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The Investigation of the E2/M1 Multipole Mixing Ratios and Deformation Parameters of Electromagnetic Transitions in Decay of ^{156}Gd

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Abstract

The extended Rotation – Vibration Model (RTV) containing different deformations for protons and neutrons is applied to ^{156}Gd nucleus. Using the Rotation-Vibration Model approach multipole mixing ratios δ(E2/M1), deformation parameter β_o, g_R factors, and quadrupole moments q_2+ and q_o were calculated.

Key Words: Rotation – Vibration Model, multipole mixing ratio, deformation parameter, quadrupole moment and electromagnetic transitions.

1. Introduction

Interest in the problem of band mixing in doubly even deformed nuclei has been rising in the last few years, stimulated by a great deal of new experimental data and, in particular, by the observation of the back- bending effect in a number of rare earth nuclei. When the selection rules allow, E2 radiation often dominates the M1 componet. This domination radiation occurs because the nuclear structure effect overrides the angular momentum dependence of the transition probabilities.

The ^{156}Gd nucleus is at the begining of the deformation region 150$\leq$ A $\leq$ 190. The nucleus is a rotor which shows a developed γ- vibrational band. To explain the form of a nucleus; the binding energy of the nucleus, the transition probabilities between different energy levels, electric and magnetic multipole moments the quadrupole moments and the rest of the observable quantities must be known properties. The pairing and the quadrupole forces are important in deformed nuclei. These forces especially influence the particals in the unfilled states. The pairing force keeps the nucleus in spherical symetry. The quadrupole charge distribution causes what is known as the quadrupole force. This force takes the nucleus to the deformed state [1]. The pairing force of protons is Gp and the pairing force of neutrons is Gn. The relation between the pairing and the quadrupole forces determines the form of the nucleus. Since Gp$\rangle$Gn , βo (p)$\langle\beta$o(n) , where, βo(p) and βo(n) are proton and neutron deformation parameters, respectively [2]. With much new experimental and theoretical work being carried out, particularly on E2 / M1 mixing ratios in even-even nuclei, a critical survey of both areas is needed to point the way for further work.

2. Theoretical Survey

In the nucleus, an electromagnetic exchange connecting a state of spin I_1 to I_2 can carry an angular momentum L between I_1+ I_2 and $|I_1 - I_2|$. In the rotation- vibration model, pioneered by Bohr and

Mottelson [3], the low-lying, even-parity states of even-even nuclei are ascribed to the collective quadrupole motion of the nucleus as a whole.

The M1-E2 mixing parameter δ is defined as

$$\delta = \pm\sqrt{\frac{T(E2)}{T(M1)}} = \pm\frac{\sqrt{3}}{10}.\frac{w}{c}\sqrt{\frac{B(E2|I \to I')}{B(M1|I \to I')}}, \tag{2.1}$$

where the $\pm$ sign must be chosen depending on the relative sign of the reduced matrix element [2]. The electric quadrupole and magnetic dipole transition probabilities T(E2) and T(M1) are, respectively,

$$\begin{aligned} T(E2|I \to I') &= \tfrac{4\pi}{75}\tfrac{1}{\hbar}(\tfrac{w}{c})^5.B(E2|I \to I'), \\ T(M1|I \to I') &= \tfrac{16\pi}{9}\tfrac{1}{\hbar}(\tfrac{w}{c})^3 B(M1|I \to I'); \end{aligned} \tag{2.2}$$

and B(E2$|I \to I'$) is the reduced E2 transition probability:

$$B(E2|I \to I') = \frac{1}{2I+1}\sum_{\mu,M,M'}\left|\langle\psi^{I'M'}|(E2,\mu)|\psi^{IM}\rangle\right|^2 . \tag{2.3}$$

The reduced transition probability M1 is given by

$$B(M1|I \to I') = \frac{3}{4\pi}(\frac{e\hbar}{2Mc})^2\frac{1}{2I+1}\sum_{\mu,M,M'}\left|\langle\psi^{IM}\middle|\mu_\sigma\middle|\psi^{I'M'}\rangle\right|^2 . \tag{2.4}$$

The relation between the total angular momentum and the magnetic moment tensor character plays an important role to find the δ/E ratios. On the other hand, the M1-E2 mixing parameter can be written as

$$(\frac{\delta}{E})_{2^{+'}\to 2^+} = \left[\frac{4\pi}{10c}\frac{1}{(}\hbar c)^2 \cdot (\frac{2Mc}{e\hbar})^2(\frac{3Z}{4\pi}R_0^2)^2\frac{\sum\limits_{M',\mu,M}|\langle 2M', n=2|\,\alpha_\mu\,|2M, n=1\rangle|^2}{\sum\limits_{M',\sigma,M}|\langle 2M', n=2|\,\mu_\sigma\,|2M, n=1\rangle|^2}\right]^{1/2} . \tag{2.5}$$

With the matrix element

$$\langle 2M', n=2|\alpha_\mu\,|2M, n=1\rangle = \sqrt{2}\sqrt{\frac{\hbar}{2Bw}}(222|M'\mu M), \tag{2.6}$$

we obtain finally the result

$$(\frac{\delta}{E})_{2^{+'}\to 2^+} = +(\frac{3.718}{5})^{1/2}.10^{-3}.\frac{A^{\frac{5}{3}}.\beta_0}{(1-2f)f}, \tag{2.7}$$

where

$$\beta_0 = \sqrt{5}.\sqrt{\frac{\hbar}{2Bw}}, \tag{2.8}$$

$$f = \frac{\beta_0 - \beta_o(p)}{\beta_0} \cong \frac{N}{A}(\frac{\beta_0(n)}{\beta_o(p)} - 1) \tag{2.9}$$

and

$$\frac{\beta_0(p)}{\beta_0(n)} = \sqrt{\frac{G_P}{G_n}}. \tag{2.10}$$

Finally, this can be used to find the expression for deformation parameter β_0:

$$\beta_0 = 10^3(\frac{\delta}{E})_{2^+\to 2^+}.\frac{\frac{N}{A}(\sqrt{\frac{Gp}{Gn}}-1)[1-2\frac{N}{A}(\sqrt{\frac{Gp}{Gn}}-1)]}{0,862.A^{5/3}}. \tag{2.11}$$

For deformed nuclei, Greiner [2] gives the g_R factors for the first three rotational bands, ground state band, beta and gamma bands, as follows:

a) Ground state band:

$$(g_R)_{|I000\rangle} = \frac{(I1I\big|I0I)\sqrt{2}(I1I\big|110)\sqrt{I(I+1)}}{I}\frac{Z}{A}(1-2f) = \frac{Z}{A}(1-2f); \tag{2.12}$$

b) Beta band:

$$(g_R)_{|I001\rangle} = \frac{(I1I\big|I0I)\sqrt{2}(I1I\big|1-10)\sqrt{I(I+1)}}{I}\frac{Z}{A}(1-2f); \tag{2.13}$$

c) Gamma band:

$$\begin{aligned}(g_R)_{|I200\rangle} &= [-\frac{1}{\sqrt{2}}(I1I\Big|112)\sqrt{(I-1)(I+2)} + \frac{1}{\sqrt{2}}(I1I\Big|3-12)\sqrt{(I+3)(I-2)} \\ &\quad +2(I1I\Big|202)(1+\frac{2}{3}f]I^{-1}(I1I\Big|I0I)\frac{Z}{A}(1-2f) \\ &= (1+\frac{8}{3}\frac{f}{I(I+1)})\frac{Z}{A}(1-2f). \end{aligned} \tag{2.14}$$

Electric multipole moments are important symptoms of nuclear deformation. q, which is a symptom of unspheral form and dimension of determined load distribution of rotational nuclei, is obtained spectroscopicaly. It plays an important role in examining deformed nuclei. If q$\rangle$0 the nucleus is prolate and if q$\langle$0 the nucleus has an oblate deformation. q = 0 shows spherical charge distribution. In the Rotational Model the spectroscopic multipole moment of the nucleus is given by the relation [4]

$$q(I) = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} \cdot q_0. \tag{2.15}$$

Here, q_0 is the intrinsic quadrupole moment and is given by the expression [5].

$$q_0 = \frac{3}{\sqrt{5\pi}} \cdot ZR^2\beta_0(1+0.36\beta_0). \tag{2.16}$$

3. Results and Discussion

We calculated the δ (E2/M1) multipole mixing ratios for the electromagnetic transitions between the energy states of the ^{156}Gd nucleus using quation (7). The calculated values and the experimantal and theoretical values from other works are given in Table 1.

It can be seen from Table 1 that the mixing ratio found for the 1040.5 keV transition is -15.89±2.60 and this value is in agreement with the experimental values of -16(+8,-∞) of Lange et al. [6] and -14(+∞,-7) of Nour El-Din et al. [8]. It can also be seen that the theoretical value of Lipas et al. [9] is far from the experimental data. The result obtained for the 1065.2 keV transition is -16.27±2.66. This value is in agreement with the experimental values -17.2(+2.2,-1.7) of Uluer et al. [7] and -18.3(+0.6,-0.7) of Nour El-Din et al.[8]. On the other hand, the −46.1 theoretical value of Gupta et al.[4] is far from the experimental data. Similar argument holds for 1159.1-959.9-1067.3 keV transitions. Of transitions on which no work has been done, such as 1218.8 and 922.3 keV, the mixing ratios for these transitions are calculated to be -10.05±1.65 and -6.29±1.03, respectively. Although there are no results to compare these, they are in accordance with the rest of our calculations.

Table 1. Multipole mixing ratios for ^{156}**Gd**

Spin Parity	Transition Energy	Mixing Ratios [δ(E2/M1)]		
$I_I \rightarrow I_f$	(keV)	Present	Experimental	Theory
$2_{\beta+} \rightarrow 2_{gr+}$	1040.5	-15.89±2.60	$-16^{+8}_{-\infty}$[6]	-24.2[9]
			$-5.9^{+1.4}_{-2.8}$[7]	
			$-14^{+\infty}_{-7}$[8]	
$2_{\gamma+} \rightarrow 2_{gr+}$	1065.2	-16.27±2.66	$-17.2^{+2.2}_{-1.7}$[8]	-46.1[4]
			-18.3±3[7]	
$3_{\gamma+} \rightarrow 2_{gr+}$	1159.1	-16.56±2.71	$-11.8^{+0.6}_{-0.7}$[7]	-57.5[4]
			$-8.9^{+2.4}_{-5.0}$[6]	
$3_{\gamma+} \rightarrow 4_{gr+}$	959.9	-10.01±1.64	$-11.7^{+2.7}_{-5.3}$[7]	-37.3[4]
			-15.0^{+4}_{-17}[6]	-26.5[9]
$4_{\gamma+} \rightarrow 4_{gr+}$	1067.3	-8.51±1.39	$-4.0^{+0.9}_{-1.6}$[7]	-15.4[4]
			$-4.5^{+1.0}_{-1.8}$[6]	
$5_{\gamma+} \rightarrow 4_{gr+}$	1218.8	-10.05±1.65	–	–
$5_{\gamma+} \rightarrow 6_{gr+}$	922.3	-6.29±1.03	–	–

The β_0 deformation parameter, the $q_{2+}$and q_0 quadrupole moments are calculated by the use of relations (7), (11) and (12). The results are shown in Table 2 together with the previous values.

Table 2. β_0, q_{2+} and q_0 parameters for ^{156}Gd

β_0	q_{2+}	q_0 (e. barn)	
	(e. barn)	Theory	Experimental
0.309±0.05(*)	-1.59±0.21(*)	5.57±0.75(*)	6.91[13]
0.302[10]	-1.93±0.4[12]	6.77±0.47[10]	
0.31[11]		6.27[13]	

(•) This work.

The calculated β_0 values for ^{156}Gd is 0.309±0.05. This result is in good agreement with result of 0.302 of Odintsova and Striganov [10] and with the 0.31 value Kalfas et al. [11]. The q_{2+} quadrupole moment is calculated as -1.59±0.21 and this agrees with the -1.93±0.4 value of Laubacher et al. [12] within the limits of error. The value found for q_0 is 5.57±0.75 and this value agrees with the theoretical and experimental results of 6.91 of Ragnarson et al .[13] and 6.77±0.47 of Odintsova and Striganov [10].

The g_R factors of ^{156}Gd are calculated through quations (8), (9) and (10). The results are given in Table 3. There is only one value for comparasion of these results and it is in agreement with our value.

Table 3. Calculation g_R factors for ^{156} Gd.

Ground State	Gamma Band		
and Beta Band	$2_{\gamma+}$	$3_{\gamma+}$	$4_{\gamma+}$
0.329(*)	0.343(*)	0.336(*)	0.333(*)
0.320[14]			

(∗) This work.

It can be seen from the tables that our results are in better agreement with the previous experimental data. Other theoretical values can not show this agreement. This certifies that the method is applicaple for the deformed region. Transitions connecting the levels of positive parity are generally in E2 character, but the existing M1 mixing show that the β and γ bands are not the quadrupole exitations of the ground state band.

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