Turk J Phy
25 (2001) , 271 – 282.

A Correlation of Thin Lens Approximation to Thick Lens Design by Using Coddington Factors in Lens Design and Manufacturing

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Received 01.11.1999

Abstract

The effect of Coddington factors on aberration functions has been analysed using thin lens approximation. Minimizing spherical aberrations of singlet lenses using Coddington factors in lens design depending on lens manufacturing is discussed. Notation of lens test plate pairs used in lens manufacturing is also presented in terms of Coddington shape factors.

Introduction

Aberrations are an inherent part of the optical system in designing spherical lenses. Minimization of aberrations is one of the most important concepts in lens design. Thin lens approximation can be applied to build a model for thick lens design. Minimization of aberrations can be realised by considering the real conditions in lens design as a part of computer integrated lens manufacturing [1]. When lens design is suitable for manufacturing conditions, it may have a wide range of usage; and when lens design fits manufacturing conditions as a result of the adaptation of already designed lenses in an optical system, the optical system complies with manufacturing conditions. Thus, lens design should be performed taking production conditions into consideration. Therefore, computations must be optimized in accordance with the constraints of available technology [2], [3]. The constraints may be defined as lens test plates, lens materials and their properties, and manufacturing conditions. Consider the case of an optical device containing a number of lenses designed with their aberrations minimized. If the computed radii and parameters of lenses do not fit the radii of the lens test plates and parameters of materials available, any subsequent modifications will necessitate recontrol of the optical system and may lead to inconvenient situations. Coddington factors which appear in aberration functions in thin lens approximation contribute to the improvement of lens design. When these contributions are kept in a specified order, the practical uses for lens design are obtained.

In this study, the application basis on the usage of Coddington factors in lens design is discussed. The effectiveness of Coddington factors is examined by using optical glass parameters with thin lens approximation. The dependence of spherical aberration on Coddington shape factor for thick lenses is discussed using exact ray tracing. Its variations are examined in the visible and infrared region. As a result of conducted studies and experiences obtained in lens production, spherical aberration variations of singlet lenses depending on Coddington shape factor are analyzed with suitable examples. In addition, a notation is introduced on the usage of lens test plate pairs used in lens manufacturing. The usage of the introduced notation for lens design can be adapted to the relevant manufacturing conditions.

This study is a typical route in this area and it satisfies lens manufacturing requirements. Similar options within different ways are available in optical design programs using test plate libraries. This present route, instead, is a correlation using Coddington factors in lens design and manufacturing. Furthermore, this correlation can even be applied in the same way to doublet lenses under more specific conditions.

2. Thin lens approximation

Aberration minimization is the most significant yet inseparable part of lens design. Corresponding wave aberrations to transverse ray aberrations are unfavourable conditions for lenses performance. Monochromatic aberrations in general, primary (fourth order) wave or third order ray aberrations can be minimized according to desired purposes. When a single lens is considered, commonly only spherical aberration computations are performed in lens design. The minimization of other aberrations is taken into account in the optical system. Principally, it is very useful to evaluate aberrations and their parameters for thin lenses in the first step. The primary wave aberration function for a thin lens which has positive focal length is given by [4-7] :

$$W(r,\theta;\eta) = C_s r^4 + C_c \eta r^3 \cos\theta + C_a \eta^2 r^2 \cos^2\theta + C_d \eta^2 r^2, \qquad (1)$$

where the exit pupil is at the lens; r and θ are the polar coordinates at the exit pupil; and η is the image height at the Gaussian image plane. The coefficients C_s, C_c, C_a, and C_d represent the coefficients of spherical aberration, coma, astigmatism, and field curvature respectively. These coefficients in Eq. (1) may be given as follows:

$$C_S = -[32n(n-1)f^3]^{-1}\left[\frac{n+2}{n-1}S^2 + (3n+2)P^2 + 4(n+1)SP + \frac{n^3}{n-1}\right]; \tag{2a}$$

$$C_C = [4\text{nif}^2]^{-1}\left[\frac{n+1}{n-1}S + (2n+1)P\right]; \tag{2b}$$

$$C_a = -[2i^2 f]^{-1}; \tag{2c}$$

$$C_d = -(n+1)[4ni^2 f]^{-1}, \tag{2d}$$

where n is the refractive index of the optical glass, f is the paraxial focal length, and i is the image distance. The coefficient of distortion or wave front tilt is non-existent taking into account the exit pupil at the lens [6]. In terms of the coefficient C_s and C_c given by Eqs. (2a) and (2b), S and P are called Coddington Shape Factor (CSF) and Coddington Position Factor (CPF), respectively. Coddington Shape Factor S determines the amount of curving as a function of the radii, R_1 and R_2, and is given by

$$S = \frac{R_2 + R_1}{R_2 - R_1}. \tag{3}$$

Each value of S describes the physical shape of the lens so that contributions of lens surfaces for refractions are determined in accordance with lenses having negative or positive focal lengths. Coddington position factor P is given by

$$P = \frac{i+o}{i-o}, \tag{4}$$

where i is the image distance and o is the object distance. Each value of P indicates the location of the usage of the lens in the optical system. As seen in Eqs. (2a) and (2b) the spherical aberration and coma of the lens depend on CSFs and CPFs. On the other hand, the coefficients of astigmatism and field curvature which are represented by Eqs. (2c) and (2d) do not depend on CSFs and CPFs for the exit pupil at the lens. Therefore, these coefficients were not considered in this study. Differentiation with respect to S of Eq. (2a) equals zero end thus gives the minimum spherical aberration corresponding to $S_{\min}$ and the relationship between $S_{\min}$ and Coddington position factor P.

The above formulation can be applied to optical glasses and other lens materials, as well as expanded and applied to thick lenses. In the visible range, the calculation of refractive indices and Abbe values of optical glasses is commonly performed at wavelength 587.5618 nm (the yellow helium line). In this case, the variations of lens characteristics for the various optical glasses were obtained at this wavelength. By using Eq.(2a) and $S_{\min}$ the variations of minimum spherical aberrations with CSF values are given in Fig. 1. As shown in Fig. 1, the parabolic curves of variations shift downward in terms of enhancement of the refractive index of optical glasses. It indicates that in thin lens approximation the best shape lenses of flint type glasses have lower spherical aberration than the best shape lenses of crown type glasses. However, zero spherical aberration for the glasses with the designations of 464658, 517642, 785258, and 952204 occurs at absolute CPF values of 4.85, 4.46, 3.31, and 2.92, respectively. Glass designation has six digit numbers to describe optical specifications: the first three digits indicate the refractive index, and the second three digits represent the Abbe value. In addition, variations of spherical aberrations of lenses having negative focal length form the symmetrical plane with respect to $S_{\min}$ axis of spherical aberration variations of lenses have positive focal length. For minimizing spherical aberration, it is useful to do the computation at infinite conjugate ratio, P = -1. In this case the variations of spherical aberrations with CSF values for various optical glasses are given in Fig. 2. As shown in this figure, every parabola has a value of ($S_{\min}$, $C_{s\,\min}$) at its vertex for parallel incident light, that the parabolas vary symmetrically, and the parabola shape and vertex strongly depend on CSF and refractive index values.

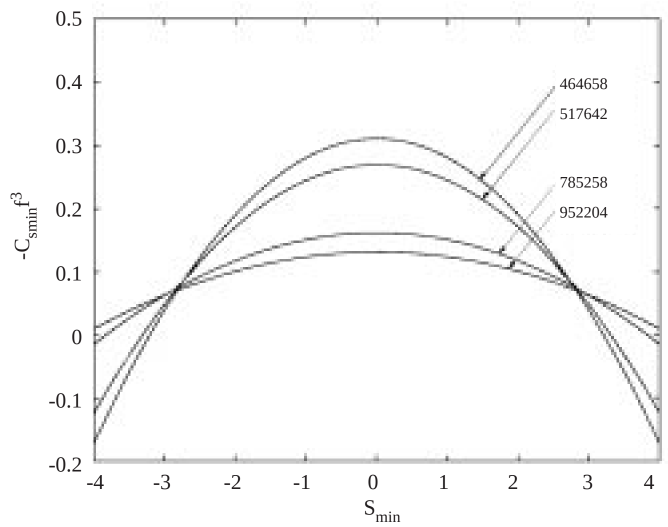

Figure 1. The variations of minimum spherical aberrations with Coddington shape factor $S_{\min}$ for crown and flint optical glasses.

In order to vanish coma as an aberration, the term in brackets at the right hand side in Eq.(2b), which equals zero, gives the spherical aberration with zero coma. Then, the relationship between the spherical aberration with zero coma S_{cz} and the Coddington position factor P is obtainned. For instance, for the parallel incident light variations of the minimum spherical aberration $S_{\min}$ and the spherical aberration with zero coma S_{cz} with optical glass refractive indices are given in Fig. 3. As a practical application, a lens which designed to obtain zero coma will have the minimum spherical aberration corresponding to $S_{\min} = S_{cz}$ [4-5]. Moreover, a lens having this condition can be used as the telescope objective to cover a rather small field of view [8]. On the other hand, the best solution may be that the lens possesses the most minimum spherical aberration and the lens is designed by considering the coma values corresponding to this case. Therefore, the residual coma values can be compensated within the optical system.

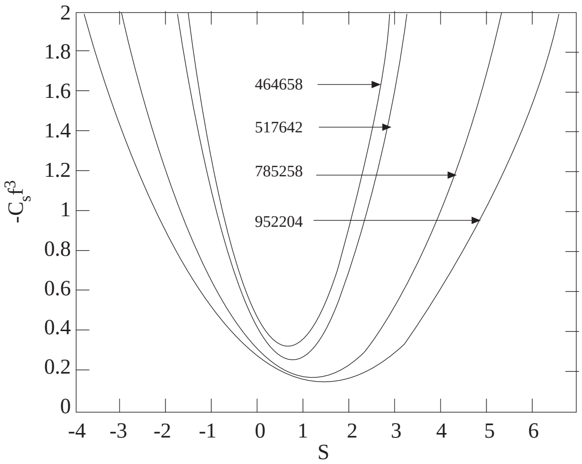

Figure 2. The variations of spherical aberrations with CSF values for various crown and flint optical glasses.

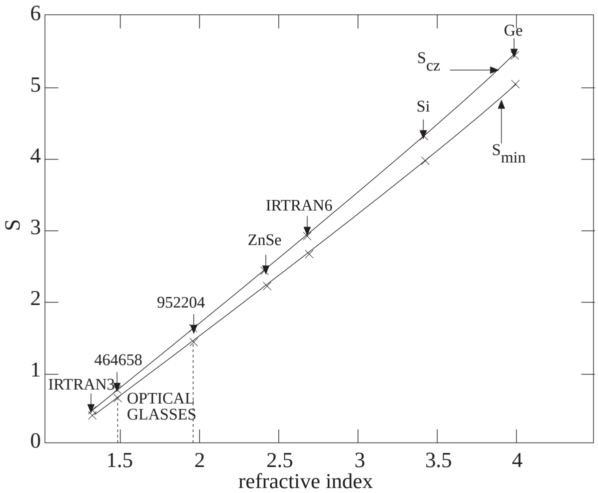

Figure 3. The variations of S_{cz} and $S_{\min}$ with refractive indices of optical glasses at the wavelength of 587.5618 nm and various infrared lens materials at the wavelength of 10.6 μm.

3. Thick lenses

3.1. Computations

The aberration values of singlet lenses have been computed by using Opticad, a software that employs ray tracing [9-11] and manufacturing optimization. A summary diagram of the software environment is given in Fig. 4.

3.2. Singlet lenses

As known, lens aberrations depend considerably on the uses of lenses, lens shapes, and lens materials. The variations of longitudinal spherical aberration (LSA) and transverse spherical aberration (TSA) with CSF values for positive and negative singlet lenses are given in Fig. 5. Positive lenses with glass designation 517642 have the same paraxial focal length, 75.00 ± 0.02 mm, and central thickness, 4.00 ± 0.01 mm, and operate at f / 3.00 (exit pupil at the lens) and an infinite conjugate ratio, $P = -1$. Negative lenses with glass designation 717295 have the same paraxial focal length, −100.00 ± 0.02 mm, central thickness of 2.00 ± 0.01 mm, operate at f / 4.00 (exit pupil at the lens) and an infinite conjugate ratio, $P = -1$. The variations of LSA with the CSF values for positive and negative singlet lenses made of various optical glasses are given in Fig. 6. As an example in this application, positive lenses with glass designations 464658, 517642, 785258, and 952204 have the same paraxial focal length, central thickness, and operate at the same relative aperture and conjugate ratio of the positive lenses in Fig. 5. Negative lenses with glass designations 517642, 673322, 762265, and 805254 have the same paraxial focal length, central thickness and operate at the same relative aperture and conjugate ratio of the negative lenses in Fig. 5. The aberrations have been also computed and observed for positive and negative singlet lenses at the same conditions in Fig. 5.

Considering Fig. 6, the evaluation of Coddington factors in the case of minimizing spherical aberration for positive singlet lenses may be outlined as follows:

The CSF values of positive lenses which have minimum spherical aberration are given in Fig. 7(a) and (b). Those lenses which have the variation of the shapes from asymmetric biconvex to meniscus form may be called the “best singlet shapes”. The asymmetric biconvex form has the CSF value of about 0.64 for 464658 glass. The meniscus form has the CSF value of about 1.44 for 952204 glass. These forms are quite acceptable for on-axis imaging. Furthermore, beyond the refractive index of optical glasses, the meniscus shape for the best shaped lenses ultimately becomes more curved in the IR region. The minimum LSAs with CSF values of positive best lens shape for common lens materials used in the IR region are given in Fig. 8 (a) and (b). In this case, the CPF has the value of -1.00. The positive singlet lenses become plano-convex for the CSF value of 1.00 and the CPF value of -1.00. As an example, best shape lenses with the optical glasses; SF (8,15), BaSF 13, KzFS 7A, BaF 50, LaF (20, N23), and LaK (9, N13, N14, 31) as indicated in the refractive index/Abbe value diagram [12] have this form. These optical glasses have the refractive index 1.69 approximately. As the dependency of the usage for

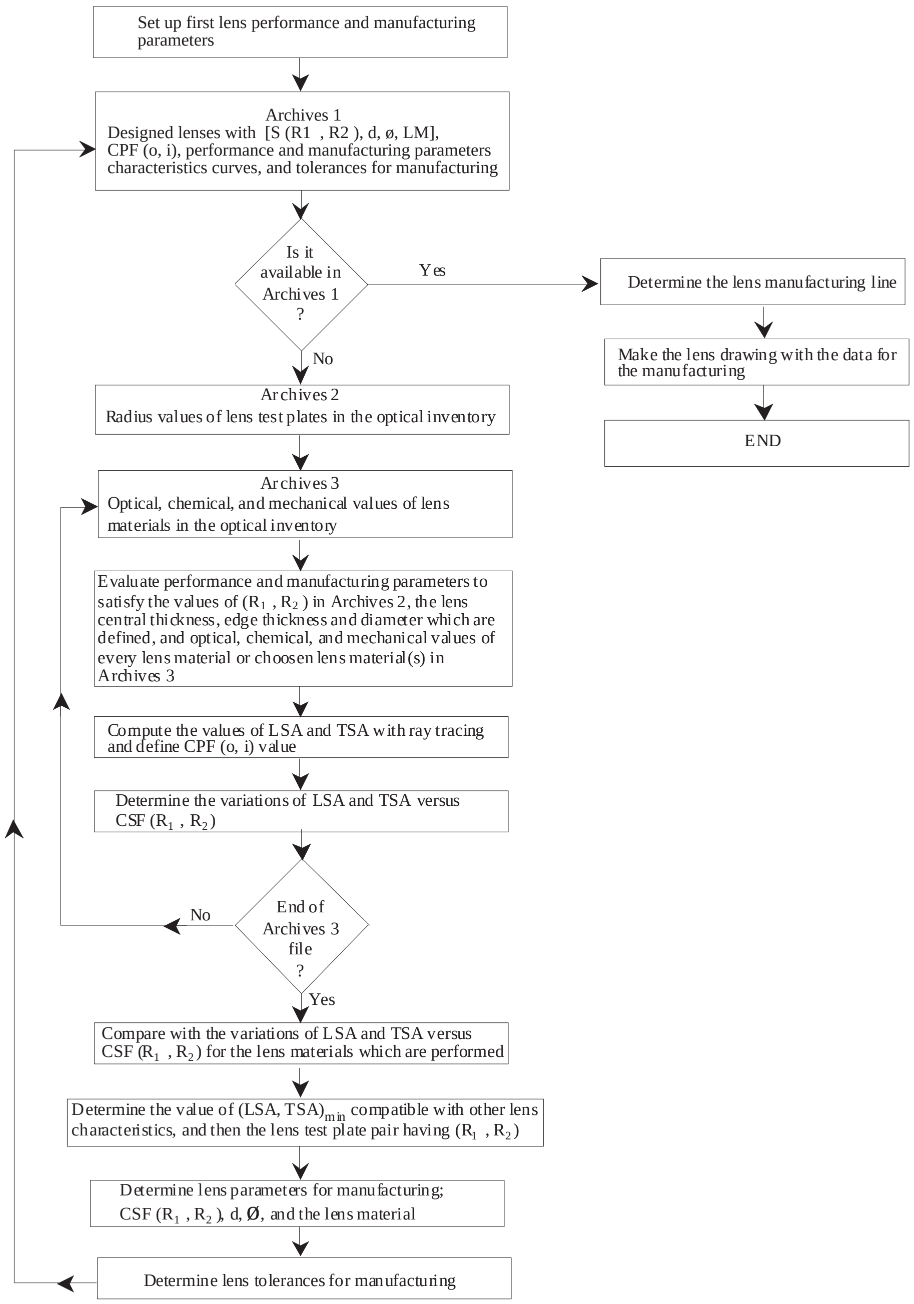

Figure 4. The summary diagram of singlet lens design and manufacturing.

the unit magnification, the symmetric biconvex lenses having the CSF value of 0.00 are the best shape. In this case, the CPF has the value of 0.00. On the other hand, the Coddington factors of negative singlet lenses may be evaluated in the similar case of positive lenses.

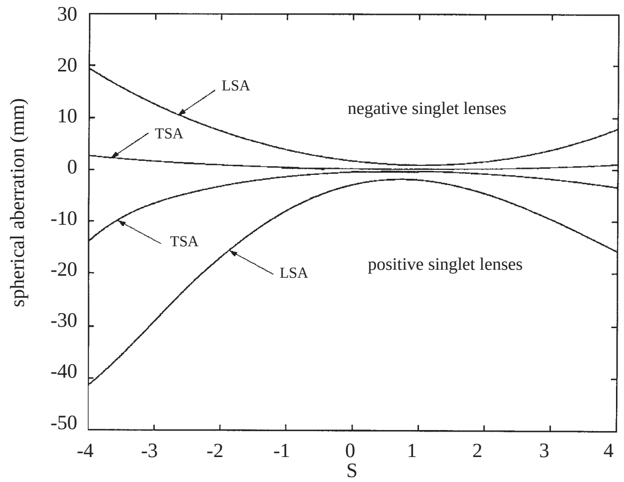

Figure 5. The variations of longitudinal and transverse spherical aberrations with CSF values for positive and negative singlet lenses. The positive and negative singlet lenses have, respectively, the same paraxial focal length, central thickness, and relative aperture (f/ϕ).

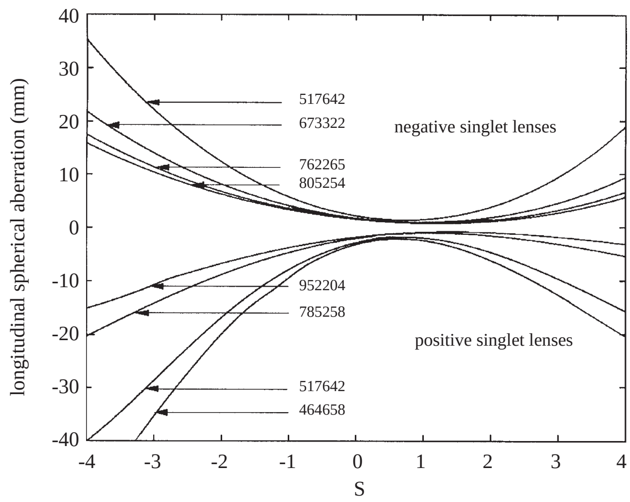

Figure 6. The variations of longitudinal spherical aberration with CSF values for positive and negative singlet lenses.

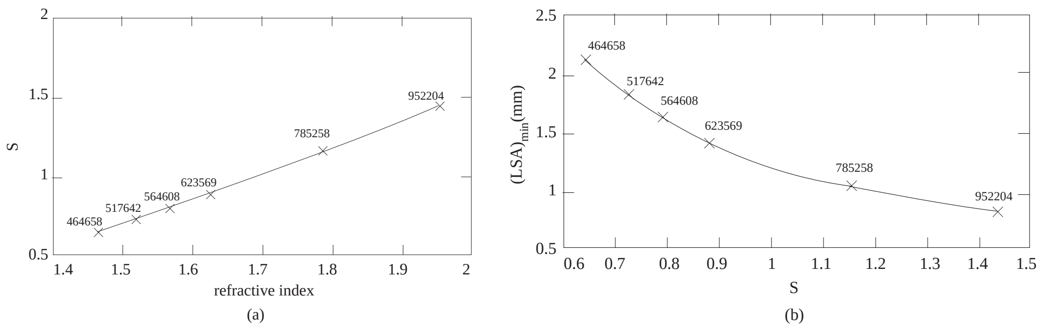

Figure 7. (a)The variation of CSF values for best lens shape with refractive indices . (b)The variation of $(LSA)_{min}$'s with CSF values for best lens shape. The optical glasses having designations of 464658, 517642, 564608, 623569, 785258, and 952204 are indicated on both curves.

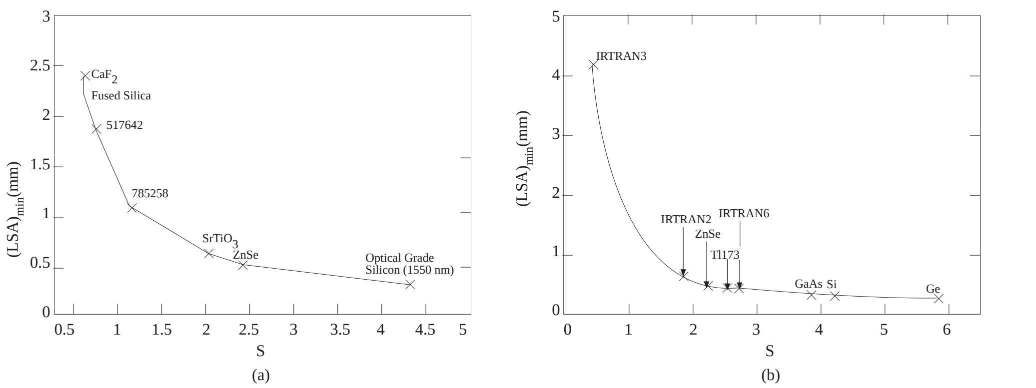

Figure 8. (a)The variations of minimum LSAs with CSF values of positive best lens shape for common IR lens materials which are used in the near IR region at the wavelength interval of 700-900 nm , and (b) the far IR region at the wavelength of 10.6 μm . All lenses have the same paraxial focal length, central thickness, clear aperture, and operate at the same conditions of positive lenses in Figure 6.

The variations of longitudinal spherical aberrations with CSF values for increasing central thickness of positive singlet lenses made of the flint glass having the designation of 785258 are given in Fig. 9. As shown in the variations, the longitudinal spherical aberration against the central thickness tends to vary nonlinearly. The disorder of the parabola symmetry is caused by the effect of CSF values and the existence of the central thickness on the spherical aberration. Lens central thickness values can be defined for the appropriate lens edge thickness values. Therefore, using these central thickness values, the curve of LSA versus S can be obtained for the purpose of optical glass optimization.

Every singlet lens which is designed on the basis of manufacturing has a lens test plate pair in connection with the spherical aberration values given in Fig. 5 and 6, for example. The lens test plate pair is a concept formed by a pair of the test plates of the two radius parameters. This can be used to manufacture a singlet lens within specified tolerances. The value of the lens test plate pair is the CSF value. This concept can be described in a group of main parameters on manufacturing basis as

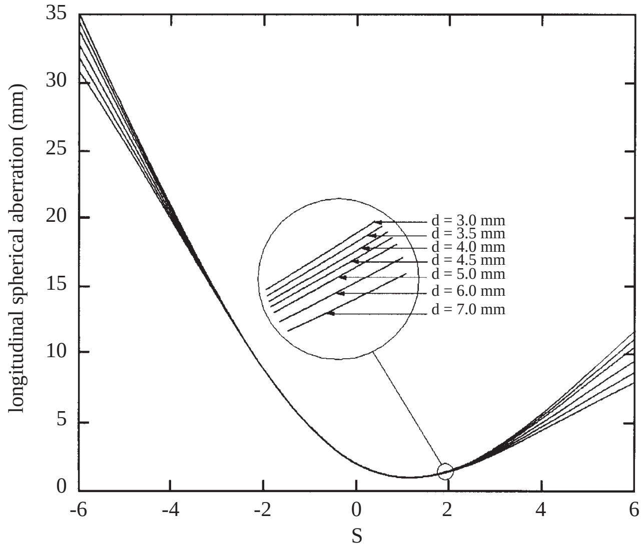

Figure 9. The variations of LSAs with CSF values for the various central thicknesses of positive singlet lenses made of the glass which have the designation of 785258. The paraxial focal lengths and clear aperture diameters of the lenses are the same at the values of 75.00 ± 0.02mm and 25.00 mm, respectively. The data was obtained at the same conditions of the positive lenses in Figure 6.

$$[S(R_1, R_2), d, \varnothing, LM], \tag{5}$$

where S is the CSF value which the lens test plate pair (R_1, R_2) has in the optical inventory. This value denotes the spherical aberration level of the lens in LSA and TSA variations.

The parameters; d, Ø, and LM are the central thickness, diameter, and material designation of the lens respectively. LM is also the designation which lens material has at the design wavelength. This notation provides lens test plates for the classification in terms of their CSF values and makes the singlet lens design easy for manufacturing. As an application of this notation, one of the positive lens samples made of the glass with the designation of 785258, particularly used in the laser diod optics is given in Fig. 10. If a lens is to be manufactured, the minimum spherical aberration depending upon the design parameters is preferred to accompany it. Therefore, each positive or negative singlet lens to be manufactured will preferably have a CSF value. If the cutting process is a part of the lens manufacture, the applications given in Figs. 5 and 6 can be carried out by using lens test plate pairs. The lens test plate controls whether the radius values of manufactured lenses after polishing processes are within tolerances or not. In this way, even if the discrete values of the radii are not similar in small establishments, they may approach one other in large establishments. In spite of this case, the applications explained above may be considered as a base concept. However, if lens manufacturing is applied by melting or hot-press process, the radii are also controlled by the lens test plates after the polishing process and the same design conditions can be valid for those processes as well.

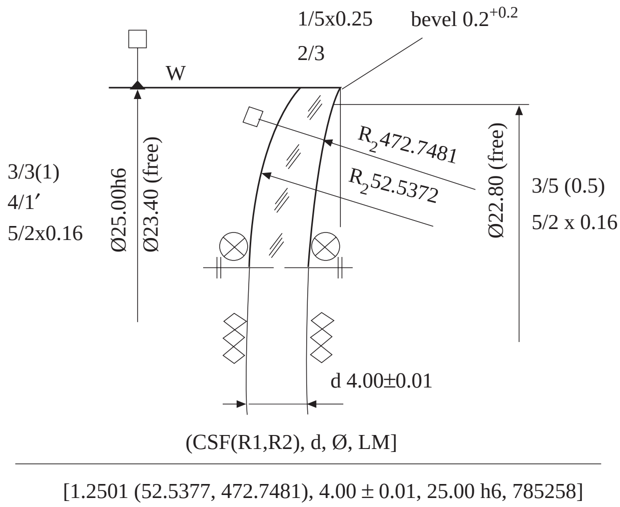

Figure 10. The drawing of one of the positive lens samples. The dimensions are in milimeters and the tolerances are in accordance with DIN 3140.

4. Conclusions

Thin lens approximation is a beneficial tool to investigate definitions, identifications and characterisations of aberrations for single lens design. For this purpose, the effects of Coddington factors on aberration functions and conditions have been illustrated. If the aperture stop is at the lens, Coddington factors are effective parameters in spherical aberration and coma functions. If coma has a minimum quantity which may be compensated in the optical system, the minimization of spherical aberrations can be carried out by using the main parameters of Coddington factors.

The exact solution for the determination of transverse ray aberrations is the ray tracing method. For this reason, as observed by ray tracing the dependence of spherical aberration on CSF values was discussed. The spherical aberrations depending on CSF values of both positive and negative singlet lens samples made of various optical glasses were computed and observed. The optical glass optimization by using CSF values have also been illustrated.

At the end of the applied studies mentioned here, the concept lens test plate pair for singlet lens design in the condition of limited lens test plates available was presented. It has been seen that this concept is effective in terms of applicability as the main parameters of the real conditions in lens design.

References

[1] R. Finkler, B.Y. Cohen, "Standards required for implementation of CIM in optics", *Proc. SPIE* **1780,** 168 - 177 (1992)

[2] T.G. Kuper and T.J. Harris,"Global optimization for lens design-an emerging technology", *Proc.SPIE* **1780**, 14-28 (1992).

[3] G.K. Hearn, "The evaluation of optimization algorithms", *Proc.SPIE* **CR41,** 54-70 (1992)

[4] V. N. Mahajan, *Aberration Theory Made Simple,* **3 - 27,** SPIE Optical Engineering Press, Bellingham, Washington (1991)

[5] V. N. Mahajan, *Optical Imaging and Aberrations,* **1,** 245 - 364, SPIE Optical Engineering Press, Bellingham, Washington (1998)

[6] M. C. Kidger, "Principles of Lens Design", *Proc. SPIE* **CR41,** 30 - 53 (1992)

[7] W. T. Welford, *Aberrations of Optical Systems,* 105 - 128, IOP Publishing Ltd., Bristol (1991).

[8] W. J. Smith, *Modern Optical Engineering,* 57 - 85, McGraw - Hill, 2nd ed. (1990)

[9] G. H. Spencer and M.V.R.K. Murty, "General Ray - Tracing Procedure", *J. Opt. Soc. Amer.* **52(6)**, 672 - 678 (1962)

[10] A. Hecht and A. Zajac, *Optics*, 167 - 175, Addison - Wesley, California (1974)

[11] A. Nussbaum and R.A. Phillips, *Contemporary Optics For Scientists and Engineers,* 36 - 95, Prentice-Hall, Englewood Cliffs, New Jersey (1984)

[12] Schott Optical Glass Catalog, Schott Glass Technologies, Duryea, PA (1992)