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Interface Roughness-Induced Intrassubband Scattering in a Quantum Well Under an Electric Field

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Abstract

The scattering rates in the lowest subband in a quantum well are calculated for interface roughness scattering when an electric field is applied perpendicular to the layer plane. It is found that the interface roughness scattering rate increases with increase in the electric field. The electric field changes the interface roughness scattering rates drastically in thick QWs compared with those for the zero-field case. This behaviour in the scattering rate gives a new degree of freedom in regions of interest in device applications.

Key Words: interface roughness, intersubband

1. Introduction

There has been considerable interest in the study of intersubband transitions in a quantum well (QW) both in the presence and in the absence of an electric field applied perpendicular to the QW layer plane [1-12]. The reason behind the interest is practical device application [3-4]. In [1] a new type of infrared laser amplifier using the intersubband transition and resonant tunneling was proposed. An infrared detector in the 10 μm has been proposed and fabricated [5] that relies on the intersubband transition and resonant tunneling between adjacent QWs.

Theoretical studies [6,7] on intersubband optical absorption under an electric field have pointed out an increase in the oscillator strength. In all the theories [6,7] the intra- and intersubband relaxation rates come into the calculation of the absorption coefficient. Various scattering mechanisms determine the intra- and intersubband relaxation rates, of which polar optic phonon scattering has been a major scattering mechanism over a considerable temperature range. The rates for such scattering have already been calculated [7]. In [8] have calculated the alloy-disorder scattering rate for two-dimensional electrons in the lowest subband of a quantum well in an applied electric field.

It is established that interface - roughness scattering dominates the low temperature mobility of two - dimensional (2D) electrons in thin QWs [13,14]. In a QW, interface-roughness may be expected to be particularly important because only a small change of barrier width can cause a large fluctuation of the mini-band width. The mobility and relaxation rates due to interface roughness scattering in QWs have been formulated previously in the absence of an applied electric field [15,16]. In the present work, we report similar calculations when an electric field is applied perpendicular to the QW layer, and investigate how the scattering rates are modified. As a first step, we consider intrasubband relaxation for the lowest subband. Using the finite potential barrier model, we have found that there is a strong electric field dependence of the scattering rate.

2. Calculations

The electrons are assumed to be quantizied along the z direction and the wave function is taken to be of the following form [2]:

$$\Phi\left(R\right)=\psi\left(z\right)\exp\left(ikr\right)=N\left(\beta\right)\left(1+\frac{\beta z}{L}\right)\psi_0\left(z\right)\exp\left(iKr\right),\qquad -L/2<z<L/2 \tag{1}$$

where $\psi_0\left(z\right)$ is the unperturbed ground-state wave function. The normalization factor of the electron wave function are

$$N^2\left(\beta\right)=\frac{\pi^3}{\pi^3+\beta^2\left(\pi^2-8\right)} \tag{2}$$

L is the thickness of the well and K and r are, respectively the 2D wave vector and position vector of the electron in the plane of free motion (x-y plane). In Eq. (1) β is variational parameter related to the electric field F as [2]

$$\beta=\frac{2|e|Fm^*L}{\hbar^2}<z^2>_0\quad,\qquad \beta^2<z^2>_0<<L^2 \tag{3}$$

where $<z^2>_0=<\psi_0\left|z^2\right|\psi_0>$.

The interface roughness is characterized by the height Δ and the lateral correlation length $\wedge$ of the Gaussian fluctuation. For the perturbing potential due to interface roughness we use the standard model [10,17], for which the random potential is assumed to have Gaussian distribution and expressed as the autocorrelation function

$$<V\left(R\right)V\left(R'\right)>=V_0^2\Delta^2\delta\left(Z-L/2\right)\delta\left(Z'-L/2\right)\exp\left(-\frac{\left(r-r'\right)^2}{\Lambda^2}\right), \tag{4}$$

where $\langle......\rangle$ denotes an ensemble average, and V_0 is the barrier height.

Using Eq. (4) we get for the square of the matrix element for roughness scattering from the k state to the k' state

$$\left|M\left(K,K'\right)\right|^2=\pi\Lambda^2\Delta^2V_0^2\psi^4\left(\frac{L}{2}\right)\exp\left(-\frac{q^2\Lambda^2}{4}\right), \tag{5}$$

where q = K′ − K is the 2D scattering wave vector and q =$|q|$. Also, as the scattering is elastic.

$$K'=K,\quad q^2=2K(1-\cos\theta)\cdot\cos\theta=\frac{KK'}{K^2}.$$

The transition probability from a state k to all other state k' is then given by

$$W_F=\frac{2\pi}{\hbar}\sum_{K'}\left|M\left(K,K'\right)\right|^2\delta\left(E_{K'}-E_K\right)=\frac{\pi\ m*\Lambda^2\Delta^2V_0^2\psi^4\left(L/2\right)}{\hbar^3}G\left(\Lambda,K\right) \tag{6}$$

where

$$G\left(\Lambda,K\right)=\frac{1}{2\pi}\int\limits_0^{2\pi}\exp[-\Lambda^2K^2\left(1-\cos\theta\right)/2]d\theta.$$

The interface roughness scattering rate is proportional to $V_0^2\psi^4\left(L/2\right)$, and using [9]

$$\psi\left(L/2\right)=\left(\frac{\hbar^2}{2m*\left(V_0-E\right)}\right)^{1/2}\frac{d\psi}{dZ}|_{z=L/2} \tag{7}$$

We obtain

$$W_F=\frac{\pi^5\Lambda^2\Delta^2\hbar}{m*L^6}\cdot\frac{V_0^2}{\left(V_0-E\right)^2}N^4\left(\beta\right)\left(1+\frac{\beta}{2}\right)^4\cdot G\left(\Lambda,K\right). \tag{8}$$

As shown in Eq. (8), the scattering rate is proportional to L^{-6}. This means that interface roughness scattering rate is much more important for narrow wells and the interface roughness scattering limited mobility in quantum wells is proportional to L^6 [15].

For the interface roughness calculation we have not bothered to take screening into account. It seems evident that screening can only reduce an effect that is already very small.

In order to facilitate comparison, a similar expression for the transition probability or scattering rate without an applied field is needed. Thus if we take the limit $V_0\to\infty$ and $\beta=0$ in Eg.(8), we may write for the scattering rate without a field, W_0(K) as [15]

$$W_0=\frac{\pi^5\Lambda^2\Delta^2\hbar}{m*L^6}G\left(\Lambda,K\right). \tag{9}$$

In the limit $V_0\to\infty$ from Eq. (8) - (9) the ratio is expressed as

$$\gamma=\frac{W_F\left(K\right)}{W_0\left(K\right)}=N^4\left(\beta\right)\left(1+\frac{\beta}{2}\right)^4. \tag{10}$$

3. Results

We have calculated the scattering rates in QWs by using the parameters characteristic of GaAs. The values of the ratio γ calculated from Eq. (10) plotted against the electric field F in Figure 1. It is found from the figure that the scattering rate increases with an increase in the electric field. However, as seen from the figure, the electric field changes the interface roughness scattering rates drastically in thick QW_s compared with those for zero-field case. These results predict qualitatively the trends observed [2]. Smaller electric field dependences can be explained by the fact that the wave function change due to the applied electric field of thin QW_s is smaller than that thick QW_s. We find that γ is also a strong function of field and this ratio becomes as small as 1.1∼1.5 below 100 kV/cm. Another observation is that although the value for γ is 1.5÷4.5 field above 400 kv/cm. In the case of polar optic phonon scattering $\gamma\approx$1.2 (F=200kV/cm) [7] and for alloy-disorder scattering $\gamma\approx$4.5 (F=400kV/cm) [8].

Previous calculations for interface-roughness scatterings show that the scattering rate is enhanced as the well width L is reduced [15,16]. When one applies an electric field perpendicular to the quantum well, electrons are pushed to one side of the well, thus the effective well width is reduced [6]. This will result in the enhancement of the interface-roughness scattering rate. Our results show that the interface-roughness scattering rate is enhanced with increasing electric field.

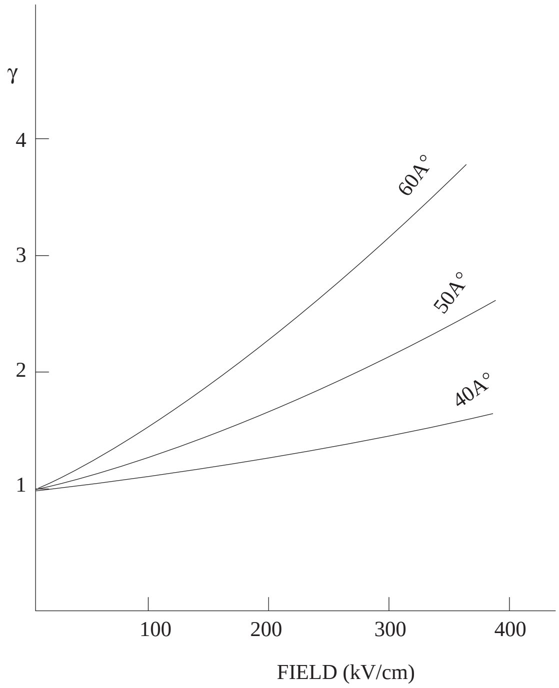

Figure 1. Variation of scattering rate $W_F(k)$ QW of 40, 50 or 60 Å. The rate W_F is normalized by the rate $W_0(k)$ in the absence of field, and the ratio is denoted by γ.

4. Conclusion

We have demonstrated that there is an increase of the scattering rate of electrons in the first subband of a QW, with increase in the perpendicular electric field, when the scattering is due to interface roughness. It permits the implementation of novel infrared and far-infrared high-speed detectors. The electric field dependence of the carrier-interface-roughness scattering should also be important in the calculation of the intersubband optical absorption in a quantum well. We find that $\mathrm{W}_f/\mathrm{W}_0$have a stronger dependence on the well width, thus this behaviour can used to study these systems in regions of interest, without the need for different samples.

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