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Field and Polarization Effects Arising in Neutrino-Lepton Processes in a Magnetic Field

V. A. GUSSEINOV
Department of General and Theoretical Physics,
Nakhichevan State University,
Nakhichevan, the Azerbaijan Republic

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Abstract

We discuss the effects arising in the neutrino-lepton processes ($\nu_\mu + e^- \rightarrow \nu_e + \mu^-$ and $\overline{\nu}_e + e^- \rightarrow \overline{\nu}_\mu + \mu^-$) in a constant uniform magnetic field in the framework of the Weinberg-Salam-Glashow model. It is shown that in the case of a superstrong magnetic field and for the muon producing on the ground Landau level the leading term of the cross section asymptotics of the ν_μ-process does not depend on the field intensity and it effectively describes the one-dimensional free process. In the ν_μ-process the muons with the spins oriented opposite to the direction of the field are mostly produced. The electrons with the spins oriented along the magnetic field mostly take part in the $\overline{\nu}_e$-process. Only left-handed electrons take part in the ν_μ-process. $\overline{\nu}_e$-process is forbidden for right-handed muons. In the strong field case only left-handed electrons and muons take part in the processes. It corresponds to the massless case. All the considered effects are significant in that case when we have dealings with strong fields and low-energy neutrions. These effects can be used for detecting of low-energy neutrinos in the experiment.

Introduction

In this paper we discuss quantum effects arising in some electroweak interaction processes such as

$$\nu_\mu + e^- \rightarrow \nu_e + \mu^-, \tag{1}$$

$$\overline{\nu}_e + e^- \rightarrow \overline{\nu}_\mu + \mu^-. \tag{2}$$

The process (1) is called an inverse muon decay which is a purely leptonic process. Its cross section can be calculated exactly, because it does not contain any contributions connected with strong interactions. The cross section of the inverse muon decay in a magnetic field in the framework of the Weinberg-Salam-Glashow model (four fermions approximation) has been given in papers [1,2]:

$$\sigma = \frac{G_F^2}{\pi^2\omega}\sum_f \int \frac{d^3k'}{\omega'}\delta(\varepsilon + q_0 - \varepsilon')\left[(kj^*)(k'j) + (k'j^*)(kj) + \frac{1}{2}q^2(j^*j) - i\varepsilon^{\alpha\beta\lambda\sigma}j_\alpha j_\beta k'_\lambda k_\sigma\right]. \tag{3}$$

Here, j^α are the components of the current:

$$j^\alpha = NF^0, j^1 = N(F_1\cos\lambda + iF_2\sin\lambda),$$
$$j^2 = N(F_1\sin\lambda - iF_2\cos\lambda), j^3 = NF^3,$$

where the functions $F_\alpha(\alpha = 0,1,2,3)$ are expressed in terms of Laguerre functions $I_{nn'}(x)$ as

$$\begin{bmatrix} F_0 \\ F_3 \end{bmatrix} = B'BI_{n,n'} \pm A'AI_{n-1,n'-1}$$
$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = A'BI_{n,n'-1} \pm B'AI_{n-1,n'}.$$

The Laguerre function is

$$I_{nn'}(x) = (n'!/n!)^{1/2}e^{-x/2}x^{(n-n')/2}L_{n'}^{n-n'}(x),$$

where $L_{n'}^{n-n'}(x)$ is the Laguerre polynomial and the argument x is

$$x = (q_x^2 + q_y^2)/2h.$$

The multiplier N is

$$N = \frac{\pi^2}{L^2}\delta(k_y + p_y - p'_y)\delta(k_z + p_z - p'_z)\times$$
$$\exp[i(n-n')(\lambda + \pi/2) - i\alpha],$$
$$tg\lambda = q_y/q_x, \ \ \alpha = q_x(p_y + p'_y)/2h.$$

In expression (3), $k = (\omega,\mathbf{k})$ and $k' = (\omega',\mathbf{k}')$ are the 4-momenta of the initial and final neutrinos. Accordingly, the 4-momentum transfer is $q = k - k' = (q_0,\mathbf{q})$. n and n' are the principial quantum numbers of the electron and the muon, respectively. $p_y(p'_y)$ and $p_z(p'_z)$ are y and z components of the momentum of the electron (the muon). $\varepsilon(\varepsilon')$ is the energy of the electron (the muon).

When the initial neutrino ν_μ is massless and the charged leptons (e^-, μ^-) are in the ground state the cross section of the process is

$$\sigma = \frac{G_F^2}{\pi^2} \int\limits_{m_\mu}^{\varepsilon_m} d\varepsilon' \frac{E\varepsilon' - m_\mu^2}{(\varepsilon'^2 - m_\mu^2)^{1/2}} \int\limits_0^{2\pi} d\varphi e^{-x}, \tag{4}$$

where m_μ is the mass of a muon,

$$x = \frac{1}{2eH}(\omega^2 + k_\perp'^2 - 2\omega k_\perp' \cos\varphi),$$

φ is the azimuthal angle of the vector of momentum $\mathbf{k}'$ of the final neutrino; $\mathrm{E} = \mathrm{m}_e + \omega$ is the total energy (the electron is in the ground state); $k_\perp'$ is the value of the transverse momentum; ε_m is the maximal energy of the muon:

$$k_\perp' = [2E(\varepsilon_m - \varepsilon')]^{1/2}, \varepsilon_m = (E^2 + m_\mu^2)/2E. \tag{5}$$

The gauge of the potential A^μ of the constant uniform magnetic field $\mathbf{H} \| Oz$ has been chosen as follows:

$$A^\mu = (0, 0, xH, 0). \tag{6}$$

The momentum of the initial neutrino is directed along the axis $\mathrm{O}x : k^\mu = \omega(1, 1, 0, 0)$.

2. Effects in one-dimension

In case of a comparatively strong field $H >> E^2/e$ the cross section of the process is

$$\sigma = \frac{1}{\pi} G_F^2 \left[E^2 - m_\mu^2 - 2m_\mu^2 \ln \frac{E}{m_\mu} + E^2 \cdot O(E^2/eH) \right]. \tag{7}$$

In this case the leading term of the cross section asymptotics does not depend on the field intensity H. The influence of the strong magnetic field are showed in one-dimension of the motion of the charged leptons: their transverse degrees of fredoom (with reference to $\mathbf{H}$) are not excited.

We can obtain the asymptotics (7) if we integrate the known square of the modulus of the amplitude of the free scattering $\nu_\mu e^- \to \nu_e \mu^-$ on the modified phase volume when $\mathbf{H} = 0$ and the electron is at rest. It should be taken into account that the spins off all the leptons are fixed and the muon only moves along the axis $\mathrm{Oz} \| \mathbf{H}$. The laws of conservation of energy and z-component of momentum are only carried out. It should be emphasized that in this case the motion of the neutrino is three-dimensional. The analogous effect of one-dimension in the muon decay has been noted in [3].

We have to note that in strong magnetic fields non-linear effects arise. But here we are not able to discuss them.

3. Effect of transverse polarization

We can obtain detailed information about the structure of weak interactions from the analysis of the polarization effects [4]. Here we consider the cross sections of the processes of high-energy muon production in scattering of low-energy neutrinos (antineutrinos) by ultrarelativistic electrons ($\nu_\mu + e^- \to \nu_e + \mu^-; \overline{\nu}_e + e^- \to \overline{\nu}_\mu + \mu^-$) in a constant uniform magnetic field.

Let the initial neutrino fly along the magnetic field $\mathbf{H} \parallel$ Oz: $k^\mu = \omega(1,0,0,1)$. Let us suppose that the electron has a large transverse - momentum ($p_\perp = (2eHn)^{1/2} >> m_\mu$) in the magnetic field $H << H_\mu = \frac{m_\mu^2}{e}$, i.e. the motion of the electron is semiclassical: $n >> 1$. n is the principal quantum number. For the muon it is also $n' >> 1$. We have to note that we use the pseudoeuclidean metric with signature $(+---)$ and the system of units where $\hbar = c = 1$.

Let us consider the asymptotic behaviours of the cross sections when the field parameter

$$\chi = \frac{H}{H_\mu}\frac{p_\perp}{m_\mu} = \frac{e}{m_\mu^3}\left[-(F_{\alpha\beta}p^\beta)^2\right]^{1/2} >> 1 \tag{8}$$

and the kinematical parameter

$$\kappa = \frac{2\omega\varepsilon}{m_\mu^2} = \frac{2kp}{m_\mu^2} > \kappa_0 = 1 - \left(\frac{m_e}{m_\mu}\right)^2, \tag{9}$$

where $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is a tensor of an external field, ω is the energy of the initial neutrino (antineutrino), and ε is the energy of the electron. In this case the cross sections, the transverse polarizations are

$$\sigma_t(\zeta',\zeta) = \frac{G_F^2}{\pi}m_\mu^2\left\{\frac{(x-1)^2}{2x} + \frac{\chi^2}{x^4}\left[1 - \frac{2}{3}x(x-1)\right] - \zeta'\chi\frac{x-1}{x^2}\right\}, \tag{10}$$

$$\begin{aligned}\sigma_t(\zeta',\zeta) = \frac{G_F^2}{\pi}m_\mu^2 &\left\{\frac{(x-1)^2}{12x^3}[x(1+2x) + \delta^2(2+x)]\right.\\ &+ \left.\frac{\chi^2}{3x^6}[2x(4-3x) + \delta^2(10 - 12x + 3x^2)] + \zeta\chi\delta\frac{x-1}{x^4}\right\}.\end{aligned} \tag{11}$$

Here, $x = u_0 + 1 = \kappa - \kappa_0 + 1 = (k+p)^2/m_\mu^2$ is the standardized Mandelstam variable.

Quantum number ζ=+1(-1) corresponds to the orientation of the spin of the electron along (against) the magnetic field direction. ζ'=+1(-1) corresponds to the muon. The spin number ζ is a normalized eigenvalue of the invariant spin operator M or Λ . M is the spin operator for the transverse polarization and Λ is the spin operator for the longitudinal polarization. In a magnetic field the spin operator of the transverse polarization is

$$M = \frac{eH}{2m_e}\mu_3, \tag{12}$$

where μ_3 is the spin operator

$$\mu_3 = \Sigma_3 + i\gamma^0\gamma^5(\Sigma \times \mathbf{P})_z = \frac{\varepsilon}{m_e}\gamma^0(\Sigma_3 + \gamma^5 p_z) \tag{13}$$

and it satisfies the following equation

$$\mu_3\psi = \frac{\varepsilon_\perp}{m_e}\zeta\psi. \tag{14}$$

Here $\mathbf{P} = -i\mathbf{\nabla} + e\mathbf{A}$ is the kinetic-momentum operator. The information about the spin operator for the longitudinal polarization will be given in the next section. For detail about the meanings of above mentioned other quantities see [5,6]. In this case we consider that the longitudinal momentum of an electron $p_z = 0$ and the energy of the initial neutrino ω is

$$eH/p_\perp << \omega << m_\mu. \tag{15}$$

When $x - 1 << \chi << 1$ and the field is comparatively weak, the cross section of the process considerably differs from the cross section of the free process which vanish at $x = 1$. In the process $\nu_\mu + e^- \to \nu_e + \mu^-$ muons with spins oriented opposite to the direction of the field $\mathbf{H}$ ($\zeta' = -1$) are mostly produced. The analogous effect of radiation polarization (the Sokolov-Ternov effect) in the synchrotron radiation of electrons and positrons has been noted in [5]. The influence of the field is more pronounced for the transversely polarized particles ($\sim \chi$) than the longitudinally polarized particles ($\sim \chi^2$).

From the formula (11) it is derived that for transversely polarized electrons the cross section of the process $\overline{\nu}_e + e^- \to \overline{\nu}_\mu + \mu^-$is larger when their spins are directed along the magnetic field (ζ=+1). It means that electrons with spins oriented along the magnetic field mostly take part in the process (2).

However, it is necessary to note that under the above mentioned condition it is experimentally impossible to detect transverse polarization.

4. Effect of longitudinal polarization

Let us consider the process (1) and (2). The process (2) is cross-symmetrical to the process (1) . When χ >>1 and $\kappa > \kappa_0$ the cross sections of the considered processes for the longitudinal polarizations are

$$\sigma_l(\zeta',\zeta) = \frac{G_F^2}{\pi} m_\mu^2 . 2\zeta_- \left\{ \zeta'_+ \left(\ln x - \frac{x-1}{x} \right) + \zeta'_-(x - 1 - \ln x) + \right.$$
$$\left. + \frac{2\chi^2}{3x^4}[\zeta'_+(3-2x) + 2\zeta'_- x(2-x)] \right\} \quad (16)$$

$$\sigma_l(\zeta',\zeta) = \frac{G_F^2}{\pi} m_\mu^2 . 2\zeta'_- \left\{ \frac{(x-1)^2}{6x^3}[\zeta_+\delta^2(2+x) + \zeta_- x(1+2x)] + \right.$$
$$\left. + \frac{2\chi^2}{3x^6}[\zeta_+\delta^2(10 - 12x + 3x^2) + 2\zeta_- x(4-3x)] \right\}, \quad (17)$$

accordingly. Here, $\delta = m_e/m_\mu, \zeta_\pm = (1 \pm \zeta)/2, \zeta'_\pm = (1 \pm \zeta')/2 \cdot \zeta = +1(-1)$ corresponds to the right-hand (left-hand) helicity.

The spin operator for the longitudinal polarization in a magnetic field is

$$\Lambda = \Sigma \cdot \mathbf{P}/m_e.$$

Here,

$$\Sigma \cdot \mathbf{P} = \gamma^5(m_e\gamma^0 - \varepsilon)$$

is the generalized helicity operator [5] which describes the longitudinal polarization and satisfies the following equation

$$(\Sigma \cdot \mathbf{P})\psi = \zeta p \psi,$$

where $p = (\varepsilon^2 - m_e^2)^{1/2}$.

From the formula (16) it is derived that only left-handed electrons take part in the process (1). The process (1) is forbidden for right-handed electrons.

The process (2) is forbidden for right-handed ($\zeta' = +1$) muons. Only left-handed ($\zeta' = -1$) muons are produced in the process (2).

5. Independence of the cross section from the mass and its consequences

The influence of an external field on the considered processes, which flow in the absence of an external field is determined with the parameter

$$\eta = \chi/|\kappa - \kappa_0|. \quad (18)$$

For the strong field case, i. e. when $\eta >> 1$ and $\chi >> 1$, the cross sections of the processes (1) and (2) contain the multiplier $m_\mu^2\chi^{2/3}$ which does not depend on a mass. For instance, the expressions of the cross sections of the process $\nu_\mu + e^- \to \nu_e + \mu^-$ are

$$\sigma_t(\zeta', \zeta) = \frac{\Gamma(2/3)}{9\pi} G_F^2 m_\mu^2 \left[(3\chi)^{2/3} - \frac{2\Gamma(1/3)}{\Gamma(2/3)} \zeta' (3\chi)^{1/3} \right], \tag{19}$$

$$\sigma_l(\zeta', \zeta) = \frac{4\Gamma(2/3)}{9\pi} G_F^2 m_\mu^2 (3\chi)^{2/3} \zeta'_- \zeta_-. \tag{20}$$

In the strong field case the processes are forbidden for the right-handed electrons and muons. Only left-handed electrons and muons take part in the processes. It corresponds to the massless case which has been noted above. In this case the muons and the electrons polarized against the magnetic field direction are mostly produced.

We used the 4- fermions approach of the Standard Model of Weinberg-Salam-Glashow. Now let us consider the condition of its applicability to weak interactions. In this case we can write the following condition

$$|q^2| << m_w^2 \tag{21}$$

for the processes (1) and (2) . Here, m_w is the mass of W-boson. The condition (21) means that the the square of the 4-momentum transfer must be relatively small. Using (9) and the fact that the initial neutrino flies along the axis Ox we can get the following condition

$$\kappa << (m_w/m_\mu)^2 \approx 5.88 \times 10^5. \tag{22}$$

All these considered effects are significant in that case when

$$\eta \geq 1. \tag{23}$$

To achieve this condition there are two possibilities:
1) to increase χ;
2) to decrease $|\kappa - \kappa_0|$ so that

$$|\kappa - \kappa_0| << 1. \tag{24}$$

The first condition depends on possibilities of increasing of the energy of the electron and the magnetic field intensity. But to achieve the second condition is a very difficult question at present. The second condition requires the fine tuning of the parameter κ so that the condition (24) is satisfied.

If we put $H = 10^8$ Gs (the impulse magnetic fields, the effective fields of monocrystals [7]) and $\varepsilon = 86$ GeV, which was the maximal energy achieved in e^-e^+ - collider LEP 2 in CERN in 1996 [8], then we obtain the estimate $\chi \approx 4 \times 10^{-8}$. The condition $\kappa \to \kappa_0$ (i.e.$|\kappa - \kappa_0|$ <<1) corresponds to the "strong influence" region. If we consider that $\kappa_0 = 1 - (m_e/m_\mu)^2 \approx 1$ we can find the relation between the energy of the electron and the neutrino:

$$\kappa = \frac{2\omega\varepsilon}{m_\mu^2} \approx 1. \tag{25}$$

The "strong influence" begins from $\eta \approx 1$. But it corresponds to the energy $\omega \approx$60 keV of the muon neutrino. This is smaller than the energies of the reactor and solar neutrinos which can be registered.

With $\varepsilon = 1$ GeV and the other parameters used above, we obtain $\chi \approx 4.7 \times 10^{-10}$. In this case we have $\omega \approx$5.6 MeV which can be registered experimentally.

The effects of the external field become more significant for the strong field case and for low-energy neutrinos. All these allow experimental detection of low-energy neutrinos.

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