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Noncommutativity in String Theory and Large Extra Dimensions*

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Abstract

Noncommutativity of space coordinates of string theory are reviewed and are related to the large extra dimension solution of the hierarchy problem

1. Introduction

In quantum mechanics, space coordinates commute. However, in the realm where quantum effects of gravity become significant, space coordinates may not commute. In string theory noncommutativity of D-brane coordinates has been recently observed. [1,2]
The appearance of noncommutativity in the presence of branes in string theory is closely tied to the nonvanishing expectation value of the NS-NS axionic two form field B with components along the brane. When the field B has non vanishing components on the D-brane, the coordinates of the open strings attached to them and the coordinates along the D-branes fail to commute. We will quickly review this development in section II.

In a different context, a novel solution to the problem of hierarchy between the Planck scale of $10^{19}GeV$ and the electroweak scale of TeV, has been recently proposed via the existence of submillimeter extra space dimensions beyond the three physical space dimensions of our universe [3]. In this scenario the extra dimensions acquire submillimeter lengths and have dramatic consequences in both classical gravity experiments and in

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future collider experiments. Studies on the effects of these extra dimensions on astrophysical and cosmological observables, in addition to the present data on classical gravity and collider experiments, have yielded bounds on the scale and the number of the extra dimensions.

We will discuss consequences of the noncommutativity of space coordinates of the branes on the scenario of large extra dimension in section III.

2. Noncommutativity for D-branes

We will start with a simple discussion of string theory.Open string action is,

$$S = \frac{1}{4\pi\alpha'} \int d\sigma d\tau \partial_a X^\mu \partial_a X^\mu$$

whose variation,

$$\delta S = \frac{1}{4\pi\alpha'} \left[\int d\tau \int_0^\pi d\sigma (2\dot{X}\delta\dot{X} - 2X'\delta X') \right]$$

$$= \frac{-2}{4\pi\alpha'} \int d\tau \int_0^\pi d\sigma (\ddot{X} - X'')\delta X$$

$$+ \frac{2}{4\pi\alpha'} \left[\int_0^\pi d\sigma \dot{X}\delta X \mid_{\tau=-\infty}^{\tau=+\infty} - \int d\tau X'\delta X \mid_{\sigma=0}^{\tau=\pi} \right]$$

gives two possibilities for the boundary condition, either

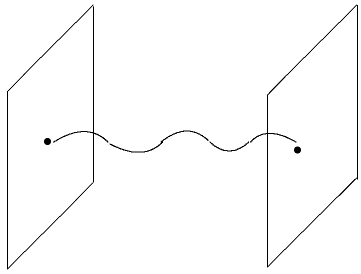
$$X'(0) = 0, \qquad Neumann$$

$$or \quad \delta X(0) = 0 \qquad Dirichlet$$

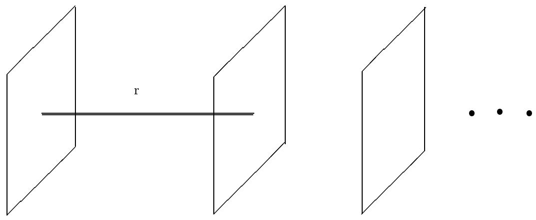
Similarly for $\sigma = \pi$.
Therefore for Dirichlet boundary condition string's ends move on a plane (D-brane).
For example: Dirichlet Boundary Condition on X^9: X=0 at $\sigma = 0$ and $X = y$ at $\sigma = \pi$
Neumann Boundary Condition on X^i $i \neq 9$.

$$X^9(\sigma,\tau) = \frac{\sigma}{\pi} Y + oscillators$$

$$X^i(\sigma\tau) = x^i + 2\alpha' p^i \tau + oscillators$$



Now on each brane there exists a U(1) gauge field, and on N parallel D-branes there are N U(1) fields with N(N-1) massive fields with mass $\sim$ length of the strings r; as $r \to 0$, we get extra massless states, and therefore a U(N) gauge field appears. The coordinates orthogonal to D-brane are the "other" components of the gauge field A, and consequently "non-commutative geometry" shows up. A different noncommutativity appears for D-branes which we will now address.



It is generally believed that there is an eleven dimensional theory called M-theory, which has the property that:
1-Compactified on the 11-th direction it is the type IIA string theory.
2-Its low energy theory is the 11-dimensional supergravity.
A couple of years ago T. Banks, W. Fischler, S.H. Shenker, and L. Susskind conjectured that M-Theory in the ∞ momentum frame, is described by the $N \to \infty$ or Do-branes with the action, (that of the SUSY quantum mechanics):

$$L = \frac{1}{2R} Tr \left(\dot{X}^a \dot{X}^a + \sum_{a,b} [X^a, X^b]^2 + \bar{\psi} i \dot{\psi} - \bar{\psi} \Gamma_a [X^\alpha, \psi] \right)$$

with $P_{11} = \frac{N}{R} \to \infty, \quad N, R \to \infty$.
Here R is the compactification radius of the 11-dimensional coordinate.

In string theory (or M theory) in order to make contact with physical world the theory has to be compactified to 4 dimensions. In a matrix model compactification is equivalent to taking the superselection sectors satisfying

$$UXU^{-1} = X + R$$

with U commuting with the uncompactified dimensions.
The solution is:

$$X = i\tilde{\partial} + A(\tilde{x}),$$

on the dual coordinate $\tilde{x}$.
Then a super symmetric gauge theory on (1+1) dimension describes the matrix theory compactified on the circle

Later A. Connes, M. Douglas, and A. Schwarz considered compactification on a noncommutative torus. They noted that to compactify on a torus,

$$UXU^{-1} = X + R_1$$

$$VYV_{-1} = Y + R_2,$$

the consistency condition, $UV = \lambda VU$, with $\lambda = e^{i\theta}$, allows a non zero θ, which defines a noncommutative torus. This procedure then produces a super Yang Mills theory on T^θ, the noncommutative torus, with field products replaced by * products as follows,

$$\varphi(x) * \varphi(x) \equiv e^{i\theta \frac{\partial}{\partial y}\frac{\partial}{\partial z}} \varphi(y)\varphi(z)\,|_{z=y=x}$$

They then conjectured that turning on the M-Theory 3 form $A^{(3)}$ with a light-like direction A^{ij-} will deform T^2 to T^θ, i.e., $\theta = \int dx^i dx^j A^{ij-}$

We noticed [1] that the above compactification has a natural setting in the so called, mixed branes, i.e., branes in the presence of the Kalb-Ramond background,

$$S = \frac{1}{4\pi\alpha'} \int d\sigma^2 \left[\eta_{\mu v}\partial_\alpha X^\mu \partial_\beta X^v g^{\alpha\beta} + \varepsilon^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta X^v \beta_{\mu v}\right].$$

The boundary condition then is:

$$X_1' + B\dot{X}_2 = 0, \qquad X_2' - B\dot{X}_1 = 0, \qquad B \equiv B_{12}$$

Upon imposition of canonical commutation relations

$$[X^\mu(\sigma\tau), P^v(\sigma', \sigma)] = i\eta^{\mu v}\delta(\sigma - \sigma'),$$

and using Fourier decomposition,

$$X^1(\sigma,\tau) = x_0^1 + p^1\tau - Bp^2\sigma + Oscillators$$

$$X^2(\sigma,\tau) = x_0^2 + p^2\tau + Bp^1\sigma + Oscillators$$

the C.M. coordinates;

$$X^i \equiv \int X^i(\sigma,\tau)d\sigma \qquad i=1,2$$

turned out, $[X^1, X^2] = i\theta$, to be noncommutative. Here $\theta = 2\pi\alpha' \frac{B}{1+B^2}$

3. Large Extra Dimensions

In the large extra dimension scenario [3] the standard model fields lie on our three dimensional world and the graviton in the bulk. For short distances classical gravity is modified, for which experiments are planned,

$$F = G_{4+n}\frac{m_1 m_2}{r^{2+n}}.$$

While for large distances it follows the Newtonian behaviour. Moreover the gravitational coupling constant is suppressed by $(M_*^n V_N)$,

$$M_4^2 = M_{4+n}^2 (M_{4+n}^n L^n).$$

So that the Planck $M_4^2 \sim L^n M^{n+2}$ is related to the bulk mass $M_* \sim 1TeV, \quad M_4 \sim 10^{19} GeV$, giving

$$L \sim 10^{\frac{30}{n}-17} cm,$$

for the scale of the large extra dimension.

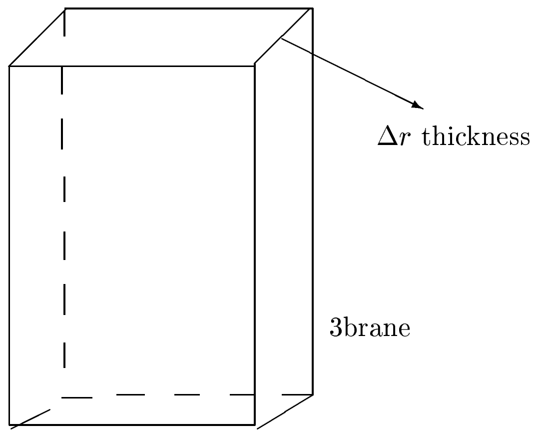
In string theory it is reasonable to assume that our world lies on a 3-brane, compactifying 6 dimension of the theory.The natural scale of string theory $l_s \sim \sqrt{\alpha'}$ is usually taken $M_{plank} \sim \frac{1}{l_s} \sim 10^{19} GeV \rightarrow l_s \sim 10^{-33} cm$, which compared to the electro-weak scale $\sim 1000 GeV$ is 16 orders of magnitude different. However, in the large extra dimension scenario the string scale is taken to be $M_* \sim \frac{1}{l_s} \sim 1TeV$, which leads to a "solution" of the "hierarchy problem".

Let us mention a few consequences of the scenario :

1- One must consider Kaluza-Klein modes of various bulk fields. Each graviton K.K. mode makes a $\frac{1}{M_*^2(\sqrt{M_*^n} v_n 0^2)} = \frac{1}{M_4^2}$ contribution which is very small. But there are many modes and they add to significant effects, which results in bounds on M_* from collider experiments (in the range of $\sim 10TeV$)

2- Graviton production carrier away energy in astrophysical objective giving similar bounds on M_*.

We will now mention a few consequences of the noncommutativity of space due to the presence of an antisymmetric field B. In analogy to the phase space noncommutativity of ordinary quantum mechanics, where $[X, P] = i\hbar$, the brane will have a thickness proportional to B. This thickness of the brane may enhance the reabsorption of the graviton and a recalculation of the cosmological consequences of the large extra dimension scenario is in order. Since brane thickness is small $\sim \frac{1}{M_*}$, graviton reabsorption in the bulk by the brane is negligable.



To stabilize the large dimensions we introduce scalar fields φ. The position of the minimum of the relevant potential will stabilize the large dimensions. Now in a 5-brane with $B \neq 0$ on two of the dimensions, coordinates do not commute, $[X_1, X_2] = 2\pi\alpha' B$, and consequently the minimum is of the form $\Delta r = \sqrt{X_1^2 + X_2^2}, \sim \sqrt{\alpha' B}$.

References

[1] F. Ardalan, H. Arfaei, and M.M. Sheikh-Jabbari, JHEP 02, 016 (1999); and "Dirac Quantization of Open Strings and Noncommutativity in Branes", hep-th/9906161.

[2] For a recent review and further references see N. Seiberg and E. Witten, "String Theory and Noncommutative Geometry", hep-th/9908142 and references there in;

[3] N. Arkani-Hamed, S. Dimopolous, and G. Dvali, Phys. Lett. B429, 263 (1998); Phys Rev D59, 086004 (1999). I. Antoniadis, N. Arkani-Hamed, S. Dimopolous, and G. Dvali, Phys. Lett