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# Nuclear Matrix Elements of Double Beta Decay in Deformed Nuclei

**Cevat SELAM, Atalay KÜÇÜKBURSA, Hasan BİRCAN**
*Dumlupınar University, Faculty of Arts and Sciences, Department of Physics, Kütahya-TURKEY*
*e-mail: atalay@dumlupinar.edu.tr*
**Hasan Ali AYGÖR, Tahsin BABACAN, İsmail MARAŞ**
*Celal Bayar University, Faculty of Arts and Sciences, Department of Physics, Manisa-TURKEY*
**Ali KÖKÇE**
*Süleyman Demirel University, Faculty of Arts and Sciences, Department of Physics, Isparta-TURKEY*

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**Abstract**

Nuclear matrix elements ($M_{GT}$) for two-neutrino double beta transitions of selected nuclei were calculated via a QRPA approach by considering the charge-exchange spin-spin interactions in the particle-hole channel among nucleons. Calculations were performed for both spherical and deformed cases of nuclei. As a result of these calculations, it has been seen that, although the value of the nuclear matrix elements in deformation case are 2-4 times smaller when compared with that of the spherical case, it is still 2-8 times greater than the experimental values.

**Key Words:** Double beta decay

## 1. Introduction

Both particle and nuclear physicists have shown much interest in the double beta decay problem and in recent years this interest has been increasing intensely. Since the double beta decay $2\beta$ is a second order weak semi-leptonic process, its matrix element is proportional to the fourth power of the interaction constant for nucleons with electron-neutrino field (Fermi coupling constant) $G_F^4$[1-5], hence its probability is very small.

The double beta decay can proceed in several ways. One of them is the two-neutrino double beta decay $2\beta_{2\nu}$

$$(A; Z) \to (A; Z + 2) + 2e^- + 2\bar{\nu}_e,$$

and another is the neutrinoless double beta decay ($2\beta_{0\nu}$)

$$(A; Z) \to (A; Z + 2) + 2e^-.$$

The most apparent difference between neutrinoless and two-neutrino modes of double beta decay is their emphasis to different physical points. For example, the former decay mode gives information about more fundamental properties such as the conservation of lepton charge, effective Majarona mass of the neutrino, and the contribution of right-handed currents to the weak interactions. On the other hand, the latter decay mode is used to test various nuclear models since it does not depend on the structure of the neutrino.

Recently there have been some intense theoretical investigations concerning $2\beta_{2\nu}$ decay. The source of this intensity is due to the consideration of the attractive charge-exchange interaction among nucleons in particle-particle channel. When the interaction studied here was taken into account, it encountered some problems within the QRPA approach. One of them is the sensitivity of the nuclear matrix elements (NME) to the attractive charge-exchange spin-spin interaction constant. The second problem is the situation in which at some critical value of this interaction constant the NME approach zero (collapse of the QRPA solution) [6-11]. It is possible to explain this phenomena from the following physical point of view: three interactions, i.e. the attractive interaction in the particle-particle channel, the interaction in the particle-hole channel and pairing interaction, can cancel out each other at some critical value of the attractive interaction constant. In this case, the system behaves as if it is a one-particle system. As it is known, in a one-particle system the value of NME is zero. Some attempts have been done to overcome the above drawbacks [12-24].

In this work, the dependence of the NME on deformation of nuclei is investigated by using the QRPA approximation and considering the charge-exchange spin-spin interaction among nucleons in the particle-hole channel.

## 2. Probability of $\beta_{2\nu}$ Decay

For a two neutrino $0^+ \to 0^+$ decay, if Fermi transitions are neglected, $2\beta_{2\nu}$ decay probability is given [1-2] by

$$\begin{aligned}\omega_{2\nu} = \tfrac{\ln 2}{T_{1/2}} = \tfrac{g_A^4 G^4}{96\pi^7} \int_{m_e}^{w-m_e} F(z,\varepsilon_1)\left|\vec{k}_1\right|\varepsilon_1 d\varepsilon_1 \cdot \int_{m_e}^{w-\varepsilon_e} F(z,\varepsilon_2)\left|\vec{k}_2\right|\varepsilon_2 d\varepsilon_2 \\ \int_{m_e}^{w-\varepsilon_e-\varepsilon_2} \nu^2 (w-\varepsilon_1-\varepsilon_2)^2 d\nu \left[K^2(\varepsilon_1,\nu)+K^2(\varepsilon_2,\nu)+K(\varepsilon_1,\nu)\cdot K(\varepsilon_2,\nu)\right]\end{aligned} \tag{1}$$

with

$$K(\varepsilon,\nu) = \sum_n \frac{(w+2\omega_n)M_{GT}(\omega_n)}{(w+\omega_n-\varepsilon-\nu)(\omega_n+\varepsilon+\nu)}$$

$$M_{GT}(\omega_n) = \langle \Psi_f(A,Z)|\vec{\sigma}\tau_-|n\rangle\langle n|\vec{\sigma}\tau_-|\Psi_i(A,Z-2)\rangle,$$

where $F(z,\varepsilon)$ is the Fermi function which depends on the nuclear charge and two electron energies $\varepsilon_1$ ,$\varepsilon_2$ with momenta $\left|\vec{k}_1\right|$ and $\left|\vec{k}_2\right|$, $\Psi_i(\Psi_f\ )$ and $|n\rangle$ are ground state wave functions of mother (daughter) nuclei and $1^+$ excited states for intermediate nuclei, $\nu(\omega_n)$ is the energy of the neutrino, and $w = E_f - E_i$ is the energy of $2\beta_{2\nu}$ decay. Using the approximation that each electron-neutrino pair will have equal energy, i.e. $\varepsilon_1+\nu_1=\varepsilon_2+\nu_2=\varepsilon+\nu=$w/2 (if $\varepsilon_1=\varepsilon_2=\varepsilon$ is supposed), the integral in Eq. (1) is reduced to the simple form

$$\omega_{2\nu} = f_{2\nu} M_{GT}^2 \ln 2, \tag{2}$$

where $f_{2\nu}$ is a phase integral which depends on charge of daughter nuclei, and is a polynomial of eleven orders with respect to $w$, and $M_{GT}$ is called the Gamow-Teller matrix element for $2\beta_{2\nu}$ decay and described by

$$M_{GT} = \sum_n \frac{\langle \Psi_f(A,Z)|\vec{\sigma}\tau_-|1_n^+\rangle\langle 1_n^+|\vec{\sigma}\tau_-|\Psi_i(A,Z-2)\rangle}{\omega_n + w/2} \quad . \tag{3}$$

The values of this phase integral of investigated nucleus are shown in Table 1 [25]. Then the half-life of the $2\beta_{2\nu}$ decay can be written as

$$T_{1/2}^{-1} = f_{2\nu} M_{GT}^2. \tag{4}$$

**Table 1.** The values of $f_{2\nu}$ investigated nucleus (year$^{-1}$MeV$^2$).

| $^{76}$Ge-$^{76}$Se | $^{82}$Se-$^{82}$Kr | $^{96}$Zr-$^{96}$Mo | $^{100}$Mo-$^{100}$Ru | $^{128}$Te-$^{128}$Xe | $^{130}$Te-$^{130}$Xe | $^{150}$Nd-$^{150}$Sm |
|---|---|---|---|---|---|---|
| 3.45 $10^{-20}$ | 1.16 $10^{-18}$ | 4.98 $10^{-18}$ | 2.43 $10^{-18}$ | 2.16 $10^{-22}$ | 1.23 $10^{-18}$ | 3.03 $10^{-17}$ |

# 3. Nuclear Matrix Elements for $\beta_{2\nu}$ Decay

We take into consideration spin-isospin interaction among nucleons in the form of Ref. [26]:

$$V_{GT} = \frac{1}{2}\chi_\beta \sum_{i\neq j} \vec{\sigma}_i\vec{\sigma}_j\vec{\tau}_i\vec{\tau}_j. \tag{5}$$

The charge-exchange part of this interaction is considered as in Ref. [27] as

$$V_{coll} = 2\chi_\beta \beta^+\beta \tag{6}$$

with

$$\beta = \sum \left(\bar{b}_{np}C_{np} - b_{np}C^\dagger_{np}\right) \tag{7}$$

$$C_{np} = \frac{1}{\sqrt{2}}\sum_\rho \alpha_{p\rho}\alpha_{n,-\rho} \ , \quad C^\dagger_{np} = \tfrac{1}{\sqrt{2}}\textstyle\sum_\rho \alpha^\dagger_{n,-\rho}\alpha^\dagger_{p\rho} \ , \mu \quad = 0,1$$

$$\left[C_{np}, C^\dagger_{n'p'}\right] \cong \delta_{nn'}\delta_{pp'} \ , \quad \left[C_{np}, C_{n'p'}\right] = \left[C^\dagger_{np}, C^\dagger_{n'p'}\right] = 0$$

$$b_{np} = \sqrt{2}\sigma^\mu_{np}u_p v_n \ , \quad \bar{b}_{np} = \sqrt{2}\sigma^\mu_{np}v_p u_n \ , \quad \sigma^\mu_{np} = \langle n+|\sigma_\mu + (-1)^\mu\sigma_{-\mu}|p+\rangle \ ,$$

where $\alpha^\dagger$ ($\alpha$) are quasiparticle creation (annihilation) operators, u and v are parameters of Bogolyubov canonic transformation and $C^\dagger_{np}$($C_{np}$) are two-quasiparticle creation (annihilation) operators.

The model Hamiltonian of the system can be written as

$$H = H_{sqp} + V_{coll}$$

with

$$H_{sqp} = \textstyle\sum_n \varepsilon_n B_{nn} + \sum_p \varepsilon_p B_{pp}$$

$$B_{nn} = \textstyle\sum_{n\rho}\alpha^\dagger_{n\rho}\alpha_{n\rho} \ , \quad B_{pp} = \sum_{p\rho}\alpha^\dagger_{p\rho}\alpha_{p\rho},$$

where $\varepsilon_n$ ($\varepsilon_p$) is the energy of the neutron (proton) quasiparticle. In RPA, a collective $1^+$ state in the intermediate odd-odd nucleus is considered as a one-phonon charge-exchange excitation described by

$$|\psi_i\rangle = Q^\dagger_i\,|\psi_0\rangle = \left[\sum_{np}\left(r^i_{np}C^\dagger_{np} - s^i_{np}C_{np}\right)\right]|\psi_0\rangle \,,$$

where $Q_i^\dagger$ is the phonon creation operator, $|\psi_0\rangle$ is the phonon vacuum which corresponds to the ground states of the mother even-even nucleus. The two quasiparticle amplitudes $r^i_{np}$ and $s^i_{np}$ are normalized by

$$\sum_{np}\left[\left(r^i_{np}\right)^2-\left(s^i_{np}\right)^2\right]=1 \quad . \tag{8}$$

Following the conventional procedure of RPA and solving the equation of motion

$$[H_{sqp}+V_{coll},Q_i^\dagger]|\psi_0\rangle=\omega_i \quad Q_i^\dagger|\psi_0\rangle,$$

we obtain the dispersion relation for the excitation energy of $1^+$ states in odd-odd nuclei:

$$\begin{aligned}&\left[1+2\chi_\beta\sum_{np}\left(\frac{\bar{b}^2_{np}}{E_{np}-\omega_i}+\frac{b^2_{np}}{E_{np}+\omega_i}\right)\right]\cdot\left[1+2\chi_\beta\sum_{np}\left(\frac{b^2_{np}}{E_{np}-\omega_i}+\frac{\bar{b}^2_{np}}{E_{np}+\omega_i}\right)\right]-\\&\left[2\chi_\beta\sum_{np}b_{np}\bar{b}_{np}\left(\frac{1}{E_{np}-\omega_i}+\frac{1}{E_{np}+\omega_i}\right)\right]^2=0,\end{aligned} \tag{9}$$

where $E_{np}=E_n+E_p$ is the two quasiparticle energy for neutron-proton pairs.

In this study, the ground states of the mother and daughter nuclei will be assumed to be same, i.e., $(|\psi_i\rangle=|\psi_f\rangle\equiv|\psi_0\rangle)$. For this approximation the matrix elements of beta transitions given in Eq. (3) takes the form of

$$\begin{aligned}M_i^+&=\left\langle 1_i^+|\vec{\sigma}\tau_+|\psi_f(A,Z)\right\rangle=\left\langle\psi_f(A,Z)|\vec{\sigma}\tau_-|1_i^+\right\rangle=\sum_{n,p}(\bar{b}_{np}r^i_{np}-b_{np}s^i_{np})\\M_i^-&=\left\langle 1_i^+|\vec{\sigma}\tau_-|\psi_i(A,Z-2)\right\rangle=-\sum_{n,p}(b_{np}r^i_{np}-\bar{b}_{np}s^i_{np}),\end{aligned}$$

where

$$r^i_{np}=\frac{\bar{b}_{np}+L(\omega_i)b_{np}}{E_{np}-\omega_i}\frac{1}{Z\left(\omega_i\right)},\quad s^i_{np}=\frac{b_{np}+L(\omega_i)\bar{b}_{np}}{E_{np}+\omega_i}\quad\frac{1}{Z\left(\omega_i\right)}$$

$$L(\omega_i)=-\left[2\chi_\beta\sum_{np}b_{np}\bar{b}_{np}\left(\frac{1}{E_{np}-\omega_i}+\frac{1}{E_{np}+\omega_i}\right)\right]\left[1+2\chi_\beta\sum_{np}\left(\frac{b^2_{np}}{E_{np}-\omega_i}+\frac{\bar{b}^2_{np}}{E_{np}+\omega_i}\right)\right]^{-1}$$

and $Z(\omega_i)$ can be calculated by using normalization condition (8).

# 4. Numerical Results and Conclusions

Numerical calculations in this work are based on the single particle Wood-Saxon potential with parametrizations given in Ref. [28] for the spherical case, and in Ref. [29] for the deformation case. Pairing correlation and deformation parameters ($\Delta_n,\Delta_p$, and $\delta$) were taken from Ref. [30]. The charge-exchange spin-spin interaction parameter $\chi_\beta$ is determined by comparing the theoretical and experimental values of $\log(ft)$ for beta transitions in deformed odd nuclei, and this value is taken as $\chi_\beta$=21/A. Calculated values of $M_{GT}$ for the investigated nuclei are shown in Table 2.

The single-quasiparticle (SQP) and QRPA values of the matrix elements for the spherical case is shown in the second and third columns of Table 2. In the fourth and fifth columns, the results of the same calculation for the deformation case are presented. In the sixth column, the experimental values for NME are given, and these values are found from Eq. (2) by using the $f_{2\nu}$ results given in Table 1 and the experimental values for the half life of $2\beta_{2\nu}$ decay $(t_{1/2})_{\text{exp}}$.

Calculations show that the single particle values of $NME(M_{GT})$ for the deformed nuclei are also zero as it is in the spherical case [31]. The SQP values of $M_{GT}$ are almost the same for both the deformation and spherical cases (see column 2 and 4) indicating the reliability of the single particle basis used in our calculations.

As it is seen from Table 2, RPA values of $M_{GT}$ in both cases are less than SQP values due to the effective spin-isospin interactions among nucleons. While this decrease is expressed in percentages in the spherical case, it is bigger in deformation case. An explanation of this decrease can be given as follows:

**Table 2.** $M_{GT}$ value for the investigated nuclei.

| Transitions | **Spherical Case** | | **Deformation Case** | | **Experiment** | **Ref.** |
|---|---|---|---|---|---|---|
| | SQP | **QRPA** | **SQP** | **QRPA** | | |
| $^{76}$Ge-$^{76}$Se | 0.561 | 0.464 | 0.718 | 0.172 | 0.127 | [32] |
| $^{82}$Se-$^{82}$Kr | 0.465 | 0.392 | 0.504 | 0.132 | 0.102 | [33] |
| $^{96}$Zr-$^{96}$Mo | 0.654 | 0.540 | 0.656 | 0.179 | 0.072 | [34] |
| $^{100}$Mo-$^{100}$Ru | 1.002 | 0.785 | 0.617 | 0.174 | 0.246 | [35] |
| $^{128}$Te-$^{128}$Xe | 0.972 | 0.727 | 0.999 | 0.190 | 0.025 | [36] |
| $^{130}$Te-$^{130}$Xe | 0.750 | 0.587 | 0.765 | 0.148 | 0.017;0.032 | [36,37] |
| $^{150}$Nd-$^{150}$Sm | 0.774 | 0.647 | 0.654 | 0.173 | 0.070 | [35] |

a) although all SQP values of the Gamow-Teller matrix element $M_i^-$ and $M_i^+$are in phase, some of their RPA values can be due to phase-out;

b) the charge exchange spin-spin interaction (in the particle-hole channel) decreases significantly the $\beta^-$ strength function amplitude in the spectroscopic region where $\beta^+$ transitions are dominant due to its repulsive character;

c) one can not find some roots for secular Equation (9) in RPA calculations (this can be considered as the error in calculation and has no physical origin).

Figures 1 and 2 show the energy dispersion of the strength functions $S_\beta^\pm$ for $^{150}$Nd isotope in both spherical and deformation case, respectively. The calculated values of these strength functions are obtained by the formula [38]

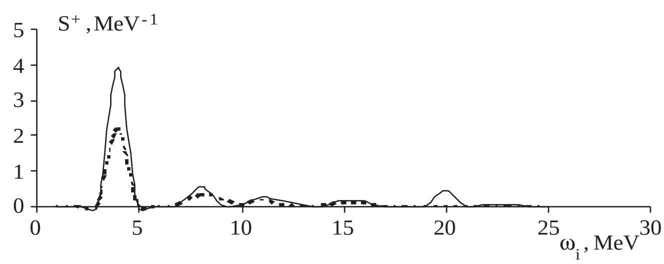


**Figure 1.** Energy dispersion of strength function $S_\beta^\pm$ for $^{150}_{60}Nd \rightarrow^{150}_{60} Sm$ (Spherical case).

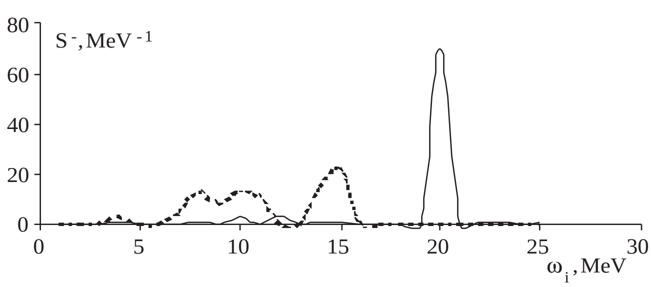


**Figure 2.** Energy dispersion of strength function $S_\beta^\pm$ for $^{150}_{60}Nd \rightarrow^{150}_{60} Sm$ (Deformation case).

$$S_{\beta}^{\pm} = \frac{1}{\Delta E} \sum_{\Delta E, i} \left| \langle 1_i^+ | \sigma \tau_{\pm} | 0^+ \rangle \right|^2, (\Delta E = 1MeV).$$

Dashed and solid lines in the graphs correspond to the SQP and RPA values of the strength functions, respectively. From Figures 1 and 2, it is seen that the effect of spin-spin interactions with charge exchange between nucleons on the strength of function $S^-$ is stronger in both the spherical and deformation cases. The value of the overlap sum will decrease since maximum points of the strength of functions ($S^-$ and $S^+$) become far away from each other by means of these interactions. Therefore, the values of the nuclear matrix elements are almost 2-4 times less than the corresponding spherical values when the deformation structure in nuclei is taken into account. The theoretically calculated values of NME studied here are still 2-8 times greater than the corresponding experimental values even though there exists such a decrease. To compensate for the difference between the theoretical and experimental values, the following situations will be considered:

- the case in which the ground states of the mother and daughter nuclei have different deformation structure;
- consideration of spin-spin interactions with charge-exchange among nucleons in particle-particle channel;
- calculation of the nuclear matrix element through the residue theory if one can not find all roots of the secular equation in Formula (9).

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