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Dependence of Neutron-Proton Entity Transfer on Superfluid Parameters

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Abstract

In this study the nuclear matrix elements of neutron-proton (n-p) entity transfer of even-even nuclei are investigated in the Tamm-Dancoff Approximation (TDA). These matrix elements are used for n-p entity stripping from even-even nuclei and n-p entity absorbing of even-even nuclei. It is shown that the matrix element of $0^+ \rightarrow 0^+$transition between $|Z, N\rangle$ and $|Z-2, N-2\rangle$ even-even nuclei via the 1^+ states of $|Z-1, N-1\rangle$ intermediate nuclei depends only on the superfluid parameters.

Key Words: Charge-exchange, particle-particle, Tamm-Dancoff, superfluid parameters

1. Introduction

In Random Phase Approximation (RPA), it was shown that charge-exchange spin-spin interaction generates a new branch of 1^+ states in odd-odd $|Z \pm 1, N \pm 1\rangle$ spherical nuclei in the particle-particle channel [1]. If a $|Z, N\rangle$ even-even nucleus absorbs a n-p entity or a deuteron, a $|Z+1, N+1\rangle$ nucleus is excited; or if a n-p entity or a deuteron is stripped from a $|Z, N\rangle$ nucleus a $|Z-1, N-1\rangle$ nucleus is produced. By the n-p transfer of $|Z, N\rangle$ even-even nucleus particle-particle type 1^+ states of neighbouring odd-odd $|Z \pm 1, N \pm 1\rangle$ nuclei are highly excited. Although particle-particle type 1^+ states are usually mixed with the other collective particle-hole type 1^+ states in the actual nuclear spectrum, the matrix elements of n-p entity transfer can be evaluated using the wave functions of the mixed 1^+ states [2].

There has been much recent interest in n-p entity transfer reactions [3,4]. The importance of investigating such reactions is that they can be used as an alternative method to the (n,p) and (p,n) reactions for calculating GT β strength, since the strength of GT β-transitions is well below the sum-rule limit. It is with this in mind that the (d, ^{2}He) reaction was investigated via E_d=260 MeV deuterons in light nuclei and it was found that the cross-section was proportional to the GT β strength [3].

In connection to n-p entity transfer reactions, it is interesting to investigate the $0^+ \rightarrow 0^+$ transition between $|Z, N\rangle$ and $|Z-2, N-2\rangle$ even-even nuclei via n-p entity stripping from $|Z, N\rangle$ nucleus, then the disintegration of excited 1^+ states of $|Z-1, N-1\rangle$ nucleus to $|Z-2, N-2\rangle$ nucleus by emission of n-p (Figure 1). In this paper, special attention will be made to the $0^+ \rightarrow 0^+$ transition and the dependence of the transition matrix elements on the parameters of the nucleus superfluid model.

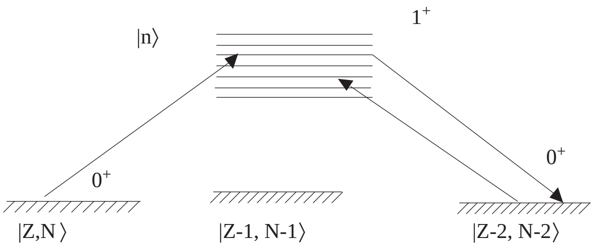

Figure 1. n-p entity stripping from$|Z,N\rangle$ nucleus and disintegration of 1^+ states by the n-p entity emission of $|Z-1,N-1\rangle$nucleus to the 0^+ states of the $|Z-2,N-2\rangle$ nucleus. $|n\rangle$represents the nth 1^+ state.

2. Description of Particle-Particle 1^+ States and $0^+ \rightarrow 1^+$ Transitions

The Hamiltonian model that represents particle-particle 1^+ excitation in spherical odd-odd nuclei in the boson representation can be written as

$$H = H_{sqp} + 2\chi_{pp}\sum_{\mu} P_{\mu}^{B+}P_{\mu}^{B}. \tag{1}$$

Here,

$$H_{sqp} = \sum_{jm}\varepsilon_{j_n}\alpha^{+}_{j_n m_n}\alpha_{j_n m_n} + \sum_{jm}\varepsilon_{j_p}\alpha^{+}_{j_p m_p}\alpha_{j_p m_p} \tag{2}$$

$$P_{\mu}^{B} = \sum_{\nu}\left\{d_{\nu}C_{\nu}(\mu) + (-1)^{\mu}\bar{d}_{\nu}C_{\nu}^{+}(-\mu)\right\}; P_{\mu}^{B+} = (P_{\mu}^{B})^{+}, \tag{3}$$

where H$_{sqp}$, describes the quasiparticle excitation with energy $\varepsilon_s = \sqrt{\Delta^2 + (E_s-\lambda)^2}$, Δ is the energy gap, λ is the chemical potential and E$_s$ is single particle energy of nucleons. χ_{pp} is the particle-particle interaction constant, $\alpha^{+}(\alpha)$ are the quasiparticle creation (annihilation) operators and C$_{\nu}^{+}$(C$_{\nu}$) are the bosonic operators representing n-p quasiparticle pairs with spin and parity 1^+ . For the sake of brevity, ν represents the quantum numbers of unlike two quasiparticle pairs. d$_{\nu}$ and $\bar{d}_{\nu}$ are defined as

$$d_{\nu} = \frac{1}{\sqrt{3}}u_{j_n}u_{j_p}\langle j_n\|\vec{\sigma}\|j_p\rangle; \bar{d}_{\nu} = \frac{1}{\sqrt{3}}v_{j_n}v_{j_p}\langle j_n\|\vec{\sigma}\|j_p\rangle, \tag{4}$$

where u$_j$ and v$_j$ are the occupation parameters of pairing theory and $\langle j_n\|\vec{\sigma}\|j_p\rangle$ are the one particle reduced matrix elements of operator $\vec{\sigma}$ [5].

In Tamm-Dancoff Approximation (TDA), the ω_n energies of 1^+ states are evaluated as the roots of the following equation for the Hamiltonian given in Eq. (1) [1]:

$$D(\omega_n) = \left\{\sum_{\nu}\frac{d_{\nu}^2}{\varepsilon_{\nu}-\omega_n} + \frac{1}{2\chi_{pp}}\right\}\left\{\sum_{\nu}\frac{\bar{d}_{\nu}^2}{\varepsilon_{\nu}-\omega_n} + \frac{1}{2\chi_{pp}}\right\} - \left\{\sum_{\nu}d_{\nu}\bar{d}_{\nu}\frac{1}{\varepsilon_{\nu}-\omega_n}\right\}^2 = 0. \tag{5}$$

In this approximation, Eq. 5 can be expressed as

$$D(\lambda) = D_1(\lambda)D_2(\lambda) - [2\chi_{pp}D_3(\lambda)]^2 = 0 \tag{6}$$

where D1 (λ), D$_2$ (λ), D$_3$ (λ) are written in the following forms:

$$D_1(\lambda) = 1 + 2\chi_{pp}\sum_{\nu}(\frac{d_{\nu}^2}{\varepsilon_{\nu}-\lambda}) \tag{7}$$

$$D_2(\lambda) = 1 + 2\chi_{pp}\sum_{\nu}(\frac{\bar{d}_\nu^2}{\varepsilon_\nu - \lambda}) \tag{8}$$

$$D_3(\lambda) = 2\chi_{pp}\sum_{\nu} d_\nu \bar{d}_\nu(\frac{1}{\varepsilon_\nu - \lambda}). \tag{9}$$

Here, $\varepsilon_{_\nu} = \varepsilon_n + \varepsilon_p$ are the energies of n-p quasiparticle states and $\omega_n = \lambda$.

The matrix elements of $0^+ \to 1^+$ transitions that correspond the n-p transfer of even-even nuclei are [1]:

$$M^B_{P_\mu} \equiv M^{0^+\to n}_{P_\mu} =< n\big|P^B_\mu\big|\psi_0 >= -\frac{L_n}{2\chi_{pp}\sqrt{Y_n}} \quad (n-p\ stripping) \tag{10}$$

$$M^{B+}_{P_\mu} \equiv M^{0^+\to n}_{P^+_\mu} =< n\big|P^{B+}_\mu\big|\psi_0 >= \frac{1}{2\chi_{pp}\sqrt{Y_n}}, \quad (n-p\ absorbing) \tag{11}$$

where L_n and Y_n are defined as

$$Y_n = \sum_{\nu}\left\{\frac{(\bar{d}_\nu - L_n}{\varepsilon_n - \omega_n}\right\}^2 \tag{12}$$

$$L_n = \frac{2\chi_{pp}D_3(\omega_n)}{D_1(\omega_n)} = \frac{D_2(\omega_n)}{2\chi_{pp}D_3(\omega_n)}. \tag{13}$$

Here ω_n are the zeros of the D(λ) functions. If

$$D'(\lambda) = \frac{\partial D(\lambda)}{\partial \lambda}, \tag{14}$$

then we can write Eq. (12) as

$$Y(\lambda) = \frac{L_n(\lambda)}{(2\chi_{pp})^2 D_3(\lambda)} D'(\lambda). \tag{15}$$

Upon the n-p stripping from $|Z,N\rangle$ nucleus, 1^+ states of $|Z-1,N+1\rangle$ are excited along with changes to corresponding matrix elements expressed with Eq. (10). Excited 1^+ states can disintegrate to 0^+ states of $|Z-2,N-2\rangle$ by n-p entity emission; the corresponding matrix element can be written approximately equal to the matrix element given by Eq. (11), which corresponds to n-p entity absorbing of $|Z-2,N-2\rangle$ nucleus (Figure 1). Nuclear matrix element of $0^+ \to 0^+$ between $|Z,N\rangle$ and $|Z-2,N-2\rangle$ nuclei are expressed as

$$M = \sum_n M^{0^+\to n}_{P^+_\mu} M^{0^+\to n}_{P_\mu} \tag{16}$$

by the terms of matrix elements given by Eqs. (10) and (11).

3. Sum Rule and Theory of Residues

In quantum mechanics, nuclear matrix elements expressing the transition of a system from one state to the another one obey special sum rules [6]. These rules come from the commutation relations between the operators that express the transitions. The sum rule for the matrix elements $M^{B+}_\mu and M^B_\mu$ can be written as

$$\langle\psi_0\big|\big[P^B_\mu, P^{B+}_\mu\big]\big|\psi_0\rangle = \sum_n\left\{\Big|M^{B+}_{P_\mu}\Big|^2 - \Big|M^B_{P_\mu}\Big|^2\right\}. \tag{17}$$

Here, $|\psi_0\rangle$ represents the ground state of even-even nucleus and $P^{B+}_\mu(P^B_\mu)$ are the operators generating 1^+ states in spherical odd-odd nuclei and is given by Eq. (3). The left side of Eq. (17) dose not depend on the model or the χ_{pp} interaction parameter, but the right side of Equation (17) does depend on the model. One can check the accuracy of RPA or TDA solutions using this sum rule equation.

The commutation relation on the left side of Eq. (17) can be written using Eq. (3) as

$$\left[P_\mu, P_\mu^+\right] = \sum_\nu (d_\nu^2 - \bar{d}_\nu^2). \tag{18}$$

The right side of the Eq. (17) can be expressed by the help of Eqs. (10), (11) and (5) as

$$\sum_n \left\{ \left[M_{P_\mu}^{B+}\right]^2 - \left[M_{P_\mu}^{B}\right]^2 \right\} = \sum_n \left[\frac{D_3(\lambda) L_n}{D'(\lambda)} - \frac{D_3(\lambda)}{D'(\lambda) L_n} \right]. \tag{19}$$

Now the basic theorem of the theory of residues [7] allows us to write this expression in the form of contour integrals:

$$\sum_n \left[\frac{D_3(\lambda) L_n(\lambda)}{D'(\lambda)} - \frac{D_3(\lambda)}{D'(\lambda) L_n(\lambda)} \right] = \frac{1}{2\pi i} \int\limits_{Z_n} \frac{D_3(\lambda)\left[L_n^2(\lambda) - 1\right]}{D(\lambda) L_n(\lambda)} d\lambda. \tag{20}$$

The contour Z_n is shown in Figure 2 and first-order singularities of the integrand at $\lambda = \omega_n$ which are the zeros of corresponding function $D(\lambda)$.

The integral given by (20) is very laborious to evaluate. However analysis show that outside Z_n the integrand has singularity at $\lambda = \infty$ (Figure 2) and the corresponding residue can be evaluated simply:

$$\underset{\lambda=\infty}{Res} \left[\frac{D_3(\lambda) L_n}{D(\lambda)} - \frac{D_3(\lambda)}{D(\lambda) L_n} \right] = \sum_\nu (d_\nu^2 - \bar{d}_\nu^2) \tag{21}$$

and

$$\sum_n \left\{ \left[M_{P_\mu}^{B+}\right]^2 - \left[M_{P_\mu}^{B}\right]^2 \right\} = \sum_\nu (d_\nu^2 - \bar{d}_\nu^2). \tag{22}$$

Thus one can use the theory of residues to check the accuracy of Eq. (17). Using a similar procedure, we can obtain from Eq. (16) the following relation corresponding to the $0^+ \to 0^+$ transition:

$$M = \sum_\nu d_\nu \bar{d}_\nu. \tag{23}$$

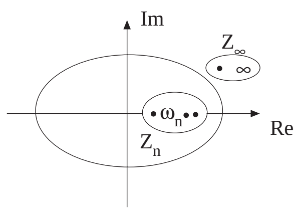

Figure 2. Contours of integration in the complex plane for Eq. (20).

4. $0^+ \to 0^+$ Transition and Conclusion

The matrix element of the $0^+ \to 0^+$transition between $|Z, N\rangle$ and $|Z-2, N-2\rangle$ nuclei can be expressed by the help of Eqs. (4) and (23) as

$$M = \frac{1}{3} \sum_{np} (u_{j_p} v_{j_p})(u_{j_n} v_{j_n}) \langle j_n \| \vec{\sigma} \| j_p \rangle^2 \tag{24}$$

where $u_{j_n} v_{j_n} = \frac{1}{2} \frac{\Delta_n}{\varepsilon_n} and u_{j_p} v_{j_p} = \frac{1}{2} \frac{\Delta_p}{\varepsilon_p}$[8]. Now

$$M = \frac{1}{6}\Delta_n\Delta_p\sum_{np}\frac{\langle j_n\|\vec{\sigma}\|j_p\rangle^2}{\varepsilon_p\varepsilon_n} \tag{25}$$

where Δ_n and Δ_p are the coupling energies (energy gaps) for the neutron and proton systems and

$$\sum_n \frac{1}{\varepsilon_n} = \frac{2}{G_n}; \sum_p \frac{1}{\varepsilon_p} = \frac{2}{G_p} \tag{26}$$

and G$_p$ and G$_n$ are the coupling constants for the proton and neutron systems [8]. Now,

$$K = \sum_{np}\langle j_n\|\vec{\sigma}\|j_p\rangle^2 = \text{ Constant}$$

and we evaluate

$$M = \frac{2}{3}K\frac{\Delta_n\Delta_p}{G_nG_p}. \tag{27}$$

Thus it is shown that the matrix element of the transition $0^+ \rightarrow 0^+$ between $|Z, N\rangle$ and $|Z-2, N-2\rangle$ nuclei via the n-p entity stripping and n-p entity emission depends only on the superfluid parameters.

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