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Nonabelian Cosmic Strings from $SO(3)$ Gauge Symmetry*

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Abstract

The nature of $SO(3)$ breaking into its closed subgroups and the emergence of nonabelian cosmic strings are discussed. Relevance to GUT breaking and thereby to cosmology is pointed out. Classification of cosmic strings are associated with the affine ADE Lie algebras.

1. Introduction

Defects in condensed matter phenomena have been the target of mathematical analysis in view of the fact that homotopy groups of the manifold lead to the classifications of point-like defects, line-defects and planar defects[1]. TWB Kibble [2] have worked out the similar problems in gauge field theories and emphasised that the cosmic strings, analogous structures of line defects in gauge field theories, may play crucial roles in cosmological models of the universe. Most of the cosmic strings in literature arising from gauge symmetry breaking studied are of abelian nature. No detailed investigation has been given for the non-abelian cosmic string other than the recent attemps of ours [3].

Since cosmic string formation occurs in a symmetry breaking where the residual little group involves disconnected group elements, that requires a detailed study of the discrete subgroups of popular gauge symmetries such as SU(5) $\approx E_4$, $SO(10) \approx E_5$, E_6 and E_8. $SU(5)$ and $SO(10)$ are out of question regarding the nonabelian cosmic strings for they do not contain sufficient group elements orthogonal to the Standard Model

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$SU(3)_c \times SU(2)_L \times U(1)_Y$. E_6 and E_8, with large number of subgroups orthogonal to the standard model, may be the best candidates for such an analysis. E_6 with its extra $SU(3)$ and E_8 with its $SU(3)\times SU(3)$ group orthogonal to the standard model may break to discrete subgroups of $SU(3)$.

A profound work will require a detailed analysis as to how a given E_6 or E_8 Higgs potential may break to the standard model with some residual discrete group elements orthogonal to the standard gauge symmetry. This can be done either breaking $SU(3)$ directly to its discrete subgroup, e.g. $PSL_2(7)$ of order 168, or through its special $SO(3)$ subgroup. Then it turns out that breaking $SO(3)$ into its discrete subgroups is not a mere academic interest but may contain some valuable ingredients applicable to cosmological models where non abelian cosmic strings naturally arise.

A systematic analysis regarding the smymmetry breaking mechanism of a gauged $SO(3)$ field theory into its finite subgroups has already began several years back where a complete analysis is avaible for the irreducible representations l=2,3. The results of l=4 and l=6 which will involve respectively the breaking into octahedral an icosahedral groups will appear soon.

In this paper we briefly describe the results of l=2 and l=3 representations which partly appeared in other publications of ours. Here we emphasize more on the classifications of the cosmic strings with the homotopy groups of the related manifolds. Therefore in Section 2 we describe the symmetry breaking mechanism for l=2 and l=3 representations. In Section 3 we give the relevant homotopy groups and illustrate their relevances to the ADE classification of the affine Lie algebras. Finally in Section 4 we discuss our results and make remarks on possible channels into which the problem may evolve.

2. $SO(3)$ Gauge theory for l=2 and $l = 3$ representations

The standard Langangian of a local gauge theory without fermions is given by

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(D_\mu\phi)^\dagger(D_\mu\phi) - V(\phi) \tag{1}$$

where the field strength $F_{\mu\nu}$ and the covariant derivative D_μ are given by

$$F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + gW_\mu \times W_\nu \tag{2}$$

$$D_\mu = \partial_\mu - igW_\mu, W_\mu = \vec{J}.\vec{W}_\mu \tag{3}$$

Here $\vec{J}$'s are the $(2l+1)\times(2l+1)$ matrix generations of $SO(3)$ for the irreducible representation l. One can label the Higgs scalars as $\phi(lm)$ which transform like spherical harmonics Y_{lm} under the group transformations.

For $l = 2$ representation the five $\phi(2m)$, m=-2,...,2 can be related to the components of a symmetric, traceless second rank tensor T_{ij} $(i,j$=1,2,3)[4]. With the tensor field T_{ij} the Higgs potential takes the form

$$V(T) = aTrT^2 + bTrT^3 + c[TrT^2]^2 \tag{4}$$

There are two little groups, D_2 and D_∞ where the vacuum expectation value of T_{ij} respectively take the values in the diagonal

$$\langle T_{ij}\rangle = (1,-1,0)v \quad \text{for} \;\; D_2 \tag{5}$$

and

$$\langle T_{ij}\rangle = (1,1,-2)v' \quad \text{for} \;\; D_\infty \tag{6}$$

The minumum of potential(4) takes places for D_2 breaking only for b=0 case where the potential possesses a larger $SO(5)$ global symmetry. It is this feature of the potential that leads to a pseudo Goldstone boson for D_2 breaking. Indeed three of the Higgs fields are absorbed by the gauge fields giving them their masses with the relation $M_{W^0} = 2M_{W^\pm}$. One of the remaining field is the genuine Higgs scalar with a mass of $\sqrt{-2a}$ whereas the other field remains as a pseudo Goldstone boson. In the case of $SO(3)$ breaking into D_∞ the potential parameters satisfy the inequalities $a < 0$, $b < 0$, $c > 0$ which leads to a result with $M_{W^\pm} \neq 0$, $M_{W^0} = 0$ and three massive Higgs fields.

Now we discuss the case for l=3 representation. The seven Higgs fields can be compactly described by a symmetric, traceless tensor T_{ijk} of rank three. The potential can be put into the form

$$V(T) = aT_{ijk}T_{ijk} + b(T_{ijk}T_{ijk})^2 + cT_{ijk}T_{ijl}T_{mnk}T_{mnl} \tag{7}$$

where the indices take values 1,2,3 and the summation over the repeated indices are implicit. We note that no third order potential term exists. It can be shown that they are identically zero. In(7) there are two fourth order terms in the potential. In fact it can be proven that all fourth order terms can be written as a linear combination of the terms in(7).We note that the first two terms in the potential has a larger symmetry of $SO(7)$ but the last term does not respect this symmetry. Therefore in the absence of the last term (c=0) one may naturally led to a result with pseudo Goldstone bosons. But with $c \neq$0 one can show that all Higgs fields gain masses. Here we give the example of the breaking of $SO(3)$ into its tetrahedral group. The other little groups of $SO(3)$ for l=3 representations will be discussed in a different publication [4]. The T_{123} component of the field tensor T_{ijk} takes a nonzero expectation value $\langle T_{123}\rangle = v \neq 0$. The three gauge bosons gain equal masses by absorbing three Higgs fields. Three of the remaining Higgs fields transforming as a three dimentional irreducible representation of the tetrahedral group gain equal masses. The fourth Higgs field is a Tetrahedral singlet with a mass of $\sqrt{-2a}$. Of course l=3 representations has many other little groups such as $SO(2)$, D_3, C_3, and C_2. Details of these breakings can be found in ref[4]. In the absence of the last term (c=0) in the potential one certainly expects pseudo Goldstone bosons for the potential possessing global $SO(7)$ symmetry. Indeed one can show that while the Higgs field transforming a Tetrahedral group singlet gain a mass, those in the triplet representation remain massless.

The l=4 is the smallest representation where $SO(3)$ can break into the octahedral group. The explicit potential in terms of the nine component Higgs field is lengthy not to produce in this short article. Nevertheless it could be put into a compact form when a symmetric, traceless tensor of rank four which we do not give here in detail is invoked [5]. If one chooses the fields V_{1133}, V_{2233} and V_{1122} taking the non-zero expectation values such that $\langle V_{1133}\rangle = \langle V_{2233}\rangle = \frac{3}{4}\langle V_{1122}\rangle = v$ then $SO(3)$ is broken into its octahedral group. The potential involving cubic and quartic potentials vialote the global $SO(9)$ symmetry which protect us from Goldstone bosons. If we want to break a $SO(3)$ gauge symmetry to its icosahedral group then the lowest dimentional representation is l=6. An invariant potential formed in terms of 13 component Higgs fields can be expressed in terms of a symmetric, traceless tensor field of rank 6. We also defer this cumbersome calculations for a future publication [5]. The general potential does not allow global $SO(13)$ so that pseudo Goldstone bosons will not arise.

3. Classification of Non-Abelian Cosmic Strings

It was suggested [1] that the disclination lines in the liquid crystals can be characterized by the conjugacy classes of the first homotopy groups of the manifolds of interest. Similar ideas can be extended to the non-abelian cosmic strings arising $SO(3)$ breaking. To be more explicit let $SO(3)$ breaks into one of its little group H. Then the first homotopy group of the manifold $SO(3)/H$ satisfies

$$\Pi_1(SO(3)/H) = \Pi_1(SU(2)/2H) = \Pi_0(2H) = 2H$$

if H is completely disconnected. Here H is one of the discrete subgroup of $SO(3)$ and $2H$ is its double cover, in other words, its image in $SU(2)$. Now we discuss those of concern in turn. For l=2 and $SO(3) \to D_2$ breaking we have

$$\Pi_1(SO(3)/D_2) = \Pi_1(SU(2)/Q) = \Pi_0(Q) = Q$$

where Q is the quaternion group $\pm 1, \pm i\sigma_1, \pm i\sigma_2, \pm i\sigma_3$. Here σ_i are the Pauli matrices. The quaternian group Q has five conjugacy classes and it is straightforward to write down the class multiplications which can be used to predict the outcome of two merging cosmic strings. For l=3 representation and $SO(3) \to$ tetrahedral group A_4 the first homotopy group is

$$\Pi_1(SO(3)/A_4) = \Pi_1(SU(2)/2A_4) = \Pi_0(2T) = 2A_4$$

where $2A_4$ denotes the binary tetrahedral group which can be represented by the 2×2 matrices $\pm 1, \pm i\sigma_1, \pm i\sigma_2, \pm i\sigma_3, \frac{1}{2}(\pm 1 \pm i\sigma_1 \pm i\sigma_2 \pm i\sigma_3)$. This group has seven conjugacy classes. The same argument can also apply to $SO(3) \to H$ where H is either octahedral group or icosahedral group. The double cover of the octahedral group is the binary octahedral group of order 48 and the $SU(2)$ image of icosahedral group is the binary icosahedral group of order 120. Binary octahedral group has eight and binary icosahedral group has nine conjugacy classes. It is intersting to observe that the McKay correspondence [6] also applies here. Namely the affine Lie algebras $SO(8), E_6, E_7$ and E_8 have respectively the structures similar to the class multiplication structures of Q(quaternion group), $2A_4$ (binary tetrahedral group), $2S_4$(binary octahedral group) and $2A_5$ (binary icosahedral group).

4. Conclusion

It is perhaps more than an academic interest to work out the spontaneous breaking mechanism of an $SO(3)$ gauge symmetry into its closed subgroups. The phenomena which is occuring in condensed matter physics may find its counterpart in gauge symmetries applied to cosmology. We know that GUT's lead to magnetic monopoles, non observation of which led to the inflationary universe models. It is equally probable that some of the GUT's may lead to cosmic string solutions where cosmic strings may be responsible of the density fluctations. In this kind of scenario breaking an $SU(3)$ component of a GUT into ist discrete subgroup creates cosmic strings. In this work we have presented a number of examples about $SO(3)$ breaking. This can be extended to the case of $SU(3)$.

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