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The Resonance Bremsstrahlung of a Fast Charged Particle in a Medium

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Abstract

The bremsstrahlung of a fast charged particle in the medium with dielectric permittivity ϵ at velocities $v \geq \frac{c}{n}(\text{Re}\epsilon = n^2)$ was considered. Bremsstrahlung radiation has singularity at $\beta = \frac{1}{n\cos\theta}$ ($\beta = \frac{v}{c}, \theta$ is an angle of the bremsstrahlung.) and is interpreted as resonance bremsstrahlung with the width characterized by $Im\epsilon = \epsilon_2$; and smaller ϵ_2 is, the higher the peak of the resonance. The angle distribution of the bremsstrahlung is determined by $\cos\theta = \frac{1}{n\beta}$ and θ coincides with the angle of Cherenkov radiation. At $\beta = \frac{1}{n}$ the resonance bremsstrahlung goes in the forward direction. The resonance bremsstrahlung depends on frequency $\omega(\epsilon \equiv \epsilon(\omega))$.

1. Introduction

In our previous work [1] we discussed the problem of a charged relativistic particle passing through a medium. It was shown that at the velocity of the particle $v \cong c'(c' = \frac{c}{n})$ the Cherenkov radiation differs from zero. This radiation is rather weak and at some velocity v, was $v < c'$, this radiation disappears. A simple mechanism proposed to avoid singularity in the electrodynamics when $v = c'$. The medium is supposed to behave as a collective with the characteristics of this medium described by dielectric permittivity ϵ. It is clear that ϵ is a function of frequency $\epsilon = \epsilon(w)$ and, at high frequencies when $\epsilon \rightarrow 1$, the medium loses the collective property and the charged particle begins to interact individually with both nuclei and electrons of the medium.

In this work we study the resonance bremsstrahlung of a charged particle passing through the medium at velocities v, $v \cong c'$ and higher. It is clear that the charged particle interacts with the medium as a collective. The property of this collective is determined by a dielectric permittivity $\epsilon = \epsilon_1 + i\epsilon_2$ ($\epsilon_1 = n^2, \epsilon = \epsilon(\omega)$).

It is interesting to remark that, after the discovery of bremsstrahlung radiation, which differs from the luminescent radiation, S. I. Vavilov [2] came to a conclusion that, more probably the new radiation is bremsstrahlung of Compton electrons knocked out by gamma–rays from atoms in the liquid.

At first, we discuss bremsstrahlung of a fast charged particle when it takes part in individual electromagnetic interactions, then discuss the resonance bremsstrahlung of a fast charged particle in the medium.

2. The Bremsstrahlung of a Fast Charged Particle

The full bremsstrahlung energy ε of a fast charged particle is determined by the following equation [3]:

$$\varepsilon = \int_{-\infty}^{\infty} J dt, \tag{1}$$

$$J = \frac{2e^2}{3m^2c^3} \frac{[\vec{E} + \frac{1}{c}[\vec{v}\vec{H}]]^2 - \frac{1}{c^2}(\vec{E}\vec{v})^2}{1 - \frac{v^2}{c^2}}, \tag{2}$$

where $\vec{E}, \vec{H}$ are the intensity of external electric and magnetic fields; and m, e are mass and charge of the particle.

If the external field is divided into two components, parallel and transversal to velocity $\vec{v}$, then for component $J_{\|}$ we obtain

$$J_{\|} = \frac{2e^2}{3m^2c^3} \vec{E}_{\|}^2, \tag{3}$$

and for the transversal component $J_{\perp}$, we obtain

$$J_{\perp} = \frac{2e^2}{3m^2c^3} \frac{\vec{E}_{\perp}^2}{1 - \frac{v^2}{c^2}}. \tag{4}$$

The angle distribution of bremsstrahlung energy is determined by the following equation:

$$I = \int \frac{dI}{d\Omega} d\Omega, \qquad d\Omega = \sin\theta d\theta d\varphi \tag{5}$$

$$\frac{dI}{d\Omega} = \frac{c^2}{4\pi c^3} \left(\frac{2(\vec{n}\vec{w})(\vec{v}\vec{w})}{c(1 - \frac{\vec{v}\vec{n}}{c})^5} + \frac{\vec{w}^2}{(1 - \frac{\vec{v}\vec{n}}{c})^4} - \frac{(1 - \frac{v^2}{c^2})(\vec{n}\vec{w})^2}{(1 - \frac{\vec{n}\vec{v}}{c})^6} \right), \tag{6}$$

where $\vec{n}$ is the direction of the bremsstrahlung.
If the velocity $\vec{v}$ and acceleration $\vec{w}$ are parallel, then for $I_{\|}$ we obtain the following equation:

$$\frac{dI_{\|}}{d\Omega} = \frac{e^2}{4\pi c^3} \frac{w^2 \sin^2\theta}{(1 - \frac{v}{c}\cos\theta)^5}. \tag{7}$$

if $\vec{v}$ and $\vec{w}$ are transversal, then for $\frac{dI_{\perp}}{d\Omega}$ we obtain the following equation:

$$\frac{dI_{\perp}}{d\Omega} = \frac{e^2}{4\pi c^3} \left(\frac{1}{(1 - \frac{v}{c}\cos\theta)^4} - \frac{(1 - \frac{v^2}{c^2})\sin^2\theta\cos^2\varphi}{(1 - \frac{v}{c}\cos\theta)^6} \right). \tag{8}$$

In the angle distribution of the bremsstrahlung of a fast charged particle, as we can see from (1)-(8) there is a peak in forward distribution and the width of this peak is determined by the following equation:

$$\theta \sim \sqrt{1 - \frac{v^2}{c^2}}. \tag{9}$$

Now we discuss the bremsstrahlung of a fast charged particle in a medium.

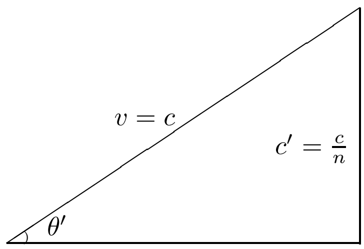
3. The Resonance Bremsstrahlung of a Fast Particle in a Medium

We study bremsstrahlung of a fast charged particle in a medium with velocities $v \geq \frac{c}{n}$. We are interested in the bremsstrahlung in the region of frequencies ω, where $n(\omega) > 1$, i.e. optics and X-ray regions. Actually, it is the region of frequencies where the charged particle interacts with the medium collectively but not individually. Then the following substitution should be made in equations (1)-(8):

$$c \to c' = \frac{c}{n}, \tag{10}$$

where $n^2 = \mathrm{Re}\epsilon$($\epsilon$ is dielectric permittivity). This substitution is implicit in equations (1)-(8) (also, see [2]).

Moreover, since the velocity of the light in the medium is $c' = \frac{c}{n}$, the field around the particle will spread out from the medium with velocity c', and the field will remain behind the relativistic particle. The triangle of the velocities of this particle in the medium has the following form:



$$\sin\theta' = \frac{c}{vn} = \frac{1}{\beta n},$$
$$\theta' = \frac{\pi}{2} - \theta.$$

So, we can see that the field around the relativistic particle ($v > c'$) has the form of a thin surface film having a rotation cone with the slope angle θ' to the direction of the moving particle, and θ' is determined by the value $\sin\theta' = \frac{1}{\beta n}$. Then, the larger the refraction coefficient n the smaller the scope angle of the surface of the rotation cone (see [1]). It is clear that the bremsstrahlung must go mainly in the transversal direction to the surface of the rotation cone.

So, equations (1)-(8) transform as:

$$\varepsilon = \int_{-\infty}^{\infty} J dt = \int_{-\infty}^{\infty} (J_{\|} + J_{\perp}) dt, \tag{11}$$

where $J_{\|}$ is:

$$J_{\|} = \frac{2e^2}{3m^2c^3} \vec{E}_{\|}^2, \tag{12}$$

and $J_{\perp}$ is:

$$J_{\perp} = \frac{2e^2}{3m^2c^3} \frac{\vec{E}_{\perp}^2}{1 - \epsilon \frac{v^2}{c^2}}. \tag{13}$$

From equation (13) we see that singularity in the medium at the charged particle velocity $v^2 = \frac{c^2}{\epsilon}$ appears there. Since ϵ is a function of ω ($\epsilon = \epsilon(\omega)$), then the singularity takes place at different velocities:

$$v^2(\omega) = \frac{c^2}{\epsilon(\omega)}. \tag{14}$$

At frequencies $\omega(\omega \to \infty)$, with $\epsilon(\omega) \to 1$, the charged particle begins to interact individually with the medium, not as a collective, and then the bremsstrahlung becomes individual.

If ϵ in (10) is a complex value $\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega)$, then (13) transforms into the following equation:

$$J_{\perp} = \frac{2e^2}{3m^2c^3} \frac{\vec{E}_{\perp}^2 [(1 - \epsilon_1 \frac{v^2}{c^2}) + i\epsilon_2 \frac{v^2}{c^2}]}{(1 - \epsilon_1 \frac{v^2}{c^2})^2 + (\epsilon_2 \frac{v^2}{c^2})^2}. \tag{15}$$

This equation has a typical resonance form (see [4]) with the point of the resonance at $\epsilon \frac{v^2}{c^2} = 1$ and with width Γ

$$\frac{\Gamma}{2} = \epsilon_2 \frac{v^2}{c^2}. \tag{16}$$

So, the charged particle bremsstrahlung in the medium depends on its velocity as:

$$J_{\perp} \sim \frac{1}{1 - \epsilon \frac{v^2}{c^2}}, \tag{17}$$

and at $\epsilon_1 \frac{v^2}{c^2} \sim 1$, it has resonance. The equation for bremsstrahlung when ϵ is a complex value, is determined by equations (12) and (13) where $J \to \mid J \mid$.

Now we consider the angle distribution of the charged particle bremsstrahlung in the medium.

If we take into account that $c \to c' = \frac{c}{\epsilon}$ in the medium, then equations (7), (8) are transformed in the following equations:

$$\frac{dI_{\parallel}}{d\Omega} = \frac{e^2}{4\pi c^3}\frac{w^2 \sin^2\theta}{(1-\sqrt{\epsilon}\frac{v}{c}\cos\theta)^5}, \tag{18}$$

$$\frac{dI_{\perp}}{d\Omega} = \frac{e^2 w^2}{4\pi c^3}\left(\frac{1}{(1-\sqrt{\epsilon}\frac{v}{c}\cos\theta)^4} - \frac{(1-\epsilon\frac{v^2}{c^2})\sin^2\theta\cos^2\varphi}{(1-\sqrt{\epsilon}\frac{v}{c}\cos\theta)^6}\right). \tag{19}$$

From equations (18), (19) we see, that their denominators are equal to zero (i.e. (18), (19) have singularity) at:

$$\frac{\sqrt{\epsilon}v}{c}\cos\theta = 1 \qquad or \qquad \cos\theta = \frac{c}{\sqrt{\epsilon}v}, \tag{20}$$

i.e. in the region of frequencies ω, where $\epsilon(\omega) > 1$, there is bremsstrahlung in direction θ, determined by equation (20). This bremsstrahlung has typical resonance of form:

$$\frac{dI_{\parallel}}{d\Omega} = \frac{e^2}{4\pi c^3}w^2\sin^2\theta\frac{[(1-\sqrt{\epsilon_1}\beta\cos\theta)+i\sqrt{\epsilon_2}\beta\cos\theta]^5}{[(1-\epsilon_1\beta^2\cos^2\theta)^2+\epsilon_2\beta^2\cos^2\theta]^5}, \tag{18'}$$

$$\frac{dI_{\perp}}{d\Omega} = \frac{e^2 w^2}{4\pi c^3}\left(\frac{[(1-\sqrt{\epsilon_1}\beta\cos\theta)+i\sqrt{\epsilon_2}\beta\cos\theta]^4}{[(1-\epsilon_1\beta^2\cos^2\theta)^2+\epsilon_2\beta^2\cos^2\theta]^4}\right) -$$
$$\frac{e^2 w^2}{4\pi c^3}\left((1-\epsilon\frac{v^2}{c^2})\sin^2\theta\cos^2\varphi\frac{[(1-\sqrt{\epsilon_1}\beta\cos\theta)+i\sqrt{\epsilon_2}\beta\cos\theta]^6}{[(1-\epsilon_1\beta^2\cos^2\theta)^2+\epsilon_2\beta^2\cos^2\theta]^6}\right). \tag{19'}$$

The equations for the resonance bremsstrahlung of the charged particle in the medium can be obtained from (18') and (19') by using the following substitution: $\frac{dI_{\parallel}}{d\Omega} \to \mid \frac{dI_{\parallel}}{d\Omega} \mid$, $\frac{dI_{\perp}}{d\Omega} \to \mid \frac{dI_{\perp}}{d\Omega} \mid$.

The integrated on φ values of $\mid \frac{dI_{\parallel}}{d\Omega} \mid$, $\mid \frac{dI_{\perp}}{d\Omega} \mid$ are:

$$\frac{d\varepsilon_{\parallel}}{dt} = \int \mid \frac{dI_{\parallel}}{d\Omega} \mid d\varphi, \frac{d\varepsilon_{\parallel}}{dx} = \frac{1}{v}\frac{d\varepsilon_{\parallel}}{dt}, \tag{21}$$

$$\frac{d\varepsilon_{\perp}}{dt} = \int \mid \frac{dI_{\perp}}{d\Omega} \mid d\varphi, \frac{d\varepsilon_{\perp}}{dx} = \frac{1}{v}\frac{d\varepsilon_{\perp}}{dt}, \tag{22}$$

where $x = vt$.

At the point of the resonance from (20), (21) we get the following equations ($\cos\theta = \frac{1}{\sqrt{\epsilon_1}\beta}, w = \omega\beta$):

$$\frac{d\varepsilon_{\parallel}}{dx} = \frac{e^2}{2c^2}\omega^2\beta\sin^2\theta(\frac{\epsilon_1(\omega)}{\epsilon_2(\omega)})^{\frac{5}{2}}, \tag{23}$$

$$\frac{d\varepsilon_{\perp}}{dx} = \frac{e^2}{2c^2}\omega^2\beta(\frac{\epsilon_1(\omega)}{\epsilon_2(\omega)})^2 + \frac{e^2}{8\pi c^2}\omega^2\beta^3\sin^2\theta\epsilon_2(\frac{\epsilon_1(\omega)}{\epsilon_2(\omega)})^3. \tag{24}$$

So, dependent on the velocity of the charged particle, the resonance bremsstrahlung in the medium goes in different directions θ (see (20)), and at the velocity $v = \frac{c}{\sqrt{\epsilon_1}}$, the resonance bremsstrahlung goes in the forward direction. When $v < \frac{c}{\sqrt{\epsilon_1}}$, the resonance bremsstrahlung disappears. It is clear that since ϵ depends on ω, then the velocity values v also depend on ω. From (18) and (19) we also see that the direction of the resonance bremsstrahlung coincides with the direction of the Cherenkov radiation [5].

The equations (23), (24) define the losses of the charged particle in the medium per unit length without taking into account the absorption part (we suppose that $\epsilon_2 << 1$), which must be proportional to $\exp(-\epsilon_2)$.

Equations (23), (24) are the same equations which are necessary to be compared with the following standard equation for the Cherenkov radiation:

$$\frac{d\varepsilon}{dx} = \frac{e^2}{2c^2}\omega^2(1 - \frac{1}{\epsilon_1 \beta^2}). \tag{25}$$

The expression for ϵ is given in [3], [6].

4. Conclusion

The bremsstrahlung of the fast charged particle in the medium with dielectric permittivity ϵ at velocities $v \geq \frac{c}{n}(\mathrm{Re}\epsilon = n^2)$ was considered. The bremsstrahlung has singularity at $\beta = \frac{1}{n\cos\theta}$ $(\beta = \frac{v}{c}, \theta$ is an angle of the bremsstrahlung). And the bremsstrahlung was interpreted as resonance bremsstrahlung with the width characterized by $\mathrm{Im}\epsilon = \epsilon_2$, and smaller ϵ_2 is, the higher the peak of the resonance. The angle distribution of the bremsstrahlung is determined by $\cos\theta = \frac{1}{n\beta}$ and this angle coincides with the angle of the Cherenkov radiation. At $\beta = \frac{1}{n}$ this resonance bremsstrahlung goes in the forward direction. The resonance bremsstrahlung depends on frequency $\omega(\epsilon \equiv \epsilon(\omega))$.

Evidently, an accurate experiment is needed to study the radiation of the fast charged particle in the medium at velocities $\beta \geq \frac{1}{n}$.

It is necessary to remark that in works [7] [8] was reported about the noticeable radiation in the forward direction in optical region, which cannot be the Cherenkov radiation, since the Cherenkov radiation in the forward direction must be equal to zero.

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References

[1] Kh.M. Beshtoev, JINR Communication E2-97-31, Dubna, 1997.

[2] S.I. Vavilov, Dokladi AC USSR, 1934, v.2, p.457.

[3] W. Panovsky, M. Phillips, Classic Electrocity and Magnetism, Addison-Wesley P.K.;
L.D. Landau, E.M. Lifschitz, The Field Theory, 1968, M., Nauka, v.2;
J. Jackson, Classical Electrodynamics, Wiley and Sons, 1962.

[4] J.M. Blatt, V.F. Weisskopf, The Theory of Nuclear Reactions, CINR T.R., 42.

[5] I.M. Frank and I.E. Tamm, Dokl. Akad. Nauk USSR, 1937, v.14, p.107;
I.E. Tamm, J. Phys. USSR, 1939, v.1, p.439;
I.M. Frank, Vavilov-Cherenkov Radiation, Theoretical Aspects, M., Nauka, 1988;
V.P. Zrelov, The Vavilov–Cherenkov Radiation, p.I, Atomizdat, M, 1968;
B.N. Bolotovski, Soviet Phys. Jorn. U.F.N., 1961, v.75, p.295.

[6] D.A. Tidman, Nuclear Phys., 1956, v.2, p.289.

[7] V.P. Zrelov, J. Ruzicka JINR Commun. P2-92-233, Dubna, 1992; Czech. J. Phys.B, 1992, v.39, p.368.

[8] A.A Tyapkin, JINR Rapid Commun. 1993, 3[60]-93, p.26.