Orbital Entanglement and Violation of Bell Inequalities in the Presence of Dephasing

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Abstract

We discuss orbital entanglement in mesoscopic conductors, focusing on the effect of dephasing. The entanglement is detected via violation of a Bell Inequality formulated in terms of zero-frequency current correlations. Following closely the recent work by Samuelsson, Sukhorukov and Büttiker[1], we investigate how the dephasing affects the possibility to violate the Bell Inequality and how system parameters can be adjusted for optimal violation.

Key Words: orbital entanglement, dephasing, Bell Inequalities, mesoscopic physics

1. Introduction

Entanglement is one of the most intriguing features predicted by quantum theory [2, 3, 4]. It leads to correlations between distant particles, which can not be described by any local, realistic theory [5]. This nonlocal property of entanglement has been demonstrated convincingly in optics [6, 7], where entangled pairs of photons have been studied over several decades [8]. Apart from the fundamental aspects, there is a growing interest in using the properties of entangled particles for quantum cryptography [9] and quantum computation [10].

Recently, much interest has been shown for entanglement of electrons in solid state systems. A controlled generation and manipulation of electronic entanglement is of importance for a large scale implementation of quantum information and computation schemes. Electrons are however, in contrast to photons, massive and electrically charged particles, which raises new fundamental questions and new experimental challenges. Most existing suggestions [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21] are based on creating, manipulating and detecting spin-entangled pairs of electrons. This requires experimental control of individual spins via spin filters or locally directed magnetic fields on a mesoscopic scale.

To overcome these difficulties, very recently spin-independent schemes for creating and detecting orbital entanglement in a mesoscopic system were proposed in normal-superconducting[1] as well as purely normal systems [22, 23]. Energy-time entanglement [24] in a Franson like geometry [25] provides another spin-independent approach. In this paper, we discuss in detail the properties of the orbital entanglement and how it can be detected via violation of a Bell Inequality. In particular, we focus on the effect of decoherence on the entanglement. We follow closely the discussion in Ref. [1], but the results are of relevance also for the orbital entanglement studied in Refs. [22, 23]. An investigation of the effect of decoherence on entanglement, discussing the setup in Ref. [22], is presented by van Velsen et al in this issue.

2. Entangled two-particle wavefunction

We consider the system shown in Fig. 1. A single superconductor (S) [the upper and lower part in the figure connected e.g. via loop] is weakly coupled to a normal conductor, a ballistic two-dimensional electron gas, via two tunnel barriers 1 and 2 with equal transparencies $\Gamma \ll 1$. The normal conductor consists of four arms, $1A, 1B, 2A$ and $2B$, with equal lengths L. The arms $1A$ and $2A$ ($1B$ and $2B$) are crossed in a beam-splitter $A(B)$ and then connected to normal reservoirs $+A$ and $-A$ ($+B$ and $-B$). We consider
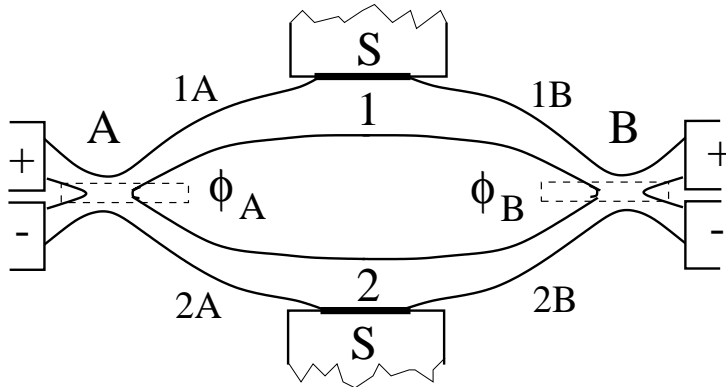

Figure 1. The system of Ref. [1]: A single superconductor (S) is connected to four normal arms via two tunnel barriers 1 and 2 (thick black lines). The arms are joined pairwise in beam splitters A and B and end in normal reservoirs + and −.

the low temperature limit, $kT \ll eV$. A negative voltage $-eV$ is applied to all the normal reservoirs and the superconductor is grounded. The voltage eV is smaller than the superconducting gap Δ, so no single particle transport takes place. It is further assumed that the size of the system is smaller than the phase breaking length.

The superconductor as a source of spin-entangled electrons has been discussed in Refs. [11, 12, 16, 21]. As shown in Ref. [1], due to the coherent properties of the superconducting condensate, the superconductor can also act as an emitter of orbitally entangled pairs of electrons. To lowest order in tunnel barrier strength $\Gamma \ll 1$, a pair of electrons is emitted either through contact 1 or contact 2, The state is thus a linear superposition of emitted pairs, an orbitally entangled two-particle state. In a second quantization language, the state of the entangled pair can be written as [1]

$$|\Psi_{out}\rangle = |\bar{0}\rangle + |\tilde{\Psi}\rangle + |\Psi\rangle, \tag{1}$$

where the states

$$
\begin{aligned}
|\tilde{\Psi}\rangle &= \frac{i\Gamma}{4} \int_0^{eV} dE \sum_{j=1,2} \sum_{\eta=A,B} \left[c_{j\eta}^{\uparrow\dagger}(E) c_{j\eta}^{\downarrow\dagger}(-E) - c_{j\eta}^{\downarrow\dagger}(E) c_{j\eta}^{\uparrow\dagger}(-E) \right] |\bar{0}\rangle, \\
|\Psi\rangle &= \frac{i\Gamma}{4} \int_{-eV}^{eV} dE \left[c_{1A}^{\uparrow\dagger}(E) c_{1B}^{\downarrow\dagger}(-E) - c_{1A}^{\downarrow\dagger}(E) c_{1B}^{\uparrow\dagger}(-E) + c_{2A}^{\uparrow\dagger}(E) c_{2B}^{\downarrow\dagger}(-E) - c_{2A}^{\downarrow\dagger}(E) c_{2B}^{\uparrow\dagger}(-E) \right] |\bar{0}\rangle. \tag{2}
\end{aligned}
$$

Here, e.g. an operator $c_{1A}^{\uparrow\dagger}(E)$ describes the creation of an electron with spin up at energy E, emitted through contact 1 and propagating towards beamsplitter A. The state $|\bar{0}\rangle$ is the groundstate, a filled Fermi sea with electrons at all energies $E < -eV$ and $|\Psi\rangle$ ($|\tilde{\Psi}\rangle$) describes orbitally entangled wavepacket-like states with two electrons propagating to different (the same) beamsplitters. As is shown in Ref. [1], the state $|\tilde{\Psi}\rangle$, where the two emitted electrons propagate towards the same beamsplitter, only contributes to the physical observables of interest to higher order in tunnel barrier transparency and can be neglected. In what follows below, we thus focus on the state $|\Psi\rangle$, describing one electron propagating towards beamsplitter A and one towards B.

To clearly visualize the orbital entanglement, we present the state $|\Psi\rangle$ in a first quantization notation. Introducing $|1A, E\rangle|\uparrow\rangle$ for the operator $c_{1A}^{\uparrow\dagger}(E)$, the properly symmetrized wavefunction $|\Psi\rangle$ is given by

$$|\Psi\rangle = \int_{-eV}^{eV} dE \left(|1A, E\rangle|1B, -E\rangle + |1B, -E\rangle|1A, E\rangle + |2A, E\rangle|2B, -E\rangle + |2B, -E\rangle|2A, E\rangle\right)$$
$$\otimes \left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right). \tag{3}$$

Here the first and second ket correspond to the first and second particle respectively. We note that the total wavefunction is a direct product of an antisymmetric (the spin singlet state of the Cooperpair) spin wavefunction and a symmetric orbital wavefunctions. The orbital entanglement with respect to emission of the pair across barrier 1 or 2 can be even more explicitly seen by writing the wave function as

$$|\Psi\rangle = |\Psi_{12}\rangle \otimes |\Psi_{AB}\rangle, \quad |\Psi_{12}\rangle = \left(|11\rangle + |22\rangle\right)/\sqrt{2},$$
$$|\Psi_{AB}\rangle = \int_{-eV}^{eV} dE \left(|A, E\rangle|B, -E\rangle + |B, -E\rangle|A, E\rangle\right) \otimes \left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right) \tag{4}$$

The state $|\Psi_{12}\rangle$ is an orbitally entangled state with respect to emission across barrier 1 and 2. This two-dimensional space, called 12-space below, plays the role of a pseudo-spin space. The state $|\Psi_{AB}\rangle$ contains all additional information, such as energy and spin dependence.

Propagating from the superconductors towards the beamsplitters the electrons pick up phases $\varphi_{1A}, \varphi_{1B}, \varphi_{2A}$ and φ_{2B}, where the index $1A$ etc. denotes the arm of the normal conductor, see Fig. 1. These phases can be taken energy independent for the system parameters considered. The beamsplitters are assumed to have an adjustable transparency and to support only one propagating mode. Such beamsplitters have been realized in recent experiments [26, 27]. The states $|+A\rangle$ and $|-A\rangle$ ($|+B\rangle$ and $|-B\rangle$) for electrons going out into the normal reservoirs and the states $|1A\rangle$ and $|2A\rangle$ ($|1B\rangle$ and $|2B\rangle$) of the electrons incident on the beasmsplitters are related via energy independent scattering matrices:

$$\begin{bmatrix} |+A\rangle \\ |-A\rangle \end{bmatrix} = \begin{pmatrix} \cos\phi_A & -\sin\phi_A \\ \sin\phi_A & \cos\phi_A \end{pmatrix} \begin{bmatrix} |2A\rangle \\ |1A\rangle \end{bmatrix}, \qquad \begin{bmatrix} |+B\rangle \\ |-B\rangle \end{bmatrix} = \begin{pmatrix} \cos\phi_B & -\sin\phi_B \\ \sin\phi_B & \cos\phi_B \end{pmatrix} \begin{bmatrix} |2B\rangle \\ |1B\rangle \end{bmatrix} \tag{5}$$

Possible scattering phases of the beamsplitters are incorporated in the phases $\varphi_{j\eta}$, with $j = 1, 2$ and $\eta = A, B$. Under these conditions the beamsplitters only affect the state in 12-space, i.e. $|\Psi_{12}\rangle$. On the normal reservoir side of the beamsplitters, the state $|\Psi_{12}\rangle$ can be written

$$|\Psi_{12}\rangle = c_{++}|++\rangle + c_{--}|--\rangle + c_{-+}|-+\rangle + c_{+-}|+-\rangle \tag{6}$$

with the coefficients

$$c_{++} = \left[\sin(\phi_A)\sin(\phi_B)e^{i(\varphi_{1A}+\varphi_{1B})} + \cos(\phi_B)\cos(\phi_A)e^{i(\varphi_{2A}+\varphi_{2B})}\right]/\sqrt{2},$$
$$c_{--} = \left[\cos(\phi_A)\cos(\phi_B)e^{i(\varphi_{1A}+\varphi_{1B})} + \sin(\phi_B)\sin(\phi_A)e^{i(\varphi_{2A}+\varphi_{2B})}\right]/\sqrt{2},$$
$$c_{-+} = \left[\cos(\phi_A)\sin(\phi_B)e^{i(\varphi_{1A}+\varphi_{1B})} - \cos(\phi_B)\sin(\phi_A)e^{i(\varphi_{2A}+\varphi_{2B})}\right]/\sqrt{2},$$
$$c_{+-} = \left[\sin(\phi_A)\cos(\phi_B)e^{i(\varphi_{1A}+\varphi_{1B})} - \sin(\phi_B)\cos(\phi_A)e^{i(\varphi_{2A}+\varphi_{2B})}\right]/\sqrt{2}. \tag{7}$$

This provides a full description of the state of the particles exiting into the reservoirs.

3.   Detection via violation of a Bell Inequality

The entanglement gives rise to a nonlocal correlation between the two electrons, i.e. the outcome of a measurement at A (e.g. electron in A+) effects the probabilities for the outcome of the measurement in B (electron in B+ or B-). The implications of such non-local correlations have been at the heart of the discussion in quantum mechanics for almost a century [2, 3, 4]. Bell was the first to show [5] that the

predictions of quantum mechanics were incompatible with a whole class of theories, trying to explain the non-local correlations with local, realistic arguments in the spirit of Einstein, Podolsky and Rosen [2]. Bells proof was formulated in terms of an inequality which could not be violated by any system described by a local, realistic theory. Systems described by quantum mechanics could however violate such inequalities, showing that local realistic theories were untenable.

Several approaches to Bell Inequalities (BI) in mesoscopic conductors have been discussed [28, 29, 30, 20]. Here we follow the approach in Ref. [1], which is closely related to Bells original derivation. Starting from the two particle state emitted from the superconductors, using the formulation in Eq. (6), we can directly calculate the probabilities for joint detection of an electron in reservoirs αA and one in βB ($\alpha, \beta = +, -$), given by the simple relation

$$P_{\alpha\beta} = |c_{\alpha\beta}|^2 \tag{8}$$

This gives directly

$$
\begin{aligned}
P_{++} = P_{--} &= \left[\cos^2(\phi_A)\cos^2(\phi_B) + \sin^2(\phi_B)\sin^2(\phi_A)\right. \\
&+ \left. \cos(2[\varphi_A - \varphi_B])\cos(\phi_A)\sin(\phi_B)\cos(\phi_B)\sin(\phi_A)\right]/2 \\
P_{+-} = P_{-+} &= \left[\cos^2(\phi_A)\sin^2(\phi_B) + \cos^2(\phi_B)\sin^2(\phi_A)\right. \\
&- \left. \cos(2[\varphi_A - \varphi_B])\cos(\phi_A)\sin(\phi_B)\cos(\phi_B)\sin(\phi_A)\right]/2
\end{aligned}
\tag{9}
$$

where we have introduced the phases $2\varphi_A = \varphi_{1A} - \varphi_{2A}$ and $2\varphi_B = \varphi_{2B} - \varphi_{1B}$. The phases φ_A and φ_B can be varied independently and locally, at beamsplitters A and B, by e.g. electrostatic sidegates changing the length of the normal arms. Note that the joint detection probabilities are normalized such that $P_{++} + P_{--} + P_{+-} + P_{-+} = 1$. Following Bell and later formulations [31, 32], we introduce the correlation function

$$E(\mathbf{a}, \mathbf{b}) = P_{++}(\mathbf{a}, \mathbf{b}) - P_{+-}(\mathbf{a}, \mathbf{b}) - P_{-+}(\mathbf{a}, \mathbf{b}) + P_{--}(\mathbf{a}, \mathbf{b}). \tag{10}$$

where $\mathbf{a}$ (**b**) collectively denotes the settings of the phase ϕ_A and angle θ_A (phase ϕ_B and angle θ_B). In terms of these correlation functions for four different measurements settings, denoted $\mathbf{a}, \mathbf{a}', \mathbf{b}$ and $\mathbf{b}'$, the Bell Inequality can be formulated as

$$-2 \leq S_B \leq 2, \qquad S_B \equiv E(\mathbf{a}, \mathbf{b}) - E(\mathbf{a}, \mathbf{b}') + E(\mathbf{a}', \mathbf{b}) + E(\mathbf{a}', \mathbf{b}'). \tag{11}$$

where S_B is the Bell parameter. This provides us with a BI in terms of the joint detection probabilities $P_{\alpha\beta}$. However, in mesoscopic systems, in contrast to optics, these quantities are not experimentally accesible, they would demand a short-time coincidence measurement. The natural observables are instead zero-frequency current correlators,

$$S_{\alpha\beta} \equiv \int_{-\infty}^{\infty} dt \langle \{\delta\hat{I}_{\alpha A}(t)\delta\hat{I}_{\beta B}(0) + \delta\hat{I}_{\beta B}(0)\delta\hat{I}_{\alpha A}(t)\}\rangle \tag{12}$$

where $\delta\hat{I}_{\alpha\eta}(t) = \hat{I}_{\alpha\eta}(t) - \langle\hat{I}_{\alpha\eta}\rangle$ is the fluctuating part of the electrical current $\hat{I}_{\alpha\eta}(t)$ in reservoir $\alpha\eta$. The zero-frequency current correlators are calculated along the lines of scattering theory [33] and are given by [1]

$$
\begin{aligned}
S_{++} = S_{--} &= \frac{4e^2}{h}|eV|P_{++} = \frac{4e^2}{h}|eV|P_{--} \\
S_{+-} = S_{-+} &= \frac{4e^2}{h}|eV|P_{+-} = \frac{4e^2}{h}|eV|P_{-+}
\end{aligned}
\tag{13}
$$

where $P_{\alpha\beta}$ are just the joint detection probabilites of Eq. (9). Thus, the zero-frequency current correlators are directly proportional to the joint detection probabilites. The simple result can be understood by considering the properties of the time-dependent correlator $\langle\delta\hat{I}_{\alpha A}(t)\delta\hat{I}_{\beta B}(0)\rangle$. It decays as $(\tau_c/t)^2$ for times $t > \tau_c$, where $\tau_c = \hbar/eV$ is the correlation time of the emitted pair. In the tunneling limit under consideration, $\Gamma \ll 1$, the correlation time is thus much smaller than the average time between the arrival of two pairs $e/I \sim \hbar/eV\Gamma^2$. As a result, only the two electrons within a pair are correlated with each other, while

electrons in different pairs are completely uncorrelated. Thus, the zero frequency current correlator in Eq. (12) is just a coincidence counting measurement running over a long time, collecting statistics over a large number of pairs. This leads to the important result that the Bell inequality, Eq. (11), can be directly formulated in terms of the zero-frequency current correlators in Eq. (12), i.e. the correlation function of Eq. (10) is given by

$$E(\mathbf{a}, \mathbf{b}) = [S_{++}(\mathbf{a}, \mathbf{b}) - S_{+-}(\mathbf{a}, \mathbf{b}) - S_{-+}(\mathbf{a}, \mathbf{b}) + S_{--}(\mathbf{a}, \mathbf{b})]/P_0 \qquad (14)$$

where $P_0 = (4e^2/h)|eV|$ is the proportionality constant between the current correlator and the joint detection probability.

It is important to note that BI's in electrical conductors quite generally have to be investigated under conditions different from the ones in optics. In optics, the discussion has to large extent been focused on the detection problem, the fact that only a fraction of the photons emitted are actually detected [31, 34, 35, 36] and how this effects the interpretation of an experimental violation of a BI. This is in general not a problem in electrical conductors, all electrons are detected in the reservoirs and contribute to the statistics. There are however different aspects of BI's to be considered in electrical conductors. First, the entagled quantities are not free electrons but quasiparticle excitations out of the Fermi sea. Second, there are several "loopholes", due to possible interaction with phonons or other electrons or coupling to the electromagnetic environment. All these phenomena open up the possibility for local realistic theories that might explain the observed phenomena. In addition, more sophisticated local realistic theories allowing for some communication (possibly at the speed of light) between the detectors or the source and the detectors, the so called locality loophole, can hardly be ruled out in any mesoscopic experiment (but have in fact been done so experimentally [7] in optics).

With these limitations in mind, a violation of a BI in an electrical conductor would neverthess provide a first indication of the nonlocal properties of correlations between electrons. More importantly, the BI treated completely within the framework of quantum mechanics can be used as a tool to extract the amount of entanglement. This becomes clear in the discussion below of the effect of dephasing on entanglement. To investigate the parameter dependence of the correlation functions in Eq. (14) we insert the expressions for the zero frequency correlators in Eq. (13) and obtain

$$E(\mathbf{a}, \mathbf{b}) = \cos(2\phi_A)\cos(2\phi_B) + \cos(2[\varphi_A - \varphi_B])\sin(2\phi_A)\sin(2\phi_B). \qquad (15)$$

We note that there are two types of parameters that govern these correlation functions, the phases φ_A and φ_B and the angles ϕ_A and ϕ_B. Putting the angles ϕ_A and ϕ_B to $\pi/4$ (a semi-transparent beam-splitter) gives rise to the correlation function in terms of the phases φ_A and φ_B

$$E(\varphi_A, \varphi_B) = \cos(2[\varphi_A - \varphi_B]). \qquad (16)$$

Second, we can instead put the phases $2[\varphi_A - \varphi_B]$ to a multiple of 2π, giving a correlation function in terms of the beam-splitter angles

$$E(\phi_A, \phi_B) = \cos(2\phi_A)\cos(2\phi_B) + \sin(2\phi_A)\sin(2\phi_B) = \cos(2[\phi_A - \phi_B]). \qquad (17)$$

This has exactly the same form as the correlation function in Eq. (16). Inserting these correlation functions into the Bell parameter in Eq. (11), and choosing e.g. $\phi_A = \pi/8, \phi_B = \pi/4, \phi'_A = 3\pi/8$ and $\phi'_B = \pi/2$ (or equivalently the same values for the phases $\varphi_{j\eta}$) we get a Bell parameter $S = 2\sqrt{2}$. This gives a maximal violation of the BI in Eq. (11).

From this, one gets the impression that it is of no importance what parameters are adjusted. However, as investigated in detail below, the two approaches lead to very different results for the ability to violate the BI in the presence of dephasing.

4.   Effect of dephasing

There are many sources of dephasing in electrical conductors. Dephasing can arise due to direct interaction between different electrons or between electrons and phonons. Another source is coupling of the electrons to the electromagnetic environment. An investigation of dephasing in mesoscopic conductors is

given by Seelig, Pilgram and Büttiker in this issue. Focusing on systems with geometries and physical properties similar to the one in Fig. 1, we note that the effect of dephasing on the phase dependence of the conductance in Mach-Zehnder interferometers was recently studied in Refs. [37, 38]. Very recently, the effect of dephasing on the phase dependence of the conductance as well as the shot noise in Mach-Zender interferometers was investigated experimentally [39]. Subsequently, a theory for the effect of dephasing on the shot noise in the same geometry was proposed [40]. We also note that the effect of dephasing on the possibility to violate a BI has been studied early in optics [41].

In this paper we study a simple model of the dephasing that nevertheless illustrates the general effects of dephasing on the entanglement and the possibility to violate the BI. The dephasing is modelled phenomenologically via a density matrix

$$\rho = [|11\rangle\langle 11| + |22\rangle\langle 22| + \gamma(|11\rangle\langle 22| + |11\rangle\langle 22|)]/2 \tag{18}$$

which only affects the entanglement in 12-space. The dephasing parameter γ can vary from 1, when the system is in a pure, coherent, fully entangled state and no dephasing is present, to 0, when the system is in a completely mixed state, fully dephased and with the entanglement suppressed. We note that this density matrix would e.g. arise from fluctuating phases $\varphi_{j\eta}$, discussed in Refs. [37, 38, 40]. The density matrix in Eq. (18) gives rise to the correlation function, a modification of Eq. (15),

$$E(\mathbf{a}, \mathbf{b}) = \cos(2\phi_A)\cos(2\phi_B) + \gamma\cos(2[\varphi_A - \varphi_B])\sin(2\phi_A)\sin(2\phi_B). \tag{19}$$

From this expression it is clear that the two approaches described above to adjust the parameters of the correlation function give very different results when trying to violate the BI. Fixing the angles ϕ_A and ϕ_B, the Eq. (19) becomes

$$E(\varphi_A, \varphi_B) = \gamma\cos(2[\varphi_A - \varphi_B]) \tag{20}$$

Thus, the dephasing parameter γ takes on the role of what is known in the optical litterature as the visibility. For a visibility smaller than $1/\sqrt{2}$, the BI can not be violated, which thus puts the rather strict condition

$$\gamma > \frac{1}{\sqrt{2}} \tag{21}$$

on the amount of dephasing allowed. Choosing the other approach above, fixing the phases φ_A and φ_B, we find Eq. (19) modified to

$$E(\phi_A, \phi_B) = \cos(2\phi_A)\cos(2\phi_B) + \gamma\sin(2\phi_A)\sin(2\phi_B). \tag{22}$$

As was pointed out in Ref. [1], this allows in principle for a violation of the BI for arbitrary dephasing. In the next section, this is investigated in detail.

5. Optimal violation

We first try to find the angles which maximize the Bell parameter in Eq. (11). By maximizing the Bell parameter with respect to first ϕ_A and ϕ'_A and then to $\phi_B - \phi'_B$, we arrive at the relations for the angles, all in the first quadrant

$$
\begin{aligned}
\tan(2\phi_A) &= -\gamma\cot(\phi_S), \\
\tan(2\phi'_A) &= \gamma\tan(\phi_S), \\
\tan(\phi_B - \phi'_B) &= \text{sign}[\cos(2\phi_A)]\sqrt{\frac{\tan^2(\phi_S) + \gamma^2}{\gamma^2\tan^2(\phi_S) + 1}}
\end{aligned}
\tag{23}
$$

where $\phi_S = \phi_B + \phi'_B$ can be chosen at will. Inserting these angles into the Bell parameter, we arrive at
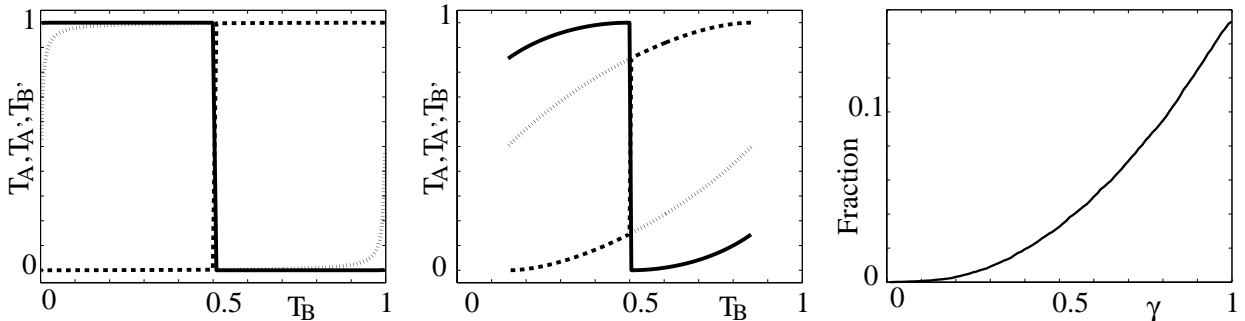
$$S_B = 2\sqrt{1 + \gamma^2} \tag{24}$$

Figure 2. Left and middle figures, from Ref. [1]: The transmission probabilities T_A (dashed), T_A' (dotted) and T_B' (solid) as a function of T_B [$T_\eta = \cos^2(\phi_\eta)$], giving optimal violation of the Bell inequalities for dephasing parameters $\gamma = 0.1$ (left) and $\gamma = 1$. (middle). Right figure: The fraction of the transmission probability space allowing for a violation, as a function of the dephasing parameter γ.

showing that for an optimal choice of the angles ϕ_A, ϕ_A', ϕ_B and ϕ_B', the BI can be violated for arbitrarily strong dephasing, $\gamma > 0$. The corresponding optimal transmission probabilities $T_\eta = \cos^2(\phi_\eta)$ are shown for $\gamma = 1$ and $\gamma = 0.1$ in Fig. 2. The BI can thus in principle be violated for any amount of dephasing. However it might be difficult to produce beam splitters which can reach all transmission probabilities between 0 and 1. This is not a serious problem in the absence of dephasing, $\gamma = 1$, a violation can be obtained for a large, order of unity, fraction of the "transmission probability space" (see Fig. 2). However, in the limit of strong dephasing, $\gamma \ll 1$, the set of transmission probabilities for optimal violation contains transmissions close to both 0 and 1, see Fig. 2. Expecting unity transmission to be most complicated to reach experimentally, we note that by instead choosing transmission probabilities $T_A = T_B = 0$, $T_B' = 1/2$ and $T_A' \ll \gamma$, the inequality in Eq. (11) becomes $2|1 + \gamma T_A'| \leq 2$. This gives a violation, although not maximal, for all $\gamma \ll 1$.

Apart from dephasing there are several other effects such as additional scattering phases, impurity scattering or asymmetric normal conductor arms that might influence the ability to violate the BI. These effects are discussed in Ref. [1]. Another possibility is that, due to asymmetries of the tunnel barriers $\Gamma_1 \neq \Gamma_2$, the amplitude for the process where the pair is emitted to $|11\rangle$ is different from the process where it is emitted to $|22\rangle$. This gives rise to a state, in 12-space,

$$|\Psi_{12}\rangle = \frac{1}{\sqrt{\Gamma_1^2 + \Gamma_2^2}} \left(\Gamma_1|11\rangle + \Gamma_2|22\rangle\right). \tag{25}$$

In this case [42, 43] we directly get the same expression for the coherence function in Eq. (19), with an effective

$$\gamma = \frac{2\Gamma_1\Gamma_2}{\Gamma_1^2 + \Gamma_2^2}. \tag{26}$$

This shows that it is in principle possible to violate BI for arbitrary asymmetry as well.

6. Conclusions

As a conclusion, we have investigated orbital entanglement in mesoscopic conductors with the focus on the effect of dephasing. The entanglement is detected via violation of a Bell Inequality formulated in terms of zero-frequency current correlations. We have investigated how the dephasing affects the possibility to violate the Bell Inequality and how system parameters can be adjusted for optimal violation.

Acknowledgements

References

[1] P. Samuelsson, E.V. Sukhorukov and M. Büttiker, Phys. Rev. Lett. (to appear), condmat/0303531.

[2] A. Einstein, B. Podolsky and N. Rosen, *Phys. Rev.*, **47**, (1935), 777.

[3] N. Bohr, *Phys. Rev.*, **48**, (1935), 696.

[4] D. Bohm and Y. Aharonov, *Phys. Rev.*, **108**, (1957), 1070.

[5] J.S. Bell, Physics **1**, (1964), 195; Rev. Mod. Phys. **38**, (1966) 447.

[6] W. Tittel, J. Brendel, H. Zbinden, and N. Gisin, *Phys. Rev. Lett.*, **81**, (1998), 3563.

[7] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter and A. Zeilinger, *Phys. Rev. Lett.*, **81**, (1998), 5039.

[8] A. Zeilinger, *Rev. Mod. Phys.*, **71**, (1999), S288.

[9] N. Gisin G. Ribordy, W. Tittel and H. Zbinden, *Rev. Mod. Phys.*, **74**, (2002), 145.

[10] A. Steane, *Rep. Prog. Phys.*, **61**, (1998), 117.

[11] P. Recher, E.V. Sukhorukov, and D. Loss, *Phys. Rev. B*, **63**, (2001), 165314.

[12] G.B. Lesovik, T. Martin and G. Blatter, *Eur. Phys. J. B,* **24**, (2001), 287.

[13] G. Burkard, D. Loss and E.V. Sukhorukov, *Phys. Rev. B*, **61**, (2000), 16303.

[14] W.D. Oliver, F. Yamaguchi and Y. Yamamoto, *Phys. Rev. Lett.*, **88**, (2002), 037901.

[15] P. Recher and D. Loss, *Phys. Rev. B*, **65**, (2002), 165327.

[16] C. Bena, S. Vishveshwara, L. Balents and M.P.A. Fisher, *Phys. Rev. Lett.*, **89**, (2002), 037901.

[17] F. Taddei and R. Fazio, *Phys. Rev. B*, **65**, (2002), 134522.

[18] J.C. Egues, G. Burkard, and D. Loss, *Phys. Rev. Lett.*, **89**, (2002), 176401.

[19] D.S. Saraga and D. Loss, *Phys. Rev. Lett.*, **90**, (2003), 166803.

[20] L. Faoro, and F. Taddei, R. Fazio, cond-mat/0306733.

[21] E. Prada and F. Sols, cond-mat/0307500.

[22] C.W.J. Beenakker, C. Emary, M. Kindermann, J.L. van Velsen, Phys. Rev. Lett. (to appear), cond-mat/0305110

[23] P. Samuelsson, E. Sukhorukov, M. Büttiker, cond-mat/0307473.

[24] V. Scarani, N. Gisin and S. Popescu, condmat/0307385.

[25] J.D. Franson, *Phys. Rev. Lett.,* **62**, (1989), 2205.

[26] R.C. Liu, B. Odom, Y. Yamamoto, and S. Tarucha, *Nature*, **391**, (1998), 263.

[27] W.D. Oliver, J. Kim, R.C. Liu, and Y. Yamamoto, *Science* **284**, (1999), 299.

[28] X. Maître, W.D. Oliver, and Y. Yamamoto, *Physica E*, **6**, (2000) 301.

[29] S. Kawabata, *J. Phys. Soc. Jpn.*, **70**, (2001), 1210.

[30] N.M. Chtchelkatchev, G. Blatter, G.B. Lesovik, and T. Martin, *Phys. Rev. B*, **66**, (2002), 161320.

[31] J.F. Clauser, M.A. Horne, A. Shimony, and R.A. Holt, *Phys. Rev. Lett.*, **23**, (1969), 880.

[32] A. Aspect, P. Grangier, and G. Roger, *Phys. Rev. Lett.*, **49**, (1982), 91.

[33] M. Büttiker, *Phys. Rev. B*, **46**, (1992), 12485.

[34] P.M. Pearle, *Phys. Rev. D*, **2**, (1970), 1418.

[35] J.F. Clauser and M.A. Horne, *Phys. Rev. B*, **10**, (1974), 526.

[36] N. Gisin and B. Gisin, *Phys. Lett. A*, **260**, (1999), 323.

[37] G. Seelig and M. Büttiker, *Phys. Rev. B*, **64**, (2001), 245313.

[38] G. Seelig, S. Pilgram, A.N. Jordan and M. Buttiker, *Phys. Rev. B*, (to appear), cond-mat/0304022.

[39] Y. Ji, Y. Chung, D. Sprinzak, M. Heiblum, D. Mahalu and H. Shtrikman, *Nature*, **422**, (2003), 415.

[40] F. Marquardt and C. Bruder, cond-mat/0306504.

[41] M.D. Reid and D.F. Walls, *Phys. Rev. A*, **34**, (1986), 1260.

[42] N. Gisin, *Phys. Lett. A*, **154**, (1991), 201.

[43] A. Abouraddy B.E.A. Saleh, A.V. Sergienko and M.C. Teich, *Phys. Rev. A*, **64**, (2001), 050101.