–spheres. All the proofs are based on theorem 1.4 using an approach which generalizes the argument we used in [Li] to obtain corollary 1.5.

Since our main tool, theorem 1.4, applies to 3–manifolds having positive scalar curvature metrics, it is natural to consider the most familiar examples of 3–manifolds with that property, i.e. the links of the classical complex 2–dimensional Kleinian singularities [Kl]. Being quotients of S^3 by the standard action of a finite subgroup of $SO(4)$, they all carry positive scalar curvature metrics. Moreover, each link, with its standard orientation, bounds the minimal resolution of the corresponding singularity [DV], which is orientation–preserving diffeomorphic to the boundaries of the smooth 4–manifolds obtained by plumbing together several copies of the (-2)–disc bundle over the 2–sphere, according to a diagram of type A_n (with n vertices), D_{n+2} (with $n+2$ vertices), E_6, E_7 or E_8 (see figure 1). The boundaries of these plumbings are all well–known 3–manifolds. For example, plumbing on A_n gives the lens space $L(n+1, n)$, while plumbing on E_8 gives the Poincaré sphere. From now on we shall simply refer to these plumbings as the "negative plumbings" on the corresponding diagrams, and we shall call "positive plumbings" those obtained in a similar way using the disc bundle with Euler number $+2$.

Observe that every link of a Kleinian singularity with its standard orientation has symplectic fillings. In fact, it is not difficult to see that the Milnor fiber of the corresponding singularity gives a symplectic filling. Alternatively, using Eliashberg's Legendrian surgery construction it is easy to show that all the minimal resolutions described above are Stein 4–manifold with boundary, and therefore are symplectic fillings of their boundaries. Thus, the answer to the existence problem for symplectic fillings of these oriented 3–manifolds

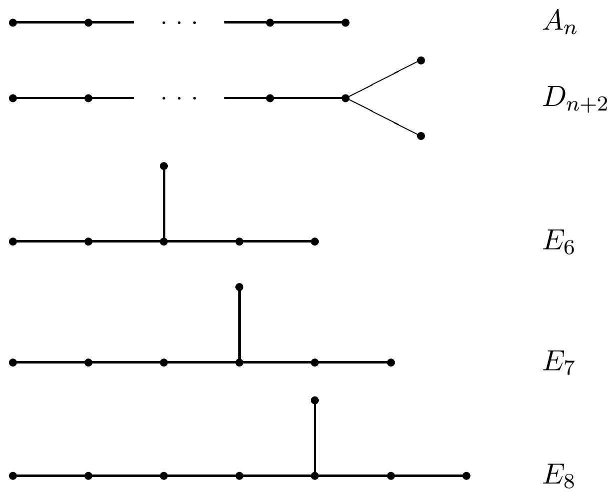

Figure 1. Resolution diagrams of the Kleinian singularities

is already well–known. For this reason, and for another reason which is related to the nature of our proofs, in this paper we shall only consider the positive plumbings, i.e. we shall study the existence of symplectic fillings for the links of the classical Kleinian singularities endowed with the orientations *opposite* to the standard ones.

Our first result generalizes corollary 1.5.

Theorem 2.1. *Let X_i, $i = 1, 2, 3$ be the oriented boundary of, respectively, the positive E_6, E_7 and E_8 plumbing. Then, for any closed oriented 3–manifold M, $X_i \# M$ is not symplectically semi–fillable.*

Remark 2.1. It follows from theorem 2.1 that, for $i = 1, 2, 3$, $X_i \# -X_i$ is not sympletically semi–fillable with any choice of orientation.

Let Y_n be the oriented boundary of the positive D_{n+2} plumbing. Using Kirby calculus it is easy to show that Y_n is also the oriented boundary of the smooth 4–manifold with boundary M_n described in figure 2. Moreover, using Eliashberg's Legendrian surgery one can show that M_n is a Stein 4–manifold with boundary (in [Go] it is explained how to do this). This implies that all the blowups of M_n are symplectic fillings of Y_n. The following theorem says that, on the other hand, the intersection lattice of any symplectic filling of Y_n is isomorphic to the intersection lattice of some blowup of M_n.

Theorem 2.2. *For any $n > 1$, let Y_n be the oriented boundary of the positive D_{n+2} plumbing. Then, the intersection lattice of any symplectic semi–filling of Y_n is isomorphic to $(\mathbb{Z}^m, \oplus_i(-1))$, for some $m \geq 0$.*

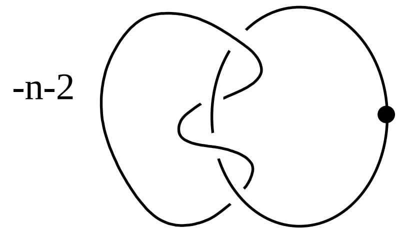

Figure 2. The manifold M_n

The oriented boundary of the positive A_n plumbing is the lens space $L(n+1,1)$. As we recalled in the introduction, Eliashberg and McDuff proved strong results on the symplectic fillings of these 3–manifolds. By applying theorem 1.4 we will prove the following theorem, which is consistent with their work.

Theorem 2.3. *The intersection lattice of any symplectic semi–filling of* $L(n,1)$ *is isomorphic to* $(\mathbb{Z}^m, (-n) \oplus_i (-1))$ *for some* $m \geq 1$ *if* $n \neq 4$. *If* $n = 4$, *then it is either isomorphic to* $(\mathbb{Z}^m, (-4) \oplus_i (-1))$ *for some* $m \geq 1$, *or to* $(\mathbb{Z}^m, \oplus_i(-1))$ *for some* $m \geq 0$.

We note that, conversely, it is well known that every intersection lattice appearing in the statement of theorem 2.3 is realized by a symplectic filling of $L(n,1)$.

Remark 2.2. Theorem 1.4 can be also applied to study the uniqueness problem for symplectic fillings of any lens space $L(p,q)$, but the combinatorial analysis needed to treat the general case is more complicated. We plan to return to this in a future paper.

3. Proofs

In this section we shall call *intersection lattice* a lattice (i.e. a finitely generated free abelian group) endowed with an integral symmetric bilinear form. By an *automorphism* of a intersection lattice we shall mean an isometric automorphism of the underlying lattice, i.e. an automorphism which preserves the intersection form.

Let $Q\colon \mathbb{Z}^4 \times \mathbb{Z}^4 \to \mathbb{Z}$ be the integral symmetric bilinear form given, with respect to the standard basis of $\mathbb{Z}^4$, by the matrix

$$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & -2 \end{pmatrix}$$

Denote by L the intersection lattice $(\mathbb{Z}^4, Q)$ and, for any $N \in \mathbb{N}$, by Λ_N the intersection lattice $\left(\mathbb{Z}^N, \oplus_i(-1)\right)$, where $\oplus_i(-1)$ is the standard negative diagonal intersection form.

Lemma 3.1. *Up to automorphisms of* Λ_N, *there is at most one isometric embedding of* L *into* Λ_N. *Such an embedding exists if and only if* $N \geq 4$ *and, if* $v_1, \ldots, v_4$ *are standard generators of* $\mathbb{Z}^4$ *and* $e_1, \ldots, e_N$ *are standard generators of* $\mathbb{Z}^N$, *it is given, up* *to automorphisms of* Λ_N*, by sending* v_1 *to* $-e_1+e_3$*,* v_2 *to* e_1+e_2*,* v_3 *to* $-e_1-e_3$ *and* v_4 *to* $-e_2+e_4$*.*

Proof. Clearly, if an embedding exists then $N \geq 4$. Moreover, by applying an automorphism of Λ_N one can change the sign of any of the e_i's as well as exchange any two of them. Therefore, with an automorphism of Λ_N one can realize any change of signs or permutation of its generators. Let $\varphi\colon L \to \Lambda_N$ be an arbitrary isometric embedding, i.e. an embedding of free abelian groups which preserves the intersection forms. We will argue that there exists an automorphism ψ of Λ_N such that $\psi \circ \varphi$ coincides with the embedding given in the second part of the statement. Since v_2 has self–intersection -2, it is clear that, up to composing with an automorphism of Λ_N, we may assume $\varphi(v_2) = e_1 + e_2$. Similarly, using the fact that the v_i's must intersect as prescribed by the matrix Q, it is an easy exercise to show that, up to sign changes and permutations of the e_i's, the images of the v_i's must be the ones given in the statement. □

Let $R = (\mathbb{Z}^6, -E_6)$ denote the intersection lattice of the negative E_6 plumbing.

Lemma 3.2. *For any* $N \geq 1$*, there exist no isometric embeddings of* R *into* Λ_N*.*

Proof. The statement is clear if $N \leq 5$. If $N \geq 6$, observe that there is an obvious isometric embedding of L into R. Arguing by contradiction, suppose there exists an isometric embedding of R into Λ_N. This induces an embedding of L into Λ_N which, up to composing with an automorphism of Λ_N, may be assumed to be the one given in the statement of lemma 3.1. Keeping the notation from that lemma and abusing it at the same time, denote by $v_1, \dots, v_4$ the standard generators of L as well their images inside Λ_N. By looking at the intersection matrix $-E_6$ one can easily see that the image of R inside Λ_N contains a vector, say v_5, having self–intersection -2 and such that $v_5 \cdot v_i = 0$ and $v_5 \cdot v_j = 1$, with $\{i, j\} = \{1, 3\}$. But since $v_1 = -e_1 + e_3$ and $v_3 = -e_1 - e_3$, this is clearly impossible. □

Proof of theorem 2.1. X_i is the quotient of S^3 by the action of a finite subgroup of $SO(4)$ acting linearly, for $i = 1, 2, 3$. This implies that X_i carries metrics with positive scalar curvature, and by theorem 1.4 if X_i is symplectically semi–fillable then it is symplectically fillable. Moreover, we claim that X_i cannot be the oriented boundary of a smooth, oriented 4–manifold W with $b_2^+(W) = 0$. Arguing by contradiction, if $\partial W = X_i$ and $\widehat{E}_{5+i}$ denotes the negative plumbing of type E_{5+i}, $i = 1, 2, 3$, then the closed, smooth oriented 4–manifold $M = W \cup_{X_i} \widehat{E}_{5+i}$ has a negative definite intersection form. By a well–known theorem of Donaldson [Do1, Do2] it follows that its intersection lattice must be isomorphic to the lattice $\Lambda_{b_2(M)} = \big(\mathbb{Z}^{b_2(M)}, \oplus_i(-1)\big)$. Therefore the lattice $\big(\mathbb{Z}^{5+i}, -E_{5+i}\big)$ embeds inside $\Lambda_{b_2(M)}$. On the other hand, for $i = 1, 2, 3$ the intersection lattice $R = \big(\mathbb{Z}^6, -E_6\big)$ clearly embeds inside $\big(\mathbb{Z}^{5+i}, -E_{5+i}\big)$. Thus, composing the two embeddings one gets an isometric embedding of R into $\Lambda_{b_2(M)}$, contradicting lemma 3.2. This proves the claim. Therefore, in view of theorem 1.4 X_i is not symplectically semi–fillable for $i = 1, 2, 3$. The statement of the theorem follows from a general result of Eliashberg ([El2], theorem 8.1):

given two closed oriented 3–manifolds X and M, if $X\#M$ is symplectically semi–fillable, then both X and M are symplectically semi–fillable. □

Let $\mathcal{D}_{n+2} = \left(\mathbb{Z}^{n+2}, -D_{n+2}\right)$ be the intersection lattice of the negative D_{n+2} plumbing.

Lemma 3.3. *Suppose that $n \geq 2$. Then, up to automorphisms of Λ_N, there is at most one isometric embedding of $\mathcal{D}_{n+2}$ into Λ_N. Such an embedding exists if and only if $N \geq n+2$, and the intersection lattice of the vectors orthogonal to the image of $\mathcal{D}_{n+2}$ is isomorphic to Λ_{N-n-2}.*

Proof. Observe that, since $n \geq 2$, $\mathcal{D}_{n+2}$ contains $\mathcal{D}_4$, which is an isometric copy of the lattice $L = (\mathbb{Z}^4, Q)$ defined at the beginning of the section. Denote by $v_1, v_2, \ldots, v_{n+2}$ standard generators of $\mathcal{D}_{n+2}$ such that $v_1, \ldots, v_4$ are the standard generators of $L = \mathcal{D}_4 \subset \mathcal{D}_{n+2}$. We shall prove by induction on $n \geq 2$ that, up to automorphisms of Λ_N, an isometric embedding of $\mathcal{D}_{n+2}$ into Λ_N exists if and only if $N \geq n+2$ and, if it does, it is given by sending v_1 to $-e_1 + e + 3$, v_2 to $e_1 + e_2$, v_3 to $-e_1 - e_3$, v_4 to $-e_2 + e_4$ and v_i to $-e_{i-1} + e_i$ for $i = 5, \ldots, n+2$. Moreover, the orthogonal lattice of the image of $\mathcal{D}_{n+2}$ is isomorphic to Λ_{N-n-2}. For $n = 2$ this is exactly the statement of lemma 3.1, apart from the claim that the orthogonal lattice is reduced to $\{0\}$, which is an easy exercise. We need to prove that the above statement holds for $\mathcal{D}_{n+3}$ if it holds for $\mathcal{D}_{n+2}$. The condition $N \geq n+3$ is clearly necessary for the existence of an isometric embedding of $\mathcal{D}_{n+3}$ into Λ_N. Given such an embedding φ, by the induction hypothesis we may assume, after possibly composing φ with an automorphism of Λ_N, that φ is given as above on the generators $v_1, \ldots, v_{n+2}$ of $\mathcal{D}_{n+2} \subset \mathcal{D}_{n+3}$. It is easy to see that, up to automorphisms of Λ_N, the image of v_{n+3} must be equal to $-e_{n+2} + e_{n+3}$. This proves the uniqueness of the embedding. To establish the statement on the orthogonal complement, observe that $\varphi(\mathcal{D}_{n+2})$ is contained inside $\Lambda_{n+2} \subset \Lambda_N$, with $\{0\}$ as its orthogonal lattice inside Λ_{n+2}, by the inductive hypothesis. Since the image of $\mathcal{D}_{n+3}$ is contained inside Λ_{n+3} and $\varphi(v_{n+3}) = -e_{n+2} + e_{n+3}$, the conclusion follows immediately. □

Proof of theorem 2.2. Y_n is the quotient of S^3 by the standard orthogonal action of the binary dihedral group of order $4n$. Therefore, it carries metrics with positive scalar curvature. It follows by theorem 1.4 that every symplectic semi–filling W of Y_n is a symplectic filling and $b_2^+(W) = 0$. As in the proof of theorem 2.1, if $\widehat{D}_{n+2}$ denotes the negative plumbing on the D_{n+2} diagram, by Donaldson's theorem the closed, negative definite 4–manifold $M = W \cup_{Y_n} \widehat{D}_{n+2}$ has standard diagonal intersection form $\Lambda_{b_2(M)}$. Therefore the intersection lattice $\mathcal{D}_{n+2}$ of $\widehat{D}_{n+2}$ embeds isometrically inside $\Lambda_{b_2(M)}$, and it is easy to check that, since $H_1(\widehat{D}_{n+2}; \mathbb{Z}) = 0$, the intersection lattice of W inside $\Lambda_{b_2(M)}$ coincides with the orthogonal to the image of $\mathcal{D}_{k+2}$. The conclusion follows by applying lemma 3.3. □

Lemma 3.4. *Let $\mathcal{A}_n = (\mathbb{Z}^n, -A_n)$ be the intersection lattice of the negative A_n plumbing, $n \geq 1$. Then, up to automorphisms of Λ_N, if $n \neq 3$ there is at most one isometric embedding of $\mathcal{A}_n$ into Λ_N, if $n = 3$ there are at most two. If $n \neq 3$ such an embedding* *exists if and only if $N \geq n+1$, if $n=3$ it exists if and only if $N \geq 3$. The intersection lattice orthogonal to the image of $\mathcal{A}_n$ is isomorphic to $\left(\mathbb{Z}^{N-n}, (-n-1) \oplus_i (-1)\right)$ if $n \neq 3$, and to either $\left(\mathbb{Z}^{N-n}, (-4) \oplus_i (-1)\right)$ or Λ_{N-4} if $n=3$.*

Proof. Let $v_1, \dots, v_n$ denote standard generators of $\mathcal{A}_n$, and $e_1, \dots, e_N$ standard generators of Λ_N. Let $\varphi\colon\ \mathcal{A}_n \to \Lambda_N$ be an isometric embedding. If n equals 1 (or 2, respectively), arguing as in the proof of lemma 3.1 it is easy to see that, up to automorphisms of Λ_N, we may assume $\varphi(v_1) = -e_1 + e_2$ (and $\varphi(v_2) = -e_2 + e_3$, respectively). If $n=3$ there are two possibilities, up to automorphisms, for $\varphi(v_3)$: either $e_1 + e_2$ or $-e_3 + e_4$. If $n \geq 4$ it is easy to check that the first possibility for $\varphi(v_3)$ cannot occur, because there would be no possible candidate for $\varphi(v_4)$. It follows that $\varphi(v_i) = -e_i + e_{i+1}$, $i = 3, \dots, n$, up to automorphisms of Λ_N. This discussion shows that for the existence of φ it is necessary and sufficient that $N \geq n+1$ if $n \neq 3$, and $N \geq 3$ if $n=3$. The statement about the orthogonal lattices can be easily established directly for $n \leq 3$, and proved by induction as in lemma 3.3 for $n \geq 4$. □

Proof of theorem 2.3. The proof is based on lemma 3.4 in the same way as the proof of theorem 2.2 is based on lemma 3.3, and is therefore omitted. □

References

[Be] D Bennequin, *Entrelacements et equations de Pfaff*, Astérisque **107–108** (1983), 83–161

[Do1] S K Donaldson, *Connections, cohomology and the intersection forms of four–manifolds*, Jour. Diff. Geom. **24** (1986) 275–341

[Do2] S K Donaldson, *The Seiberg–Witten equations and 4–manifold topology*, Bull. Am. Math. Soc. **33** (1996) 45–70

[DV] P Du Val, *On isolated singularities of surfaces that do not affect the condition of adjunction I, II, III*, Proc. Cambridge Phil. Soc. **30** (1934) 483–491

[El1] Y Eliashberg, *Topological characterization of Stein manifolds of dimension* > 2, Intern. Journal of Math. **1**, No. 1 (1990) 29–46

[El2] Y Eliashberg, *Filling by holomorphic discs and its applications*, London Math. Soc. Lecture Notes Series **151** (1991) 45–67

[El3] Y Eliashberg, *Contact 3–manifolds twenty years since J. Martinet's work*, Ann. Inst. Fourier **42** (1992) 165–192

[El4] Y Eliashberg, *Unique holomorphically fillable contact structure on the 3–torus*, Intern. Math. Res. Not. **2** (1996) 77–82

[Gi] E Giroux, *Topologie de contact en dimension* 3, Séminaire Bourbaki **760** (1992-93), 7–33

[Go] R E Gompf, *Handlebody construction of Stein surfaces*, Ann. of Math., to appear.

[Ki] R Kirby, *Problems in Low–Dimensional Topology.* In W H Kazez (Ed.), Geometric Topology, Proc of the 1993 Georgia International Topology Conference, AMS/IP Studies in Advanced Mathematics, pp. 35–473, Amer. Math. Soc. & International Press, 1997

[Kl] F Klein, *Vorlesungen über das Ikosaeder und die Auflösung der Gleichungen vom fünften Grade*, Teubner, Leipzig, 1884

[KM] P B Kronheimer, T S Mrowka, *Monopoles and contact structures*, Invent. math. **130** (1997) 209–256

[La] F Laudenbach, *Orbites périodiques et courbes pseudo–holomorphes, application à la conjecture de Weinstein en dimension* 3 *[d'après H. Hofer et al.]*, Astérisque **227** (1995) 309–333

[Li] P Lisca, *Symplectic fillings and positive scalar curvature*, Geometry and Topology **2** (1998) 103–116.

[LM] P Lisca, G Matić, *Tight contact structures and Seiberg–Witten invariants*, Invent. math. **129** (1997) 509–525

[Ma] J Martinet, *Formes de contact sur les variètès de dimension* 3, Lecture Notes in Mathematics **209**, Springer–Verlag (1971) 142–163

[McD] D McDuff, *The structure of Rational and Ruled symplectic* 4*–manifolds*, J. Am. Math. Soc. **3** (1990) 679–712

[Sl] P Slodowy, *Platonic solids, Kleinian singularities, and Lie groups*, Lecture Notes in Math. **1008**, pp. 102–138.

[Ta1] C H Taubes, *The Seiberg–Witten invariants and symplectic forms*, Math. Res. Lett. **1** (1995) 809–822

[Ta2] C H Taubes, *More constraints on symplectic manifolds from Seiberg–Witten equations*, Math. Res. Lett. **2** (1995), 9–14

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