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K-Vacancy Production in the Collision of Highly Charged Relativistic Ions With Heavy Atoms

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Abstract

A general expression for the cross section of the inelastic collision of relativistic highly charged ion with heavy (relativistic) atoms is obtained using the generalized eikonal approximation. In the ultrarelativistic limit, the obtained formula coincides with a known exact one. As an application of the obtained result, probability and cross section of the K-vacany production in the $U^{92+} - U^{91+}$ collision are calculated.

Inelastic processes in the collision of fast and relativistic highly charged ions with atoms have become the subject of extensive investigation in recent years [1, 2, 3] as expressed in the great number of papers devoted to the theoretical investigation of these collisions. (See for example [1, 4] and references therein.)

Cross sections of such inelastic processes are large enough and therefore their associated processes are interesting in the applied plan [1]. As is well known [4], perturbation theory begins to break down for relativistic ion-atom collisions with large projectile charges (for Z≥75). For example, a well known Born approximation leads to the result in which (for small impact parameters) ionisation probability may exceed unity. For this reason there appear the need to calculate the process in nonperturbative methods. Presently, only a few nonperturbative results are available. Becker et. al. [5] have used a finite difference method to solve the Dirac equation for $U^{92+} - U^{91+}$ collisions at 1 GeV/u on a discretised grid. Recently Baltz [6] obtained an exact solution to the Dirac equation for relativistic heavy ion collisions in the ultrarelativistic limit. Another sucessive nonperturbative method giving the cross section of these inelastic processes is the Glauber approximation [8, 9]. In [9], we sucessfully applied this method to relativistic heavy ion-light atom collisions. In this work, using the eikonal approximation, we obtain a genaral expression for the cross section of an inelastic collision of a fast and relativistic highly charged ion with a comlex atom. In the ultrarelativistic limit this expression coincides with the known exact expression in [6]. As an application of this formula, we have calculated the probablitiy and cross section of K-vacancy production in the collision of relativistic $U^{92+} - U^{91+}$.

A general expression for inelastic amplitude of transition from state $\mid \Phi_i >$ to state $\mid \Phi_f >$ for the collision of relativistic highly charged ion with a light (nonrelativistic) atom in the Glauber approximation can be obtained as in [9] (by following [7]):

$$f_{if}(\mathbf{q}) = \frac{ik_i}{2\pi} \int e^{-iqb} < \Phi_f \mid \left[1 - \exp\{-\frac{i}{v} \int U dx\} \right] \mid \Phi_i > d^2 b, \tag{1}$$

where $\mathbf{q} = \mathbf{k}_f$ - $\mathbf{k}_i$ is the momentum transfer. The scattering potential $U = U(x, \mathbf{b}; \{\mathbf{r}_a\})$ is a function of ion's coordinate as well as coordinates of atomic electrons $\mathbf{R}$ = (x,$\mathbf{b}$), which we denote as { $\mathbf{r}_a$ }, a = 1,2,... N, where N is the number of electrons.

To generlize this eikonal approximation for the case of relativistic ion heavy (relativistic) atom collision, one should account for the following: a) behaviour of atomic electrons as described by the Dirac equation; b) in the Glauber approximation, $U(x, \mathbf{b}; \{\mathbf{r'}_a\})$ is the static Coulomb potential induced by the atomic nucleus and the electrons which are in fixed (and simultaneous from the projectile viewpoint) points $r_a = (x'_a, y'_a, z'_a)$. Then

$$\frac{1}{v} \int_{-\infty}^{+\infty} U dx = \sum_{a=1}^{N} \chi_a(\mathbf{b}, \mathbf{s'}_a), \;\; \chi_a(\mathbf{b}, \mathbf{s'}_a) = \frac{2Z}{v} \ln \frac{\mid \mathbf{b} - \mathbf{s'}_a \mid}{b},$$

where axis for x is along $\mathbf{k}_i$ and $\mathbf{s}'_a = (y'_a, z'_a)$ specifies the perpendicular vector. To be definiteness, let us consider electrons in the instantaneous positions $\mathbf{r}'_a$ at the moment $t' = 0$ in the rest frame of the ion and corresponding wave function is $\psi'(\mathbf{r'}_a, t')$. Then instead of Eqn[1] we have

$$f_{if}(\mathbf{q}) = \frac{ik_i}{2\pi} \int \psi'^{+}_f(\mathbf{r'}_a, t' = 0) \left[1 - \exp\{-\frac{i}{v} \int U(x, \mathbf{b}; \{\mathbf{r'}_a\}) dx\} \right] \times$$
$$\times \psi'_i(\mathbf{r'}_a, t' = 0) \exp(-i\mathbf{qb}) d^2 b \prod_{a=1}^{N} d^3 r'_a.$$

In the rest frame of the atom we have for $t' = 0$:

$$x_a = \gamma x'_a, \mathbf{s}_a = \mathbf{s'}_a, \;\; t = -x_a \frac{v}{c^2};$$
$$\psi(\mathbf{r_a} t) = \psi(\mathbf{r_a}) \exp(-iEt) = \psi(\mathbf{r_a}) \exp(iEx_a \frac{v}{c^2}) = S_a^{-1} \psi'$$
$$d^3 r_a = dx_a dy_a dz_a = \gamma d^3 r'_a = \gamma dx'_a dy'_a dz'_a,$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$, and S_a is the Lorentz matrix which transofrms the wave function the ion's to the atoms rest frame. It acts only to bispinor indices related to atomic electrons with number a (corresponding Dirac matrices are α_a) written as

$$S_a^2 = \gamma(1 - \mathbf{v}\alpha_a/c).$$

Thus in the rest frame of the atom the transition amplitude of from state $\mid \psi_i >$ to state $\mid \psi_f >$ can be written in the Glauber approximation as:

$$f_{if}(\mathbf{q}) = \frac{ik_i}{2\pi}\int < \psi_f \mid [1 - \exp\{-i\sum_{a=1}^{N}\chi_a(\mathbf{b},\mathbf{s}_a)\}]\times$$
$$\times S^2 \exp\,[i\sum_a \frac{x_a}{c}(E_f - E_i)] \mid \psi_i > \exp\,(-i\mathbf{qb})d^2b, \tag{2}$$

where $S^2 = \prod_{a=1}^{N} S_a^2$. This is the final expression for transition amplitude which should be used in the collision of a relativistic ion with a heavy (relativistic) atom. If we are not interesting in scattering angles one can perform integrating over this angles. So, for small angles one has

$$d\Omega \approx d^2q/(k_ik_f) \approx d^2q/k^2. \tag{3}$$

Integrating (2) over d^2b' and d^2q by using (3) and the integral representation of δ-function, we obtain the cross section of transition from state $\mid \psi_i >$ to state $\mid \psi_f >$ in the relativistic ion-heavy atom collision:

$$\sigma = \int d^2b \, |< \psi_f \mid [1 - \exp\{-i\sum_{a=1}^{N}\chi_a(\mathbf{b},\mathbf{s}_a)\}]\times$$
$$\times S^2 \exp\,[i\sum_a \frac{x_a}{c}(E_f - E_i)] \mid \psi_i >|^2\,. \tag{4}$$

In this expression integrand is interpreted as the probability of transition from state $\mid \psi_i >$ to state $\mid \psi_f >$ during the collision with the impact parameter $\mathbf{b}$. One should note that in this form this probability coincides with the exact one obtained in [6] for the ultrarelativistic case. For long-range potentials the integral in (4) diverges for large impact parameters. However, as is known, such divergence is not considerable [3,10], since for large impact parameters the Born approximation is less applicable, a way from the region in which the applicabilty of the Born and eikonal approximation overlap one another. This provides a correct matching of cross sections over the impact parameter. Consider this match in the case of K-vacancy production in the collision of relativistic ions with heavy atoms when transition of the atomic K-shell-electron from the state $\mid i >$ to the continuum state $\mid \mathbf{k} >$ with momentum $\mathbf{k}$ occures. Denote b_0 as the upper integration limit over the impact parameter b. For $b \gg s$ and orthogonal $\mid \mathbf{k} >$ and $\mid i >$ the inelastic formfactor

$$< f \mid \exp\,\{-i\frac{2Z}{v}\,\ln\frac{\mid \mathbf{b}-\mathbf{s} \mid}{b}\} \mid i > \approx < f \mid \exp\{i\mathbf{qr}\} \mid i > \tag{5}$$

tends (for small $\mathbf{q}$) to $i\mathbf{q} < f \mid r \mid i >$, where $\mathbf{q} = 2Z\mathbf{b}/(vb^2)$. Therefore integral in (4) over d^2b depends logarithmically on b_0 and for this reason the contribution of the region $b < b_0$ to the cross section can be written as

$$\sigma(b < b_0) = 8\pi \frac{Z^2}{v^2} \lambda_i \ln \frac{2\alpha_i}{q_0}, q_0 = 2Z/(vb_0), \tag{6}$$

where

$$\lambda_i = \int d^3k \mid < \mathbf{k} \mid \mathbf{r} \mid i > \mid^2 /\mathbf{3}, \tag{7}$$

$$\alpha_i = lim_{q_0 \to 0} \frac{q_0}{2} \exp \left\{ \frac{1}{\lambda_i} \int_{q_0}^{\infty} \frac{dq}{q^3} \int d^3k \mid < \mathbf{k} \mid \exp(-i\mathbf{q}\mathbf{r}) \mid i > \mid^2 \right\}. \tag{8}$$

In the region $b > b_0$ the field of the ion is a weak perturbation and one can use in this region the so-called Bethe asymptotics:

$$\sigma_i(b > b_0) = 8\pi \frac{Z^2}{v^2} \lambda_i \left(\ln \frac{2v}{\eta b_0 \omega_i \sqrt{1 - \beta^2}} - \frac{\beta^2}{2} \right), \tag{9}$$

where $\eta = e^B = 1.781$, B $= 0.5772$ is the Euler constant, and ω_i is the average ionization frequency:

$$\ln\omega_i = \frac{\int d^3k \mid < \mathbf{k} \mid \mathbf{r} \mid i > \mid^2 \ln \Omega_{ki}}{\int d^3k \mid < \mathbf{k} \mid \mathbf{r} \mid i > \mid^2}. \tag{10}$$

Here, $\Omega_{ki} = \epsilon_k - \epsilon_i$ is the transition frequency. Summing (6) and (9) we obtain the total K-shell ionzation cross section:

$$\sigma_i = 8\pi \frac{Z^2}{v^2} \lambda_i \left(\ln \frac{2\alpha_i v^2}{\eta Z \omega_i \sqrt{1 - \beta^2}} - \frac{\beta^2}{2} \right). \tag{11}$$

Quantities $\lambda_i \alpha_i$ and ω_i are calculated numerically using formulas (7), (8) and (10). Note that the dependence on the cut-off parameter b_0 disappears after mathching.

Thus we have derived general formulas for cross section which are applicable in the case of collisions of atoms with ions of arbitirary charge. Now we apply the above results for the calculation of probability and cross section of K-vacancy production in the $U^{92+} - U^{91+}$ collision. In the case of large ion's charges the inelastic cross sections are large enough in comparison with the atomic sizes. Therefore one can use, for the calculation of transition amplitude, large impact parameter approximation (5). For wave functions $\mid \mathbf{k} >$ and $\mid i >$ we use Darwin and Zommerfeld-Mau wave functions [4]. Ionization probability as a function of impact parameter for $U^{92+} - U^{91+}$ collision is given in Figure 1. As is seen from this figure our approach gives for small impact parameters ionization probability which is less than unity. In Figure 2 the dependence of K-vacancy production cross section on the relativistic factor γ is given for the $U^{92+} - U^{91+}$ collision.

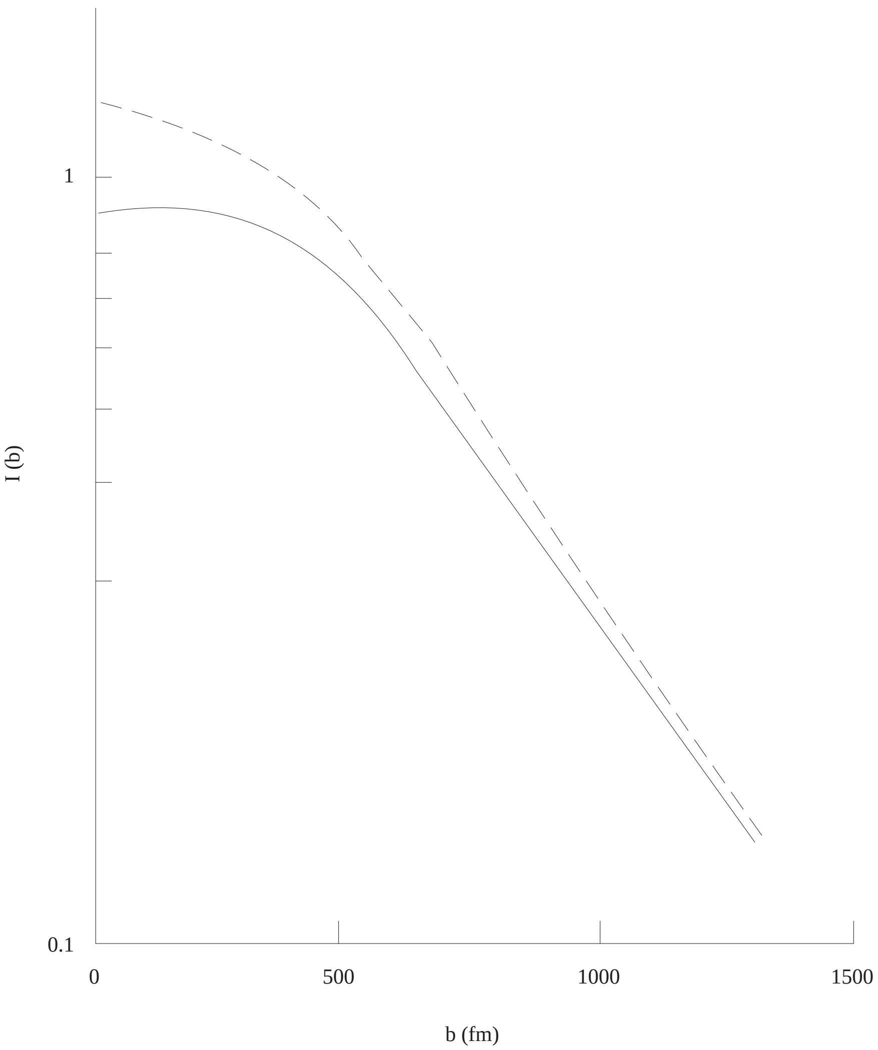

Figure 1. K-vacancy production probability as a function of the impact parameter b for $U^{92+}cU^{91+}$ collision. Solid line is our result and the dashed line is the result of [10].

Thus the above calculations of cross section and ionization probability, using the eikonal approximation, provides the ability to avoid some difficulties which appear in the case of the application of perturbation theory to the collision of relativistic highly charged ion with heavy atom and leads to a result, in the ultrarelativistic limit, coinciding with the exact one.

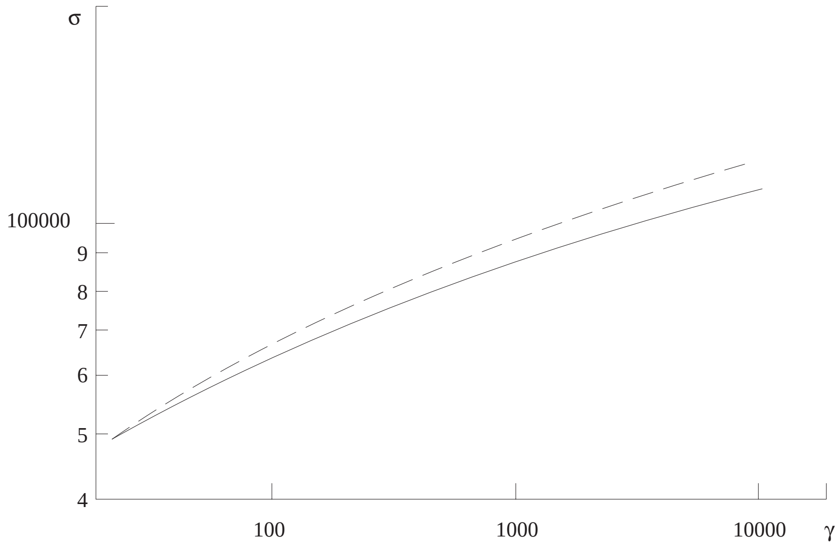

Figure 2. K-vacancy production cross section as a function of relativistic factor γ for $U^{92}+U^{91}$ collision. Solid line is the result of our calculations; the dashed one is the result from [9]. The cross section is given in barns.

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