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Kerr Spinning Particle*

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Abstract

We give a brief review of the problems connected with the model of spinning particle based on the Kerr geometry. We consider peculiarities of the Kerr rotating black hole solution in gravity, supergravity and low energy string theory. A non-trivial supergeneralization of the Kerr-Newman solution to broken N=2 supergravity is considered.

The problem of source of the Kerr geometry is discussed, and we consider some arguments in favor of the supersymmetric Fermi-ball model for the Kerr source based on the Witten $U(1) \times U(\tilde{1})$ field model for superconducting strings.

1. Introduction

The Kerr solution is well known as a field of the rotating black hole. However, for the case of a large angular momentum L; $\mid a \mid = L/m \geq m$ all the horizons of the Kerr metric are absent, and the naked ring-like singularity is appeared. This naked singularity has many unpleasant manifestations and must be hidden inside a rotating disk-like source. The Kerr solution with $\mid a \mid \gg m$ displays some remarkable features indicating a relation to the structure of the spinning elementary particles.

In the 1969 Carter [1] observed, that if three parameters of the Kerr - Newman metric are adopted to be ($\hbar$=c=1) $e^2 \approx 1/137$, $m \approx 10^{-22}$, $a \approx 10^{22}$, $ma = 1/2$, then one obtains a model for the four parameters of the electron: charge, mass, spin and magnetic moment, and the giromagnetic ratio is automatically the same as that of the Dirac electron. Keres [2], and then Israel [3] have introduced a disk-like source for the Kerr field, and it was shown by Hamity [4] that this source represents a rigid

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relativistic rotator. A model of "microgeon" with Kerr metric was suggested [5] and an analogy of this model with string models [6]. Then a model of the Kerr-Newman source in the form of oblate spheroid was suggested [7]. It was shown that material of the source must have very exotic properties: null energy density and negative pressure. The electromagnetic properties of the material are close to those of a superconductor [8,7,9], that allows to consider singular ring of the Kerr source as a closed vortex string like the Nielsen-Olesen [10] and Witten [11] superconducting strings. Since 1992 black holes have paid attention of string theory. In 1992 the Kerr solution was generalized by Sen to a solution to low energy string theory [12]. It was shown that black holes can be considered as fundamental string states, and the point of view has appeared that some of black holes can be treated as elementary particles [13]. Here we consider the obtained recently super-Kerr-Newman solution [14,15] representing a natural combination of the Kerr spinning particle and superparticle models, and also consider briefly one new idea concerning the possible structure of the disk-like "core" of the Kerr solution.

2. Main Peculiarities of the Kerr Solution

Kerr singular ring

The Kerr singular ring appears in the rotating BH solutions instead of the point-like singularity of the non-rotating BH. The simplest solution possessing the Kerr singular ring was obtained by Appel in 1887 (!) [16]. It can be considered as a Newton or a Coulomb analogue to the Kerr solution.

On the real space-time the singular ring arises in the Coulomb solution $f = e/\tilde{r}$, where $\tilde{r} = \sqrt{(x-x_o)^2 + (y-y_o)^2 + (z-z_o)^2}$, when the point-like source is shifted to a complex point of space $(x_o(t), y_o(t), z_o(t)) \rightarrow (0, 0, ia)$. Radial distance $\tilde{r}$ is complex in this case and can be expressed in the oblated spheroidal coordinates r and θ as $\tilde{r} = r + ia\cos\theta$. Singular ring corresponds to complex equation $\tilde{r} = 0$ representing intersection of plane and sphere in Cartesian coordinates. The Kerr singular ring is a branch line of the space on two sheets: "positive" one, the real world covered by $r \geq 0$, and "negative" one (an anti-world) covered by $r \leq 0$. The sheets are connected by disk $r = 0$ spanned by singular ring. The physical fields change signs and directions on the "negative "sheet. To avoid this twosheetedness, one can truncate the negative sheet. In this case the fields will acquire a shock crossing the disk, and some material sources (masse and charge) have to be spread on the disk surface to satisfy the field equations. The structure of electromagnetic field near the disk suggests then that the "negative" sheet of space can be considered as an image of the real world in the rotating superconducting mirror.

Complex interpretation

Like the Appel solution, the source of Kerr-Newman solution can be considered from complex point of view as a "particle" propagating along a complex world-line [17,18,20,21] parametrized by complex time. The objects described by the complex world-lines occupy an intermediate position between particle and string. Like the string they form the two-dimensional surfaces or the world-sheets in the space-time. It was shown that the complex Kerr source may be considered as a complex hyperbolic string which requires an orbifold-like structure of the world-sheet. It induces a related orbifold-like structure of the Kerr geometry [17,18] which is closely connected with the above mentioned twosheetedness.

Principal null congruence (PNC)

Next remarkable peculiarity of the Kerr solution is the PNC which can be considered as a vortex of null radiation. This vortex propagates via disk from negative sheet of space onto positive one forming a caustic at singular ring. PNC plays a fundamental role in the structure of the Kerr geometry. The Kerr metric can be represented in the Kerr-Schild form

$$g_{\mu\nu} = \eta_{\mu\nu} + 2he^3_\mu e^3_\nu, \tag{1}$$

where η is metric of an auxiliary Minkowski space and h is a scalar function. Vector field e^3 is null, $e^3_\mu e^{3\mu} = 0$, and tangent to PNC of the Kerr geometry. The Kerr PNC has the properties to be twisting, geodesic and shear free [19]. All the PNC with such properties are described by the Kerr theorem via a complex function $Y(x)$

$$e^3 = du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y\bar{Y} dv, \tag{2}$$

where the null Cartesian coordinates are used

$$2^{\frac{1}{2}}\zeta = x + iy, \qquad 2^{\frac{1}{2}}\bar{\zeta} = x - iy, \qquad 2^{\frac{1}{2}}u = z + t, \qquad 2^{\frac{1}{2}}v = z - t. \tag{3}$$

The Kerr theorem

The Kerr theorem [20,17,18,19] gives a general rule to construct the twisting, geodesic and shear free congruences in twistor terms. PNC is determined by function $Y(x)$ which is a solution of the equation

$$F = 0,$$

where $F(l_1, l_2, Y)$ is an arbitrary analytic function of projective twistor coordinates

$$l_1 = \zeta - Yv, \qquad l_2 = u + Y\bar{\zeta}, \qquad Y. \tag{4}$$

The singularities of the solutions are caustics of the congruence and they are defined by the system of equations

$$F = 0, \qquad \partial_Y F = 0. \tag{5}$$

The Kerr PNC may be obtained from the complex source by a retarded-time construction. The rays of PNC are the tracks of null planes of the complex light cones emanated from the complex world line [18,20,21]. The complex light cone with the vertex at some point x_0 of the complex world line $x_0^\mu(\tau)$

$$(x_\mu - x_{0\mu})(x^\mu - x_0^\mu) = 0, \tag{6}$$

can be split into two families of null planes: "left" planes

$$x_L = x_0(\tau) + \alpha e^1 + \beta e^3, \tag{7}$$

spanned by null vectors e^1 and e^3, and"right"planes

$$x_R = x_0(\tau) + \alpha e^2 + \beta e^3, \tag{8}$$

spanned by null vectors e^2 and e^3. [1] The Kerr PNC arises as the real slice of the family of the "left" null planes of the complex light cones which vertices lie on the straight complex world line $x_0(\tau)$.

Stringy Suggestions

In 1974, the model of microgeon with the Kerr-Newman metric was considered [5], where singular ring was used as a waveguide for wave excitations. It was recognized soon [6] that singular ring represents in fact a string with traveling waves. Further, in dilaton gravity, the string solutions with traveling waves have paid considerable attention [22].

In 1992 Sen generalized the Kerr solution to low energy string theory with axion and dilaton [12]. This solution was analyzed in [23]. It was shown that in spite of the strong deformation of metric by dilaton (leading to a change the type of metric from type D to type I) the Kerr PN congruence survives in the Kerr-Sen solution and retains the properties to be geodesic and shear free. It means that the Kerr theorem and the above complex representation are valid for the Kerr-Sen solution too. It has also been obtained that the field of the Kerr-Sen solution near the Kerr singular ring is similar to the field around the fundamental heterotic string that suggested stringy interpretation of the Kerr singular ring.

3. Non-trivial Super-Kerr-Newman Solution

Description of spinning particle based only on the bosonic fields cannot be complete, and involving fermionic degrees of freedom is required. The most natural way to involve fermions is to treat corresponding super black holes in supergravity. On the other hand the models of spinning particles and superparticles based on Grassmann coordinates have paid great attention. A natural way to combine the Kerr spinning particle and superparticle, leading to a non-trivial supergeneralization of the Kerr-Newman solution to N=2 supergravity broken by Goldstone fermion, was suggested in [14,15].

Problem of Triviality for Super-Black-Hole Solutions

Consistent supergravity [24] represents an unification of the gravitational field $g_{ik} = e_i^a e_k^a$, with a spin 3/2 Rarita-Schwinger field ψ_i . The combined Lagrangian has the form

$$\mathcal{L}_{sg} = -eR/2k^2 - \frac{i}{2}\epsilon^{ijkl}\bar{\psi}_i\gamma_5\gamma_j\mathcal{D}_k\psi_l, \tag{9}$$

[1] The null tetrad vector $e^1 = d\zeta - Y dv$, and $e^2 = d\bar{\zeta} - \bar{Y} dv$.

where $e = \det e_{ia}$; $\mathcal{D}_i = \partial_i + nonlin.terms$. Corresponding action $I = \int \mathcal{L}_{sg} d^4x$ is invariant under the local supersymmetry transformations, and all the supergauge-related solutions are physically equivalent. It leads to the problem of obtaining non-trivial supergravity solutions [25]. Any exact solution of the Einstein gravity is indeed a trivial solution of supergravity field equations with a zero field ψ_i Ṡtarting from an exact solution of Einstein gravity, and using the supergauge freedom one can easily turn the gravity solution into a form containing spin-3/2 field ψ_i satisfying the supergravity field equations. However, since this spin-3/2 field can be gauged away by the reverse transformation, such supersolutions have to be considered as *trivial.* It was shown [14] that *non-trivial* super-Kerr geometry can be obtained by a *trivial* supershift of the Kerr solution taking into account some non-linear body-slice (B-slice) constraints selecting the "body-part" of the bosonic space-time coordinates x^i similarly to real slice of the complex Kerr geometry discussed above.

Hints from the complex structure of the Kerr geometry

The considered above complex representation of the Kerr geometry shows that from complex point of view the Schwarzschild and Kerr geometries are equivalent and connected by a *trivial* complex shift.

The *non-trivial* twisting structure of the Kerr geometry arises as a result of the *shifted real slice* of the complex retarded-time construction [17,20]. If the real slice is passing via 'center' of the solution x_0 there appears a usual spherical symmetry of the Schwarzschild geometry. The specific twisting structure results from the complex shift of the real slice regarding the source.

Similarly, it is possible to turn a *trivial* super black hole solution into a *non-trivial* if one finds an analogue to the real slice in superspace.

The *trivial supershift* can be represented as a replacement of the complex world line by a superworldline

$$X_0^i(\tau) = x_0^i(\tau) - i\theta\sigma^i\bar{\zeta} + i\zeta\sigma^i\bar{\theta}, \tag{10}$$

parametrized by Grassmann coordinates ζ, $\bar{\zeta}$, or as a corresponding coordinate replacement in the Kerr solution

$$x'^i = x^i + i\theta\sigma^i\bar{\zeta} - i\zeta\sigma^i\bar{\theta}; \qquad \theta' = \theta + \zeta, \quad \bar{\theta}' = \bar{\theta} + \bar{\zeta}, \tag{11}$$

Assuming that coordinates x^i before the supershift are the usual c-number coordinates one sees that coordinates acquire nilpotent Grassmann contributions after supertranslations. Therefore, there appears a natural splitting of the space-time coordinates on the c-number 'body'-part and a nilpotent part - the so called 'soul'. The 'body' subspace of superspace, or B-slice, is a submanifold where the nilpotent part is equal to zero, and it is a natural analogue to the real slice in complex case.

Superlightcone Constraints, and Supergravity Broken by Goldstone Fermion

Reproducing the real slice procedure of the Kerr geometry in superspace one has to consider superlightcone constraints [2]

$$s^2 = [x_i - X_{0i}(\tau)][x^i - X_0^i(\tau)] = 0, \tag{12}$$

and B-slice, where coordinates x^i do not contain nilpotent contributions. Selecting the body and nilpotent parts of this equation we obtain three equations. The first one is the discussed above real slice condition of the complex Kerr geometry claiming that complex light cones can reach the real slice.

The nilpotent part of (12) yields two B-slice conditions

$$[x^i - x_0^i(\tau)](\theta\sigma_i\bar\zeta - \zeta\sigma_i\bar\theta) = 0; \tag{13}$$

$$(\theta\sigma\bar\zeta - \zeta\sigma\bar\theta)^2 = 0. \tag{14}$$

These equations can be resolved by representing the complex light cone equation via the commuting two-component spinors Ψ and $\tilde\Psi$

$$x_i = x_{0i} + \Psi\sigma_i\tilde\Psi. \tag{15}$$

"Right" (or "left") null planes of the complex light cone can be obtained keeping Ψ constant and varying $\tilde\Psi$ (or keeping $\tilde\Psi$ constant and varying Ψ.) As a result we obtain the equation

$$\bar\Psi\bar\theta = 0, \qquad \bar\Psi\bar\zeta = 0, \tag{16}$$

which in turn is a condition of proportionality of the commuting spinors $\bar\Psi(x)$ determining the PNC of the Kerr geometry and anticommuting spinors $\bar\theta$ and $\bar\zeta$, this condition providing the left null superplanes of the supercones to reach B-slice.

Finally, by introducing the Kerr projective spinor coordinate $Y(x)$ we have $\bar\Psi^{\dot 2} = Y(x), \quad \bar\Psi^{\dot 1} = 1$, and we obtain [3]

$$\bar\theta^{\dot\alpha} = \begin{pmatrix} \bar\theta^{\dot 1} \\ Y(x)\bar\theta^{\dot 1} \end{pmatrix}, \tag{17}$$

$$\bar\zeta^{\dot\alpha} = \begin{pmatrix} \bar\zeta^{\dot 1} \\ Y(x)\bar\zeta^{\dot 1} \end{pmatrix}. \tag{18}$$

It also leads to $\bar\theta\bar\theta = \bar\zeta\bar\zeta = 0$, and equation (14) is satisfied automatically.

Thus, as a consequence of the B-slice and superlightcone constraints we obtain a non-linear submanifold of superspace $\theta = \theta(x), \quad \bar\theta = \bar\theta(x)$. The original four-dimensional supersymmetry is broken, and the initial supergauge freedom which allowed to turn the super geometry into trivial one is lost. Nevertheless, there is a residual supersymmetry based on free Grassmann parameters $\theta^1, \quad \bar\theta^{\dot 1}$.

[2] These constraints are similar to the complex light cone constraints of the standard Kerr geometry and demand existence of the body-slice for the superlightcones placed at the points of the superworldline.

[3] Vector e^3 can be expressed now via Ψ as $e_i^3(x) = \Psi\sigma_i\bar\Psi/(\Psi^1\bar\Psi^{\dot 1})$.

The above B-slice constraints yield in fact the non-linear realization of broken supersymmetry introduced by Volkov and Akulov [26,27] and considered in N=1 supergravity by Deser and Zumino [28]. It is assumed that this construction is similar to the Higgs mechanism of the usual gauge theories and $\zeta^\alpha(x), \quad \bar{\zeta}^{\dot\alpha}(x)$ represent Goldstone fermion which can be eaten by appropriate local supertransformation $\epsilon(x)$ with a corresponding redefinition of the tetrad and spin-3/2 field. It means that starting from the gravity solution with zero spin-3/2 field and some Goldstone fermion field λ one can obtain in such a way a non-trivial supergravity solution with non-linear realization of broken supersymmetry.

There are two obstacles for indirect application of this scheme to the Kerr-Newman case. First one is the electromagnetic charge which demands to change the expression for supercovariant derivative that leads to non-Majorana values for spin-3/2 field. The second one is the complex character of supertranslations in the Kerr case that also yields the non-Majorana supershifts. Thus, this scheme has to be extended to N=2 supergravity.

4. Self-Consistent Super-Kerr-Newman Solutions to Broken N=2 Supergravity

The generalized to broken N=2 supergravity Deser-Zumino Lagrangian [28] takes the form

$$\mathcal{L} = -(i/2)[\bar\lambda\gamma\tilde{\mathcal{D}}\lambda - \bar{\tilde{\mathcal{D}}}\bar\lambda\gamma\lambda] - (i/2b)[\bar\lambda\gamma^i\chi_i - \bar\chi_i\gamma^i\lambda] + \mathcal{L}_{2-sg}, \tag{19}$$

where the N=2 supergravity Lagrangian

$$\mathcal{L}_{2-sg} = -eR/2k^2 - 1/4F_{ij}F^{ij} - i\epsilon^{ijkl}\bar\chi_i\gamma_5\gamma_j\tilde{\mathcal{D}}_k\chi_l \tag{20}$$

was given by Ferrara and Nieuwenhuizen [29]. The spin-3/2 field χ_i is a complex combination of two Majorana fields $\chi_i = \psi_i^1 + i\psi_i^2$. It follows from (19) that the self-consistent solutions to broken N=2 supergravity has to take into account the energy-momentum tensor of the Grassmann fields. In particular, when considering the initiate *trivial* solutions in the super-gauge with zero Rarita–Schwinger field, one can use this Lagrangian with $\chi = 0$ that yields the Einstein–Maxwell–Dirac system of equations. We note that the energy-momentum tensor of the Goldstone field λ acts here as fermionic matter. However, when using the *exact* Kerr-Newman solution as trivial one to perform the super-gauge with absorption of the Goldstone fermion, we do not take into account the energy-momentum tensor of the Goldstone field. Therefore, in general case, the obtained by a supershift super-geometries cannot be treated as self-consistent. However, one exclusive case can be selected when the self-consistency is guaranteed. It takes place for the ghost Goldstone field possessing the zero energy-momentum tensor.

In this case, starting from the Lagrangian with $\chi = 0$, we have in fact the Einstein-Maxwell system of equation leading to the exact Kerr-Newman solution and the Dirac equation (on the Kerr-Newman background) for the Goldstone fermion λ.

This solution can be considered as an exact super-solution to N=2 supergravity coupled to Goldstone field. Then, absorption of the Goldstone field by the complex Rarita-

Schwinger field χ turns this solution into self-consistent solution with broken N=2 supersymmetry.

In the aligned to e^3 case

$$\lambda = \begin{pmatrix} 0 \\ B \\ C \\ 0 \end{pmatrix}. \tag{21}$$

The solution of the massless Dirac equation on the Kerr-Newman background gives for the functions B and C following expressions

$$B = \bar{Z} f_B(\bar{Y}, \bar{\tau})/P, \qquad C = Z f_C(Y, \tau)/P, \tag{22}$$

where f_B and f_C are arbitrary analytic functions of the complex angular variable

$$Y = e^{i\phi} \tan(\theta/2), \tag{23}$$

and the retarded-time

$$\tau = t - r - ia\cos\theta \tag{24}$$

satisfies the relations $\tau,_2 = \tau,_4 = 0$, and $Y,_2 = Y,_4 = 0$.

Details of this treatment can be found in [15].

Peculiarities of the Super-Kerr-Newman Solution.

Performing calculations one sees that the energy-momentum tensor of the Goldstone field λ with $C = \bar{B}$ cancels, $T_{ik} = 0$, and the field takes the ghost character. The torsion and Grassmann contributions to tetrad cancel, and metric takes the exact Kerr-Newman form.

The main features of the resulting super-Kerr-Newman solutions are the extra wave fields on the bosonic Kerr-Newman background: the complex Rarita-Schwinger field χ_i and a nilpotent contributions to electromagnetic field.

The expressions for B and C are singular on the Kerr singular ring, $Z^{-1} \equiv P^{-1}(r + ia\cos\theta) = 0$, and contain traveling waves if there is an oscillating dependence on complex time parameter τ. Indeed, near the Kerr singular ring $\tan\theta \simeq 1$, and angular dependence of these solutions on ϕ is determined by the degree of function $Y = e^{i\phi}\tan(\theta/2)$. One sees that any non-trivial analytic dependence on Y will lead to a singularity in θ. Thus, besides the Kerr singular ring the solutions contain an extra axial singularity which is coupled topologically with singular ring threading it. One should also note that this 'axial' singularity in some solutions can change the position in time scanning the space-time.

Elementary fermionic wave excitations have the form

$$C = \bar{B} = (Z/P) Y^n e^{i\omega\tau} = \frac{1}{r + ia\cos\theta} e^{in\phi} \tan^n(\theta/2). \tag{25}$$

The first factor contains the singular branch line corresponding to the known twofoldedness of the Kerr geometry. Because of that parameter n can take both integer and half-integer values. Electromagnetic field contains the term representing the null electromagnetic field propagating along PNC. Near the Kerr singular ring PNC is tangent to ring leading to traveling waves propagating along the ring.

The 'axial' singularity coincides with z-axis and can be placed either at $\theta = 0 \quad (Y = 0)$ or at $\theta = \pi \quad (Y = \infty)$. It is a half-infinite line threading the Kerr singular ring and passing to 'negative' sheet of the Kerr geometry. Its position and character depend on the values of n. By introducing the distance from 'axial' singularity $\rho = \sqrt{x^2 + y^2}$, one can describe its behavior in the asymptotic region of large r by the following expressions:

- if $\theta \simeq 0$ then

$$\delta_g A \sim \rho^{2n} r^{-3-2n}(dx + idy),$$
$$\delta_F A \sim \rho^{2n} r^{-4-2n}(dz + dt),$$

- if $\theta \simeq \pi$ then

$$\delta_g A \sim \rho^{-2n-2} r^{2n-1}(dx - idy),$$
$$\delta_F A \sim \rho^{-2n-2} r^{2n-2}(dz - dt).$$

One sees that this singularity can be increasing or decreasing function of distance r. For some n (for example n=1/2,-3/2) dependence on r can disappear. The solutions with 'increasing' and 'even' singularities cannot be stable. In the cases $n = 0$ and $n = -1$ singularity represents a 'decreasing' half-infinite line like the string of the Dirac monopole. The case $n = -1/2$ is exclusive: there are two 'decreasing' singularities which are situated symmetrically at $\theta = 0$ and $\theta = \pi$. The space part of the null vector e^3 is tangent to axial singularity, and electromagnetic field grows near this singularity and contains in asymptotic region the leading term in the form of the null traveling wave

$$F \simeq -(\bar{s} s 2^{3/2}/k)[C^2 \tilde{r}^{-1} \omega e^1 \wedge e^3 + c.c.term]. \tag{26}$$

5. Problem of the Kerr Source, and Fermi Ball Models

The above consideration of super-Kerr-Newman solution is based on the massless fields providing description of the rotating super-black-hole with a source hidden behind horizon.

However, for the known parameters of spinning particles, the value of angular momentum is very high, regarding the mass parameter. In this case the black hole horizons disappear, and the Kerr singular ring is naked. Besides there appears a narrow region of causality violation near the Kerr singular ring. This region has to be covered by a matter source. The massless fields of the above super-black-hole solution have to get a mass in this region of matter source. Structure of the Kerr source represents an old problem, and few distinct models of this source were suggested.

Kerr Ring Model.

The line source generated by the Kerr singular ring was considered by Israel [30], and also was suggested by microgeon model [5] and by stringy interpretation of the Kerr singular ring [6], in particular, by low energy string theory [23]. This source represents an 'Alice' string [11,31] which is a branch line of space leading to the topological twosheetedness of the Kerr space.

Thin Shell Model.

The Kerr twosheetedness can be removed by a truncation the negative sheet. As a result there appears a source distribution on the surface of truncation. Truncation at $r = 0$ (ellipsoidal coordinate) gives the model of the rotating infinitely thin disk considered by Israel [3] as a model of electron. The model of source was then specified by Hamity [4] who obtained the exotic structure of matter of the source and pointed out that the Kerr disk represents a rigid and relativistic rotator. In the development of model given by López [7], negative sheet is truncated at the coordinate surface $r = r_e = \frac{e^2}{2m}$. As a result the region of causality violation is covered by source in the form of the highly oblated elliptic shell. It has the Compton radius $a = \frac{1}{2m}$ and thickness of the classical Dirac electron. The resulting oblateness of the disk is $a/r_e \approx 137$. For small angular momentum the source takes the spherical form of the Dirac electron model. The fields out of the shell have the exact Kerr-Newman form. Interior of the shell is flat. The shell is in rigid relativistic rotation, charged and built of a superconducting matter with negative pressure and zero energy density in corotating frame.

Solid Disk Model

Attempt to find a physical reason for origin of the exotic matter of the Kerr source have led to the model of solid disk built of a superdense matter confined by the vacuum Casimir energy [9]. Negative contribution of the Casimir energy is diverging, however, for superdense and superconducting matter it can be regularized by a cutoff parameter which is determined by interparticle distances of constituents. It was shown that for the limiting ultrahigh densities of matter, when the interparticle distances tend to their Compton sizes, the negative Casimir contribution to energy is extremely strong and achieves the energy density of matter. As a result, the superdense matter turns into a pseudovacuum state with zero energy density. Constituents became massless in superdense region that provides confinement. The unique predicted difference of this pseudovacuum state from the true vacuum is its impenetrability for electromagnetic field. It was proposed that the Kerr source represents a rotating bag filled by a matter in such pseudovacuum state.

Fermi ball Model

The above models look incompatible. We will consider briefly here one new proposal concerning the structure of the Kerr source which, probably, will allows one to remove incompatibility of these models. The new approach is based on a similarity of the structure of the Kerr source with the structure of cosmological Fermi-ball models.

Fermi ball model [32,33] represents a bag of false vacuum that is inhabited by Fermi gas. Bag is restricted by tiny wall trapping fermions which became massless near the wall. One can see that the shell and disk-like bag models of the Kerr source can be combined forming the Fermi ball model. Unfortunately, analytic solutions for the fermi balls exist only for the simplest field models [32,33] and in the case of the simplest geometry corresponding to the flat wall surface. However, like the diverse field models for superconducting strings [10,11], the Fermi ball models can also be considered on the base of diverse field models. Taking into account superconducting character of the Kerr source, one sees that many of the field models used for the modelling superconducting strings can be also used for modelling the Fermi balls. Supersymmetric generalizations of these models [33,34] have a special interest since they allow one to get the supersymmetric vacuum providing the flat asymptotical solutions in supergravity. Because of the necessity to have the long range electromagnetic field in the region out of the Kerr source, the flavor has to be given to a super-version of the Witten $U(1) \times U(\tilde{1})$ field model. Besides the unification of the bag and shell models, the Fermi ball models give some suggestions in the flavor of the existence of a central singularity inside the ball allowing one to involve in the model also the stringy source. In the simplest field models with a flat wall surface the vacuum states are homogenous at the distance from wall. However, if the wall surface is not flat, there appears an instability of the vacuum regarding the appearance of singularity inside the ball.

In the case of the Kerr geometry, it shall lead to the appearance of the Kerr singularity inside the bag that is compatible with the above stringy model of the Kerr source. Because of the high oblateness of the Kerr disk $\sim \alpha^{-1} \approx 137$, the Kerr singularity will be placed very close to the shell, at the distance $\alpha^2 a \approx 10^{-4} a$. It can be a trapping zone of the wall where the fermions are massless. Therefore, fermions will be trapped by the Kerr singular ring reproducing the superconducting string effect described by Witten [11].

We propose that fermi ball model of the Kerr source shall allow one to unite the bag, shell and string models. Experience of the string-like solutions does not give a hope on analytic solution of this problem even for the simplest case of spherical bag. At first stage, a detailed analysis of the different field models and models of broken gauge symmetry and supersymmetry will be necessary with the attempt to get a spherical Fermi ball model with appropriate properties of the vacuum and matter fields inside and out of bag.

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References

[1] Carter B., Phys. Rev. **174**(1968) 1559;

[2] Keres H. Zh. Eksp. Teor. Fiz. **25**(1967) 54;

[3] Israel W., Phys. Rev. **D2** (1970) 641;

[4] Hamity V. Phys. Lett. **A 56**(1976)77;

[5] Burinskii A., Sov. Phys. JETP **39**(1974) 193;

[6] Ivanenko D. and Burinskii A., Izv. VUZ Fiz.**5**(1975)135; Sov. Phys. J. (USA).,nr.**7** (1978)113;

[7] López C.A., Phys. Rev. **D30** (1984) 313;

[8] Tiomno I., Phys. Rev.**D7**(1973) 992.

[9] Burinskii A. Phys. Lett. **B 216**(1989)123;

[10] H.B.Nielsen and P. Olesen, Nucl. Phys.,**B61**(1973)45.

[11] Witten E., Nucl. Phys.,**B249**(1985)557.

[12] Sen A., Phys.Rev.Lett., **69**(1992)1006;

[13] Dabholkar A., Gauntlett J., Harvey J. and Waldram D., Nucl. Phys. **B 474**(1996) 85; Sen A. Modern Phys. Lett. **A 10**(1995)2081; Nucl.Phys **B46** (Proc.Suppl)(1996)198; Horowitz G.T. and Sen A., Phys. Rev. **D 53** (1996)808; 't Hooft G., Nucl. Phys. **B335**(1990)138; Holzhey C. and Wilczek F., Nucl. Phys. **B380**(1992)447; hep-th/9202014; J. Preskill, P.Schwarz, A. Shapere, S.Trivedi and F.Wilczek, Mod. Phys. Lett. **A6**, 2353 (1991);

[14] Burinskii A., Phys.Rev.D 57(1998)2392, hep-th/9704102; Burinskii A., in Proc. of the International Symposium on Frontiers of Fundamental Physics (Hyderabad 1997), edited by B.G.Sidharth and A. Burinskii, Universities Press, Hyderabad 1998;

[15] Burinskii A. Class.Quant.Grav.**16**(1999)3497, hep-th/9903032;

[16] E.T. Whittacker and G.N. Watson, A Course of Modern Analysis, Cambrige Univ. Press London/New York,p.400, 1969 .

[17] Burinskii A., String - like Structures in Complex Kerr Geometry, Proc. of the Fourth Hungarian Relativity Workshop Edited by R.P. Kerr and Z. Perjés, Academiai Kiado, Budapest 1994, gr-qc/9303003;

[18] Burinskii A. Espec. Space Explorations, **9 (C2)**(1995) 60, Moscow, Belka, hep-th/9503094;

[19] Debney G.C., Kerr R.P., Schild A., J.Math.Phys.,**10**(1969)1842;

[20] Burinskii A., Kerr R.P. and Perjes Z., 1995, Nonstationary Kerr Congruences, gr-qc/9501012;

[21] Burinskii A., Phys.Lett. **A 185**(1994)441;

[22] D. Garfinkel, Black String Traveling Waves, gr-qc/9209002;

[23] Burinskii A., Phys.Rev.**D 52**(1995)5826, hep-th/9504139;

[24] Deser S. and Zumino B., Phys. Lett., **B 62** (1976) 335;

[25] Aichelburg P.C. and Güven R. Phys.Rev. **D24** (1981) 2066;
Dereli T. and Aichelburg P.C. Phys.Lett.**B 80** (1979) 357;
Aichelburg P.C. and Güven R. Phys.Rev. **D 27** (1983)456;

[26] Volkov D.V. and Akulov V.P. Pis'ma Zh. Eksp.Teor.Fiz. **16**621(1972);

[27] Wess J. and Bagger J. Supersymmetry and Supergravity, Princeton, New Jersey 1983;

[28] Deser S. and Zumino B. Phys. Rev. Lett., **38**(1977) 1433;

[29] Ferrara S. and van Nieuwenhuizen P., Phys.Rev. Lett., **37** (1976) 1669;

[30] Israel W. Phys.Rev.**D 15**(1977) 935;

[31] Schwarz A.S., Nucl. Phys. **B208**(1982) 141;

[32] MacPherson A. and Campbell, Phys. Rev. Lett., **B 347**(1995) 205; hep-ph/9408387;

[33] Morris J. and Bazeia D. Phys.Rev. **D54** (1996) 5217;

[34] Morris J. Phys.Rev. **D53** (1996) 2078.