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Mesoscopic d-Wave Qubits: Can High-T$_c$ Cuprates Play a Role in Quantum Computing?

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Abstract

Due to nontrivial orbital pairing symmetry, surfaces and interfaces of high-T$_c$ superconductors support states which violate time-reversal ($\mathcal{T}$-) symmetry. Such naturally degenerate states, useful as working states of a qubit, are standard for atomic or molecular-size qubit prototypes (e.g. based on nuclear spins), but exceptional for mesoscopic qubits. (In particular, they hold promise of a better scalability.) In these lectures I review the physics of $\mathcal{T}$-breaking on surfaces and interfaces of high-T$_c$ superconductors; then describe existing proposals for high-T$_c$ based qubits and the current state of experiments; finally, I discuss the decoherence sources in the system, open questions, and future research directions.

Key Words: high-T$_c$, Josephson, $\mathcal{T}$-breaking, d-wave, qubit, decoherence, design, scalability.

A researcher, in deep concentration,
Performed quantum gate computation,
But Erwin, his cat,
Qubit him so bad,
That he lost most of his reputation.

1. Time-reversal symmetry breaking on surfaces and interfaces of high-T$_c$ superconductors

1.1. Description of transport in high-T$_c$ structures.

The lack of accepted microscopic theory of superconductivity in high-T$_c$ cuprates did not prevent successful research in this field since 1986. We have in several respects the repetition of the situation ca 1938, but with a clear advantage of already having BCS theory to provide insight and language for phenomenological treatment.

On this level, high-T$_c$ superconductors are successfully described by Gor'kov equations for normal and anomalous Green's functions[1], which in Matsubara representation are defined in the usual way:

$$\begin{aligned} G_{\alpha\beta}(\mathbf{k},\tau;\mathbf{k}',\tau') &= -\left\langle \mathcal{T}_\tau a_\alpha(\tau) a^\dagger_\beta(\tau')\right\rangle, \\ F_{\alpha\beta} = (\mathbf{k},\tau;\mathbf{k}',\tau') &= \left\langle \mathcal{T}_\tau a_{\mathbf{k}\alpha}(\tau) a_{-\mathbf{k}'\beta}(\tau')\right\rangle, \\ F^+_{\alpha\beta} = (\mathbf{k},\tau;\mathbf{k}',\tau') &= \left\langle \mathcal{T}_\tau a^+_{-\mathbf{k}\alpha}(\tau) a^\dagger_{\mathbf{k}'\beta}(\tau')\right\rangle. \end{aligned} \tag{1}$$

The only difference from conventional superconductivity is in the nontrivial symmetry of the pairing potential,

$$H_{\rm int} = \frac{1}{2}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V_{\alpha\beta,\lambda\mu}(\mathbf{k},\mathbf{k}')a^{\dagger}_{-\mathbf{k}+\mathbf{q}/2,\alpha}a^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\beta}a_{\mathbf{k}'+\mathbf{q}/2,\lambda}a_{-\mathbf{k}'+\mathbf{q}/2,\mu}. \tag{2}$$

As usual, we write equations of motion for each of these functions and use the "anomalous mean field" recipe to decouple the four-operator products, $\langle a^\dagger a^\dagger a a\rangle \to \langle a^\dagger a^\dagger\rangle\langle aa\rangle$. This brings out modified self-consistency relations for the order parameter,

$$\Delta_{\alpha\beta}(\mathbf{k},\mathbf{q}) = -\sum_{\mathbf{k}'} V_{\beta\alpha,\lambda\mu}(\mathbf{k},\mathbf{k}')F_{\lambda\mu}(\mathbf{k}'+\mathbf{q}/2,\tau;\mathbf{k}'-\mathbf{q}/2,\tau),$$
$$\Delta^{+}_{\lambda\mu}(\mathbf{k},\mathbf{q}) = -\sum_{\mathbf{k}'} V_{\alpha\beta,\mu\lambda}(\mathbf{k}',\mathbf{k})F^{+}_{\alpha\beta}(\mathbf{k}'-\mathbf{q}/2,\tau;\mathbf{k}'+\mathbf{q}/2,\tau), \tag{3}$$

and Gor'kov equations, which in the spatially uniform, stationary system read:

$$\begin{aligned}(i\omega_n-\xi_k)G_{\alpha\beta}(\mathbf{k},\omega_n)+\Delta_{\alpha\gamma}F^{+}_{\gamma\beta}(\mathbf{k},\omega_n)&=\delta_{\alpha\beta};\\ (i\omega_n+\xi_k)F^{+}_{\alpha\beta}(\mathbf{k},\omega_n)+\Delta^{+}_{\alpha\gamma}G_{\gamma\beta}(\mathbf{k},\omega_n)&=0;\\ (i\omega_n-\xi_k)F_{\alpha\beta}(\mathbf{k},\omega_n)-\Delta_{\alpha\gamma}G_{\beta\gamma}(-\mathbf{k},-\omega_n)&=0.\end{aligned} \tag{4}$$

Here ξ_k is the Fourier transform of the kinetic energy operator, $\hat{\xi}=(2m)^{-1}\hat{p}^2-\mu$, μ being the chemical potential.

It was established, that in high-T$_c$ cuprates, like YBCO, the order parameter is a spin singlet with d-wave orbital symmetry,

$$\Delta_{\alpha\beta}(\mathbf{k}) = \delta_{\alpha\beta}\Delta(\mathbf{k}),\;\; \Delta(\mathbf{k})\propto\cos^2(k_x)-\cos^2(k_y) \tag{5}$$

(with axes chosen along crystallographic directions (1,0,0), (0,1,0) in the cuprate layer)[11, 12, 13]. For the following, the most important consequence of this symmetry is the sign change of $\Delta(\mathbf{k})$ for certain directions. This means, first, that there exists an *intrinsic phase shift* of π between different directions in the crystal; second, that in certain (nodal) directions the order parameter is zero, and therefore the quasiparticle excitation spectrum is not gapped.

On the spatial scale exceeding the coherence length, ξ_0, it is more convenient to use the Eilenberger equations[2], which follow from (4) in the quasiclassical limit. This is certainly justified in high-T$_c$ cuprates with their small ξ_0.

The Eilenberger equations are conveniently written in matrix form,

$$\mathbf{v}_F\cdot\hat{G}(\omega_n)+[\omega_n\hat{\tau}_3+\hat{\Delta},\hat{G}(\omega_n)]=0. \tag{6}$$

Here the matrix Green's function and order parameter,

$$\hat{G}(\mathbf{v}_F,\mathbf{r};\omega_n)=\begin{pmatrix} g_{\omega_n}(\mathbf{v}_F,\mathbf{r}) & f_{\omega_n}(\mathbf{v}_F,\mathbf{r})\\ f^{+}_{\omega_n}(\mathbf{v}_F,\mathbf{r}) & g_{\omega_n}(\mathbf{v}_F,\mathbf{r})\end{pmatrix};\;\; \hat{\Delta}(\mathbf{v}_F,\mathbf{r};\omega_n)=\begin{pmatrix} 0 & \Delta(\mathbf{v}_F,\mathbf{r})\\ \Delta^{+}(\mathbf{v}_F,\mathbf{r}) & 0\end{pmatrix}, \tag{7}$$

depend both on position $\mathbf{r}$ and on (direction of) Fermi velocity $\mathbf{v}_F$. (The layered structure of high-T$_c$ cuprates allows us to reduce the problem to two dimensions to a good accuracy, unless we have to consider, for example, a twist junction[3], or tunneling in the c-direction.) The components of $\hat{G}$, obtained from Gor'kov's functions by integration over energies, satisfy the normalization condition, $g_{\omega_n}=\sqrt{1-f^{+}_{\omega_n}f_{\omega_n}}$, and the self-consistency relation (3) becomes

$$\Delta(\mathbf{v}_F,\mathbf{r}) = 2\pi N(0)T\sum_{\omega_n>0}\langle V(\mathbf{v}_F,\mathbf{v}'_F)f_{\omega_n}(\mathbf{v}_F,\mathbf{r})\rangle_\theta\,. \tag{8}$$

Here the angle averaging $\langle\rangle_\theta=\int_0^{2\pi}\frac{d\theta}{2\pi}$.

In a little different language, the same results are obtained using the Andreev approximation in the Bogoliubov-de Gennes equations for the components of the single-bogolon wave function, $(u(\mathbf{r}),v(\mathbf{r}))^\top$. The original Bogoliubov-de Gennes equations are obtained in the process of diagonalization of pairing BCS Hamiltonian [4]:

$$\begin{pmatrix} -\frac{1}{2m}\nabla^2-\mu & \Delta_k(\mathbf{r}) \\ \Delta_k^*(\mathbf{r}) & \frac{1}{2m}\nabla^2+\mu \end{pmatrix}\begin{pmatrix} u_k(\mathbf{r}) \\ v_k(\mathbf{r}) \end{pmatrix} \approx \begin{pmatrix} -\mathbf{v}_F\cdot\nabla & \Delta_k(\mathbf{r}) \\ \Delta_k^*(\mathbf{r}) & \mathbf{v}_F\cdot\nabla \end{pmatrix}\begin{pmatrix} u_k(\mathbf{r}) \\ v_k(\mathbf{r}) \end{pmatrix} = E_k\begin{pmatrix} u_k(\mathbf{r}) \\ v_k(\mathbf{r}) \end{pmatrix}. \tag{9}$$

Here the wave vector $\mathbf{k}$ labels the bogolon state, E_k is the excitation energy, and the self-consistency relation reads

$$\Delta_k^*(\mathbf{r},T) = \sum_{k'} V(\mathbf{k},\mathbf{k}')u_{k'}^*(\mathbf{r})v_{k'}(\mathbf{r})\tanh\frac{E_{k'}[\Delta_{k'}(\mathbf{r})]}{2T}. \tag{10}$$

Unlike the Gor'kov equations (4) (or the initial Bogoliubov-de Gennes equations), the equations (6,9) are of the first order in gradients, which allows us to introduce quasiclassical trajectories (characteristics) along $\mathbf{v}_F$ and solve the corresponding equations by integration along these trajectories, with proper boundary conditions.

It is known that a quasiparticle (electron or hole, described, in terms of Bogoliubov-de Gennes equations, by a vector $\vec{\psi}_k = (1,0)^\top\exp(i\mathbf{kr})$ $((0,1)^\top\exp(i\mathbf{kr}))$ respectively) impinging on the superconductor from the normal metal can undergo an *Andreev reflection*, switching the branch of the excitation spectrum, acquiring an additional phase, and almost exactly reversing the direction of its group velocity (this happens because an electron and a hole with the same momentum $\mathbf{k}$ have opposite group velocities):

$$\vec{\psi} \to \hat{\mathcal{R}}_A\cdot\vec{\psi}, \tag{11}$$

where

$$\hat{\mathcal{R}}_A = \begin{pmatrix} 0 & e^{-i\pi/2+i\chi} \\ e^{-i\pi/2-i\chi} & 0 \end{pmatrix}. \tag{12}$$

The phase χ is the phase of the superconducting order parameter; the $(-\pi/2)$-shift is exact in the limit when the quasiparticle energy is much less than the superconducting gap (generally it is some energy-dependent function $\delta(E)$).

Now consider a slab of normal conductor sandwiched between two superconductors (*SNS junction*). If we neglect the spatial dependence of the order parameter in superconductors, we don't need to solve the self-consistency equations (3,10). Therefore the problem reduces to a single-particle one and is most naturally solved in Bogoliubov-de Gennes language (Eq.(9) becomes a Schrödinger equation for a two-component wave function).

Solutions of this equation with the boundary condition (12) are standing waves. Obviously in the act of Andreev reflection a charge of $\pm 2e$ is transferred to the superconductor, therefore every standing wave (Andreev level) carries supercurrent.

Quasiclassically, in order to find Andreev levels in a normal layer of thickness L, sandwiched between "left" and "right" superconductors, with phase difference χ, we write the Bohr-Sommerfeld quantization condition,

$$\oint p(E)dq \pm \chi + \delta_l(E) + \delta_r(E) = 2\pi n. \tag{13}$$

Here the kinematic phase gain of the quasiparticle along the closed trajectory, $\oint p(E)dq = \int_l^r p_e(E)dq + \int_r^l p_h(E)dq = \int_l^r (p_e(E)-p_h(E))dq$, takes into account electron-hole (or vice versa) conversion.

The positions of levels, and therefore the supercurrent, depend on the phase difference χ between the superconducting banks, and we arrive at Josephson effect in SNS structures [5, 6, 7]. Actually, the language of Andreev levels can be successfully used to describe the Josephson effect in general (for a review see [8]).

1.2. π-junctions and time-reversal symmetry breaking

The crucial experiments([14, 15, 16]; see also review[13]) which confirmed d-wave pairing symmetry in high-T$_c$ cuprates were directed at catching the intrinsic phase π-shift. The general idea of the experiment follows from the fluxoid quantization condition in a superconductor: if a superconducting contour $\mathcal{C}$ is penetrated by the magnetic flux Φ, then [17]

$$2\pi\frac{\Phi}{\Phi_0} + \oint_{\mathcal{C}} d\mathbf{s}\cdot\nabla\phi = 2\pi n,\;\; \Phi_0 \equiv \frac{hc}{2e} \tag{14}$$

(in CGS units; $\Phi_0 \approx 2\cdot 10^{-15}$Wb in SI).

In the case of a massive superconducting ring the contour can be chosen well inside the superconductor, where there is no current and therefore no superconducting phase gradient. Then the magnetic flux is quantized in units of Φ_0.

If there is a Josephson junction in the ring, the phase change will concentrate there, yielding

$$2\pi\frac{\Phi}{\Phi_0} + \chi = 2\pi n. \tag{15}$$

Here χ is the phase difference across the junction.

The equilibrium value of χ is determined by the interplay between Josephson and magnetic energy of the system. The Josephson energy is related to the Josephson current via

$$I(\chi) = 2e\frac{\partial E(\chi)}{\partial\chi}, \tag{16}$$

and in the simplest case of a tunneling junction, $I(\chi) = I_c\sin\chi$, and $E(\chi) = -(I_c/2e)\cos\chi$. For a conventional Josephson junction $E(\chi)$ is at a minimum when $\chi = 0$; therefore in the absence of an external field the total energy of the ring,

$$U(\chi) = E(\chi) + E_{\mathrm{magn.}}(\Phi) = -\frac{I_c}{2e}\cos\chi + \left(\frac{\chi}{2\pi}\right)^2\frac{\Phi_0^2}{2L}, \tag{17}$$

has a single global minimum at $\chi = 0$ for any value of $2\pi LI_c/\Phi_0$ (here L is the self-inductance of the ring).

Not so if the pairing symmetry is d-wave: if we choose the configuration of the loop in such a way, that the opposite sign lobes contact across the junction (so called π-junction), the current-phase relation switches to $I(\chi) = I_c\sin(\chi+\pi) = -I_c\sin\chi$, and $E(\chi) = +(I_c/2e)\cos\chi$. Therefore if $2\pi LI_c/\Phi_0 > 1$, the system has two degenerate minima; if this ratio is so big that the magnetic energy can be neglected compared to the Josephson energy, the equilibrium phase difference is $\chi_0 = \pi$, and we obtain from (15), that

$$\Phi = \left(n + \frac{1}{2}\right)\Phi_0. \tag{18}$$

Thus in equilibrium there is a spontaneous flux $\Phi_0/2$ in the ring. Its direction (up or down) allows us to distinguish the two ground states of the system, where the *time-reversal symmetry is thus broken.*

Such behaviour was indeed observed in an experiment [16]: a tri-crystal ring of YBCO generated a half-flux quantum, which was detected by SQUID microscopy.

A natural question is whether π-junctions are the only possibility provided by d-wave symmetry, and whether only $\Phi_0/2$ fluxes can be spontaneously generated. The answer is no: in principle, any equilibrium phase difference can be realized in a d-wave junction, and time-reversal symmetry breaking can be accompanied by generation of an arbitrary magnetic flux, or none at all.

1.3. Josephson effect and T-breaking in SND and DND junctions

Let us return to an SNS junction (assuming a rectangular normal part, $L\times W$). Each quasiclassical trajectory connecting two superconductors acts as a conduit of supercurrent between them, an "Andreev tube" of diameter $\sim\lambda_F$, which carries supercurrent, determined by the phase difference between its ends. If we neglect the normal scattering at NS interfaces, there is no mixing between different trajectories, since Andreev reflections simply reverse the velocity, sending the reflected particle along the same path. It can be shown, that if $L \gg \xi_0$, every such trajectory passing through the point $\mathbf{r}$ contributes a partial supercurrent density[4, 9]

$$\mathbf{j}(\mathbf{r}, \mathbf{v}_F) = \frac{2e\mathbf{v}_F}{\lambda_F W} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\mathcal{L}(\mathbf{v}_F)}{l_T} \frac{\sin(n(\phi_1 - \phi_2))}{\sinh(n\mathcal{L}(\mathbf{v}_F)/l_T)} e^{-2n\mathcal{L}(\mathbf{v}_F)/v_F\tau_i}. \tag{19}$$

In Eq.(19), $\mathcal{L}(\mathbf{v}_F)$ is the length of the trajectory, and $l_T = v_F/2\pi k_B T$ is the so called normal metal coherence length. (We also included effects of weak impurity scattering with scattering time τ_i.) The physical meaning of l_T is that in a clean normal metal an electron and Andreev-reflected hole (or vice versa) with energy $k_B T$ maintain phase coherence across distance l_T simply because they travel along the same trajectory. Indeed, the momentum of an electron (hole) with energy $k_B T \ll E_F$ is $p_{e,h} \approx p_F \pm (\partial_E p_F) k_B T = p_F \pm k_B T/v_F$. At a distance l from the point of Andreev reflection (measured along the trajectory) they would gain phase difference $(p_e - p_h)l = 2k_B T l/v_F$. If the phase difference is of order π, the coherence is effectively lost, so we get $l_{T,\text{ballistic}} \sim v_F/k_B T$. The factor of $(2\pi)^{-1}$ appears in accurate treatment, like in Eq.(19).

As an aside, in case of very strong scattering in the normal part of the system, when the motion of electrons/holes is diffusive with diffusion coefficient D, we can still use the same argument. Now the observable length scale is given by the displacement of quasiparticle, $l^2 = Dt$, while the phase difference between the electron and the hole is gained along the crooked path of length $l' = v_F t = v_F l^2/D$ they take. So the condition $2k_B T l'/v_F \sim \pi$ yields $l_{T,\text{diffusive}} \sim \sqrt{D/k_B T}$.

Such coherence in a normal metal is a purely *kinematic* effect, since there is no interaction in the normal (non-magnetic) metal, which would either support or suppress superconductivity. ("Normal metal is neutral with respect to superconducting correlations."-C.W.J. Beenakker.) Nevertheless its effects are quite real, e.g. the very possibility of coherent supercurrent flow (Josephson effect) in SNS junctions.

Let us apply the approach of (19) to calculation of Josephson current in a clean SNS contact. In every point of NS boundary (e.g. $x = L$), we integrate (19) over directions of $\mathbf{v}_F$ (such that $v_{Fx} > 0$). As it should be expected, the result reproduces Ishii's sawtooth [6]: in the limit of zero temperature and no scattering inside the normal layer ($\chi \equiv \phi_1 - \phi_2$),

$$I(\chi) = \int dy \sum_{v_{Fx}>0} j_x(y, \mathbf{v}_F) \propto \frac{2}{\pi} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin(n\chi)}{n} = \frac{\chi}{\pi}, \;\; |\chi| < \pi \tag{20}$$

(periodically extended).

Consider now an SND junction, that is, an SNS contact, where one of the superconductors has d-wave symmetry [10, 4]. Now, when integrating over the directions of $\mathbf{v}_F$ we encounter two kinds of trajectories: "zero"- and "π"-trajectories, which link the conventional superconductor with the lobes of the d-wave order parameter of opposite sign (intrinsic phase π-shift). The Josephson current is therefore a sum of two contributions like (20), one of which has the phase argument $\chi + \pi$. The relative weight of these contributions depends on the orientation of the SD boundary with respect to the crystal axes of the cuprate (to which the order parameter is nailed). In the case of a 45° orientation, when the boundary is along the (110) plane and is therefore normal to the nodal direction of the d-wave order parameter, the two groups contribute equally, yielding ($\cos\theta = v_{Fx}/v_F$)

$$I(\chi) = \sum_{v_{Fx}>0}^{\text{zero levels}} \frac{ev_F \cos\theta}{L} \frac{2}{\pi} \sum_{n=1}^{\infty} (-1)^{n+1} e^{-2nL/l_i \cos\theta} \frac{L}{l_T \cos\theta} \frac{\sin(n\chi) + \sin(n(\chi + \pi))}{\sinh(nL/l_T \cos\theta)}. \tag{21}$$

All odd harmonics cancel, and we obtain a π-periodic sawtooth: at $T = 0$, $l_i \to \infty$

$$I(\chi) \propto \frac{\chi - \pi/2}{\pi/2}, \;\; 0 < \chi < \pi, \tag{22}$$

(periodically extended). (This is simply a sum of two identical, 2π-periodic sawtooth functions, shifted by π, see Fig.2.) There are two stable equilibrium phase differences across the junction: $\chi = \pm\pi/2$. This means, that the time-reversal symmetry is broken. (In addition, the frequency of ac Josephson effect in the system will be doubled [10, 20]).

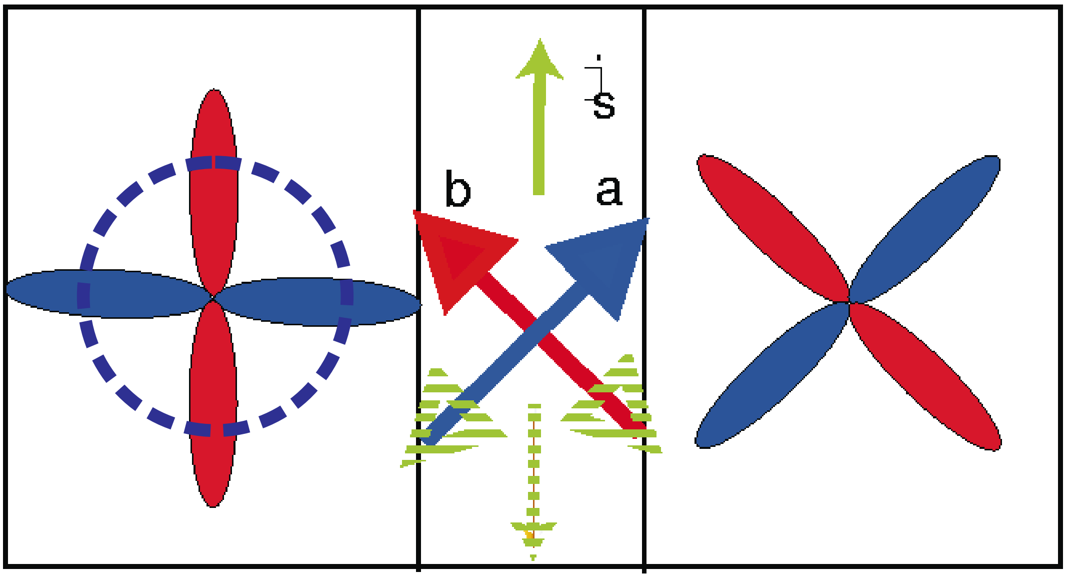

Figure 1. Josephson and spontaneous currents in SND and DND junctions. Arrows a and b indicate Andreev zero- and π-levels. In equilibrium their contributions to the Josephson current (normal to the NS boundary) cancel. By switching current directions (dashed arrows) we come to the other ground state, with the opposite direction of the spontaneous current, j_S.

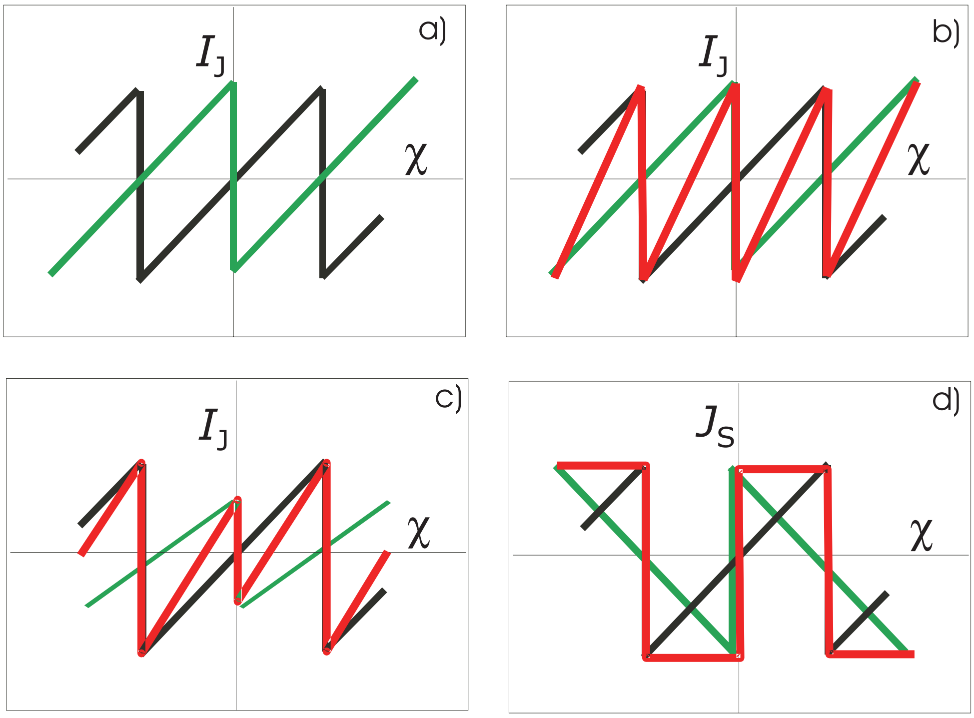

Figure 2. Josephson and spontaneous currents in SND and DND junctions. a)Contributions of zero- and π-levels to the Josephson current in a 0-45° DD(SD) junction. b)Resulting current-phase dependence. c) Same in more general case. d) Spontaneous current in a 0-45° junction.

The SND junction may seem simply a "superposition" of two SNS junctions, with phase differences shifted by π, but the situation is more interesting. In equilibrium, spontaneous currents flow in the normal layer, parallel to the boundary[18]. This is clear from our "Andreev tube" treatment. Take, for example, two trajectories, with $\theta = +\alpha$ and $\theta = -\alpha$, carrying equal partial currents. Their contribution to the Josephson current (normal to the boundary) is zero, since their currents' projections on this direction cancel each other. On the contrary, the projections on the direction, parallel to the boundary, add. The phase dependence of this current, $I_s(\chi)$, can be obtained in the same way as for the Josephson current. In the limit of zero temperature and clean normal layer it reduces to a *difference* of two sawtooth functions, yielding

$$I_s(\chi) \propto \mathrm{sgn}\chi, \;\; |\chi| < \pi, \tag{23}$$

again periodically extended. States with $\pi/2$ and $-\pi/2$ thus carry spontaneous currents in opposite directions. If an SND junction is closed on itself (annular geometry), these currents translate into spontaneous magnetic fluxes, normal to the plane of the system [19].

Note that all of the above is only possible if higher harmonics of current-phase dependence are not all zero, since all the odd harmonics (including the standard Josephson term, $\sin\chi$) cancel. (The expected "total depairing" in the (110) plane.)

In the case of arbitrary orientation of the ND plane, the weights of zero- and π-sawtooth functions will be different (Fig.2c). Then the current-phase dependence regains 2π-periodicity. At zero temperature and in the absence of scattering, the T-breaking is generally still present. The equilibrium phase difference is no longer $\pm\pi/2$ and depends on the geometry. (The exception is when the boundary is along (100) or (010). Then the contribution of only one group survives, and we have either standard, or π-junction. Of course, it is impossible to tell, whether a single junction is a π-junction - it is necessary to look at the contour, in which it is included[12].) Finite temperature and scattering suppress higher harmonics first, and therefore the transition to non-T-breaking state is possible [18].

Essentially the same interplay of zero- and π-levels takes place in DND junctions. There we have a somewhat richer picture. For example, the time-reversal symmetry can be broken without producing spontaneous currents (Fig.3c).

One of the reliable methods of fabrication of Josephson devices in high-T_c cuprates is based on forming grain boundary junctions (see e.g. [25]). The order parameter is suppressed within $\sim \xi_0$ around the boundary, and this region can be considered "normal". Therefore the SND/DND model applies, but only qualitatively, since the above equations are derived in the limit $L \gg \xi_0$. A more accurate approach, based on Eilenberger equations (6), confirms the qualitative similarity between DND and DD junctions[21].

Integrating the current-phase dependences of Fig.2, we obtain the Josephson energy of the junction, Eq.(16), which has two minima, corresponding to its degenerate ground states. This bistability plays the crucial role in qubit applications of high-T_c superconductors. This prediction was confirmed in grain boundary YBCO junctions[26] (Fig.6).

So far we did not take into account normal scattering on NS interfaces, and therefore missed an important point. Let us first consider a boundary of a d-wave superconductor with a vacuum or an insulator. The order parameter near the surface is suppressed, and a qualitative understanding of the situation can be obtained by "inserting" a normal layer of thickness $\sim \xi_0$ between the insulator and the bulk superconductor, similar to the DND model[27]. Energy levels in such a layer can be found from (13) for every quasiparticle trajectory (assuming specular reflection), Fig.7a:

$$\oint p(E)dq + 2\delta(E) + \pi s = 2\pi n. \tag{24}$$

Here $s = 0$ or $s = 1$ depending on whether the trajectory connects the lobes of the same or opposite sign. Note that $\oint p(0)dq = 0$ and $2\delta(0) = \pi$. Therefore solution $E = 0$ exists if and only if $s = 1$. This is the zero-energy, or midgap, state (ZES, MGS), which obviously cannot exist in conventional superconductors, where s always equalis zero (see review [27]).

Now consider a DD junction with finite transparency, $\mathcal{D} < 1$ (instead of the case of ideal transparency, $\mathcal{D} = 1$, which we dealt with earlier). We can use the DND model, inserting in the middle of the normal layer an infinitely thin barrier with transparency $\mathcal{D}(\theta)$, dependent on the incidence angle of the quasiparticle trajectory[27].

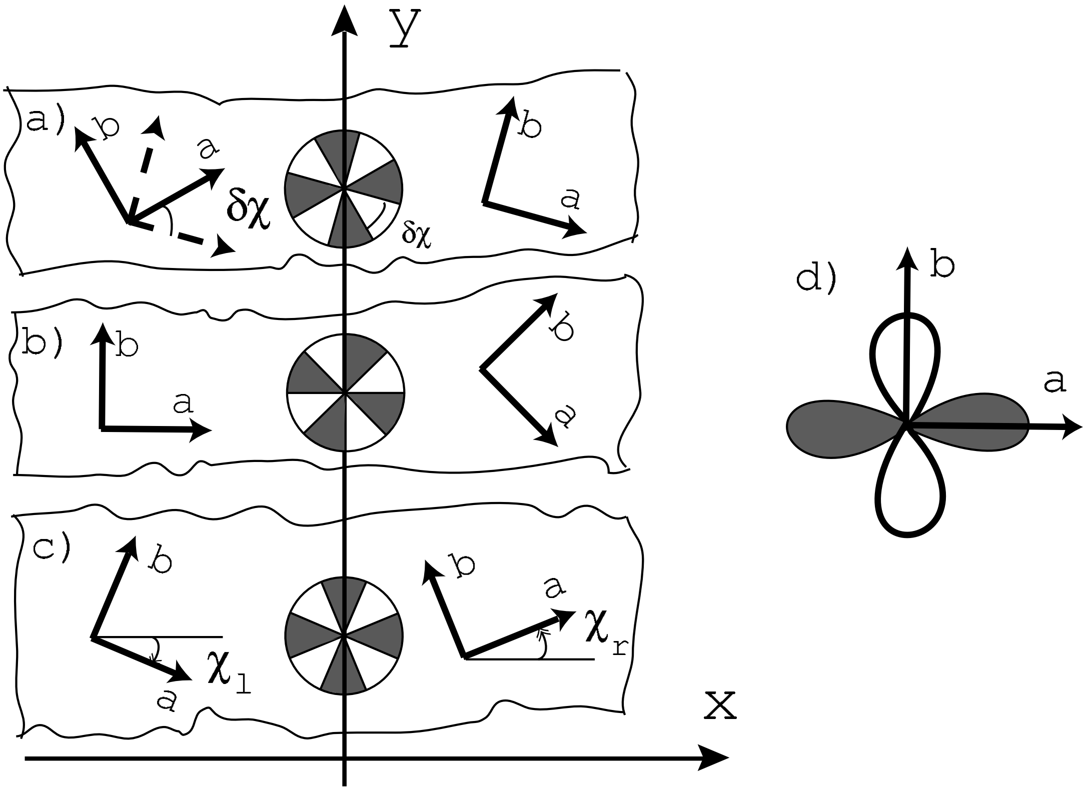

Figure 3. Josephson and spontaneous currents in DND junctions [21, 22]. a) Arbitrary case. b) Asymmetric (0-45°) junction. c) Symmetric (22.5°-22.5°) junction. Directions corresponding to zero-levels are shaded. In the case c) the contributions of zero- and π-levels to the current in y-direction are identically zero; therefore the spontaneous current is absent, but the time-reversal symmetry is still broken. d) Orientation of the d-wave order parameter with respect to the crystalline axes.

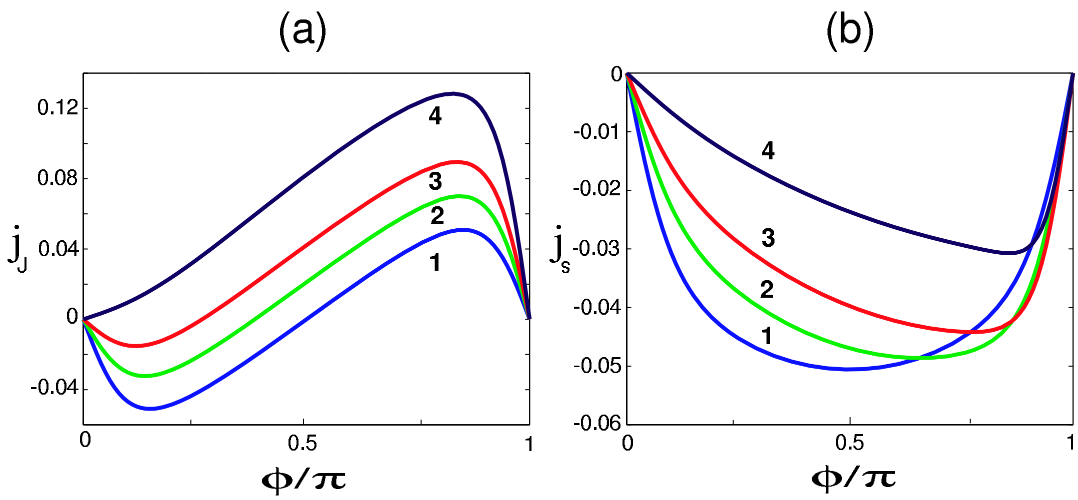

Figure 4. Josephson current (a) and spontaneous current (b) versus the phase difference in a clean DD grain boundary junction calculated in the non-self-consistent approximation. Temperature is $T = 0.1T_c$. The mismatch angles are $\chi_l = 0$ and $\chi_r = 45°$ (1), 40° (2), 34° (3), 22.5° (4) [22].

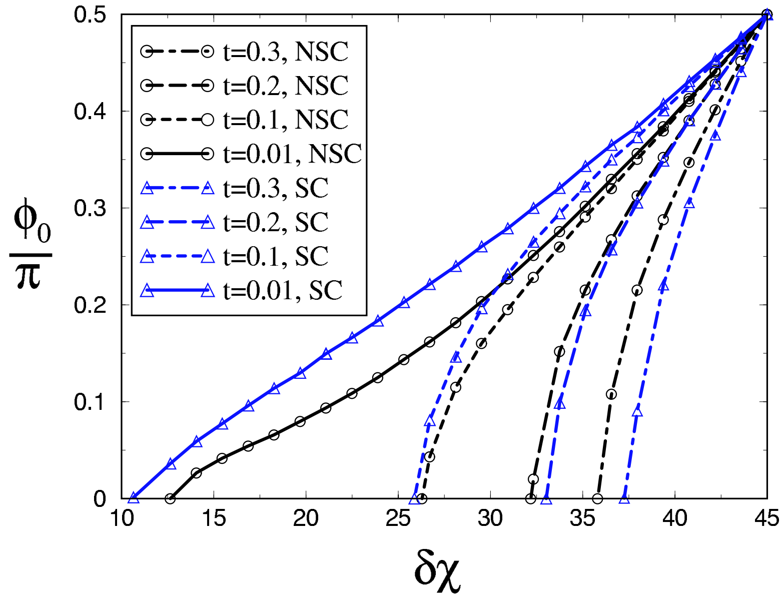

Figure 5. Equilibrium phase difference ϕ_0 in a clean grain boundary junction as a function of the mismatch angle: $\chi_l = 0$; $\chi_r = \delta\chi$ (Fig. 3). Parameter $t = T/T_c$ is the dimensionless temperature. Triangles and circles correspond to self-consistent and non-self-consistent solutions of Eilenberger equations, respectively. The 0-45° junction is a $\pi/2$-junction at any temperature, since by symmetry all odd components of the Josephson current are cancelled. Compare this to the behaviour observed in YBCO junctions (Fig.6).

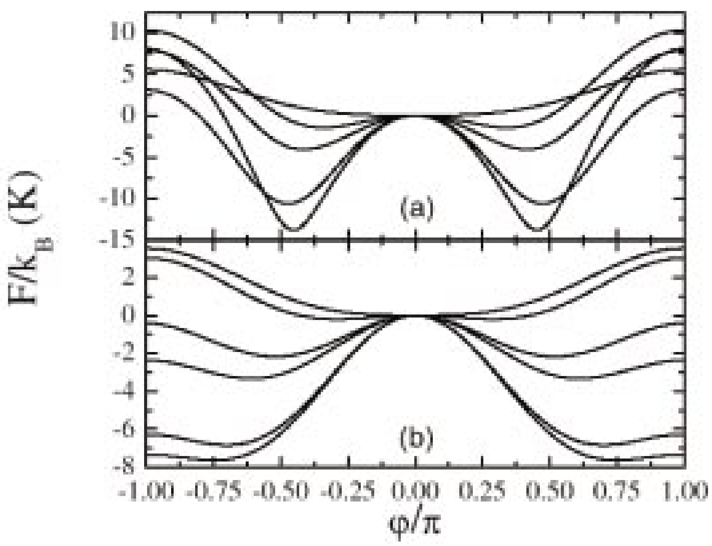

Figure 6. Bistability in a grain boundary YBCO junction[26]. Free energy, restored from Josephson current-phase dependence, is plotted vs. phase for (a) 0-45° and (b) symmetric (22.5°-22.5°) junction. Curves (top to bottom) correspond to temperatures (a) 30, 20, 15, 10, and 4.2 K; (b) 20, 15, 11, 10, 5, and 1.6 K, respectively.

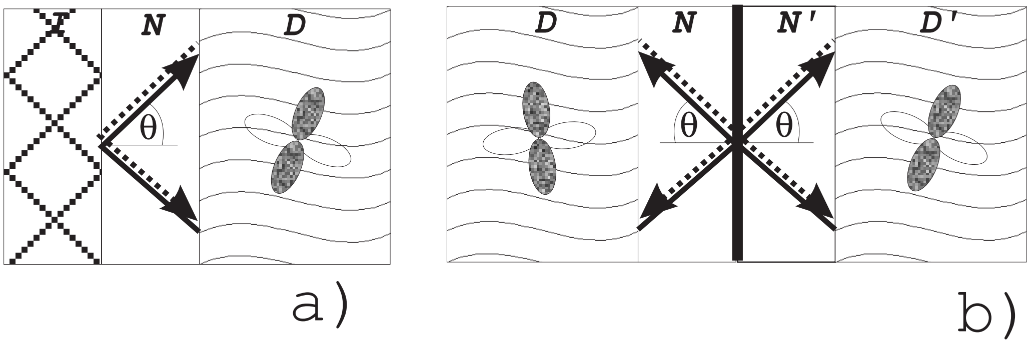

Figure 7. Surface bound state in a d-wave superconductor (a) and model of a DD junction with finite transparency (b) (after [27]) .

Bohr-Sommerfeld quantization conditions for quasiparticle trajectories shown in Fig.7b yield the energy levels of the bound states in the junction. In the limit of low transparency, $\mathcal{D} \ll 1$, the critical current is much larger ($O(\sqrt{\mathcal{D}})$, not $O(\mathcal{D})$), if the orientations of the d-wave order parameter allow formation of zero energy states on both sides of the barrier.

Even when transparency is not small, the presence of ZES may be qualitatively important. For example, the junction can become a π-junction at low enough temperature (see review [27]); you can see such behaviour in Fig.6b, where below 11 K the potential wells are centered around π rather than zero.

The appearance of spontaneous currents in SD and DD junctions can be considered as due to a time-reversal symmetry breaking order parameter with $s + e^{i\chi_0}d$ or $d + e^{i\chi_0}d'$ symmetry, which is formed in the junction area due to the proximity effect (here $\chi_0 \neq 0, \pi$ is the equilibrium phase difference across such junction). In certain conditions, such combinations could appear near the surface of a d-wave superconductor, leading to spontaneous currents and magnetic moments. So far there is no conclusive evidence for such currents (see [23] and references therein).

2. Mesosopic d-wave qubits

2.1. Flux qubits with conventional superconductors

Qubits are the basic building blocks of future quantum computers. Essentially they are two-state quantum systems which can be put in an arbitrary superposition of states ("initialized"), coupled to each other, undergo desired quantum evolution and measured ("read out") before losing quantum coherence. Here we concentrate on *superconducting phase qubits*.

The simplest example of such a qubit is an RF SQUID, that is, a loop with a single Josephson junction (like in Section 1.2). We saw, that in certain circumstances the system has two degenerate minima, corresponding to a flux of $\pm\Phi_0/2$ through the loop.

The Hamiltonian of such a system is

$$H = U_J(\chi) + U_C(\hat{Q}), \tag{25}$$

where U_C is the Coulomb energy of charge Q on the junction (which has some finite capacitance C). The charge operator $\hat{Q}$ can be expressed in terms of the phase difference across the junction as $\hat{Q} = -i\partial_\chi$ (see e.g. [17, 4]). Due to the presence of the Coulomb term, phase is no longer a "good" quantum variable, and the eigenstates of the Hamiltonian (25) become linear combinations of "up" and "down", or "left" and "right", states (with spontaneous flux $\pm\Phi_0/2$). In other words, the qubit can now tunnel between the wells of the Josephson potential, corresponding to certain phases (Fig.6).[1]

[1]This description is appropriate as long as the Coulomb energy does not exceed the Josephson energy, otherwise the natural starting point would be the states with definite *charge* on the junction; we do not consider such systems (*charge* qubits) here,

Coherent quantum tunneling was indeed observed in an RF SQUID[29]. Simultaneously, this effect was obtained in a different system, consisting of a small inductance loop with *three* Josephson junctions (the so called persistent current qubit, [30]).

The advantage of the latter design is as follows. As we have seen in Section 1.2, the degenerate states appear in the RF SQUID only if the self-inductance of the loop is large enough, and they carry comparatively large spontaneous fluxes, $\pm\Phi_0/2$. Therefore they will couple to the external degrees of freedom and reduce the time τ_d during which the system maintain its quantum coherence. Even more important is the fact that the resulting potential barrier is comparatively high, which may prohibit the tunneling we are after. In an experiment [29] the coherent tunneling was indeed observed not between the lowest, but between the excited states in the wells.

In case of the three-junction loop of negligible self-inductance, the fluxoid quantization condition (15) leaves two independent Josephson phase differences in the circuit:

$$\chi_1 + \chi_2 + \chi_2 + 2\pi \frac{\Phi_{\text{external}}}{\Phi_0} = 2\pi n. \tag{26}$$

The Josephson energy,

$$U_J = -E_{J1}\cos(\chi_1) - E_{J2}\cos(\chi_2) - E_{J3}\cos(\chi_3) \tag{27}$$

of the system forms a 2D potential profile (e.g. as a function of χ_1, χ_2), which depends on the external flux $\Phi_{\text{external}} \equiv f \times \Phi_0$ as a parameter.

If $f = 0.5$, and the E_J's are comparable, this potential has two degenerate minima; unlike the case of the RF SQUID, the potential barrier can be small, as are the spontaneous fluxes corresponding to the "left" and "right" states.

2.2. Rationale and proposed designs for qubits with d-wave superconductors

One of the main problems with the designs of the previous section is the necessity to artificially break the T-symmetry of the system by putting a half flux quantum through it. Estimates show [31] that the required relative accuracy is $10^{-5} - 10^{-6}$. The micron-size qubits must be positioned close enough to each other to make possible their coupling; the dispersion of qubit parameters means that applied fields must be locally calibrated; this is a formidable task given such sources of field fluctuations as fields generated by persistent currents in qubits themselves, which depend on the state of the qubit; field creep in the shielding; captured fluxes and magnetic impurities. Moreover, the circuitry which produces and tunes the bias fields is an additional source of decoherence in the system.

These problems are avoided if the qubit is *intrinsically bistable.* The most straightforward way to achieve this is to substitute the external flux by a static *phase shifter*, a Josephson junction with unconventional superconductors with nonzero equilibrium phase shift χ_0. For example, three-junction (persistent current, Mooij) qubits would require an extra π-junction ($\chi_0 = \pi$) [31]. The only difference compared to the case of an external magnetic field bias is in the prevalent decoherence sources: instead of noise from field-generating circuits we will have to take into account intrinsic decoherence from nodal quasiparticles and interface bound states (see below).

Another suggestion [32] is based on the same tricrystal high-T$_c$ ring geometry as in experiments[16, 13] (Fig. 8). The spontaneous flux $\pm\Phi_0$ generated by such a structure labels the qubit states $|0\rangle$, $|1\rangle$. Tunnelling between them, necessary for quantum operations, is made possible by applying a magnetic field in the plane of the system. Indeed, then the states $|0\rangle$, $|1\rangle$ are no longer the eigenstates of the Hamiltonian.

The tricrystal ring D is surrounded by an s-wave superconductor ring S, aimed at screening the spontaneous flux from the environment (including other qubits). Indeed, due to the fluxoid quantization condition (14) in the ring S, the total flux through it must be an *integer* multiple of Φ_0, and therefore the states of the rings D and S become entangled (e.g. $\alpha|0\rangle_D \bigotimes |1\rangle_S + \beta|1\rangle_D \bigotimes |0\rangle_S$).

This entanglement puts forward an interesting problem. In order to perform two-qubit operations, as well as initialization and readout, it is necessary to make the qubits "visible" to each other and the outside world. To do this, it is suggested in [32] to locally destroy superconductivity in the screening ring by using

but they were successfully implemented experimentally[28].

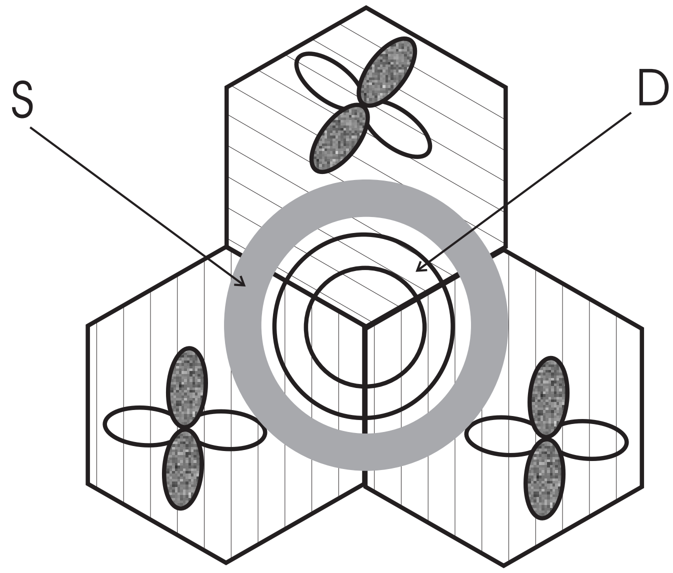

Figure 8. A tricrystal high-T$_c$ qubit [32]. The ring D formed of high-T$_c$ film contains one π-junction and therefore supports a spontaneous flux $\pm\Phi_0/2$[16]. The ring S of conventional superconductor screens this flux from the environment.

a superconducting field effect transistor (SUFET) (not shown) (i.e. by applying a gate voltage to the part of the screening ring). Will this transition collapse the wave function of the qubit, destroying quantum coherence between $|0\rangle_D$ and $|1\rangle_D$?

Now let us consider the case when the bistable d-wave system is employed *dynamically*, that is, when its phase is allowed to tunnel between the degenerate values. In a so called "quiet" qubit [33] an SDS' junction (effectively two SD junctions in the (110) direction) put in a small-inductance SQUID loop in parallel with a conventional Josephson junction and a large capacitor, Fig.9. One of the SD junctions plays the role of a $\pi/2$-phase shifter. The other junction's capacitance C is small enough to make possible tunneling between the $\pi/2$ and $-\pi/2$-states due to the Coulomb energy $Q^2/2C$. Two consecutive SD junctions are effectively a single junction with equilibrium phases 0 and π (which are chosen as working states of the qubit, $|0\rangle$ and $|1\rangle$). The proposed control mechanisms are based on switches $c,\ s$. Switch c connects the small S'D junction to a large capacitor, thus suppressing the tunneling. Connecting s for the duration Δt creates an energy difference ΔE between $|0\rangle$ and $|1\rangle$, because in the latter case we have a frustrated SQUID with 0- and π- junctions, which generates $\Phi_0/2$ spontaneous flux. This is a generalization of applying a σ_z operation to the qubit. Finally, if switch c is open, the phase of the small junction can tunnel between 0 and π. Entanglement between different qubits is realized by connecting them through another Josephson junction in a bigger SQUID loop.[2]

Due to the absence of currents through the loop during tunneling between $|0\rangle$ and $|1\rangle$ the authors called it "quiet", though as we have seen small currents and fluxes are still generated near SD boundaries.

Another design based on the same bistability [24, 34] only requires one SD or DD boundary (Fig.10). Here a small island contacts a massive superconductor, and the angle between the orientation of the d-wave order parameter and the direction of the boundary can be arbitrary (as long as it is compatible with bistability [21, 22], Fig.5). The advantage of such a design is, that the potential barrier can be to a certain extent controlled and suppressed; moreover, in general the two "working" minima ($-\phi_0,\ \phi_0$; the phase of the bulk superconductor across the boundary is zero) will be separated from each other by a smaller barrier,

[2] The switches in question are in no way trivial, since they must operate without destroying quantum coherence of the system. One possible solution is to use a frustrated dc-SQUID[33], that is, insert in the wire a small inductance loop with two equivalent Josephson junctions in parallel. The total supercurrent through the switch, $I = I_1(\chi_1) + I_2(\chi_2)$, goes to zero if the phase difference (tunable by external magnetic flux) $\chi_1 - \chi_2 = \pi$. Modifications of this scheme are discussed in [31]. Another possibility is to use superconducting single-electron transistors (SSETs, "parity keys")[24].

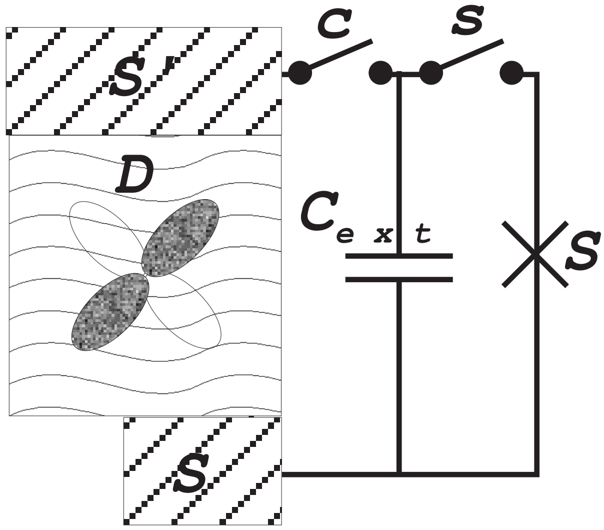

Figure 9. "Quiet" SDS' qubit (after [33]). Switches c and s connect the SDS' structure to a conventional Josephson junction, S, and a large capacitor, C_{ext}.

than from the equivalent states differing by $2\pi n$. This allows us to disregard the "leakage" of the qubit state from the working space spanned by ($|0\rangle$,$|1\rangle$), which cannot be done in the "quiet" design with the exact π-periodicity of the potential profile.

Qubit operations in this system are realized by connecting qubits to each other and to the ground electrode (normal or superconducting) through SSETs (or other kind of switches). When isolated, a qubit undergoes natural evolution between $|0\rangle$ (state with phase $-\phi_0$) and $|1\rangle$ (phase ϕ_0), which realizes the σ_x operation. The σ_z operation (that is, adding a controllable phase shift to one of the states with respect to the other) can be realized by e.g. connecting the island through a SSET to the massive superconductor ("bus"), the phase of which $\chi \neq 0$. The same operation repeated periodically can be used to block unwanted tunneling (so called "bang-bang" technique) [34]: physically, if we keep shifting the levels in the right and left well with respect to each other, the tunneling is suppressed, since they are practically never in resonance.

A different design was suggested in [38] ("silent qubit"). Here two small bistable d-wave grain boundary junctions with a small superconducting island between them are set in a SQUID loop (Fig. 11). As usual, "small" means that the total capacitance of the system allows phase tunneling: the Coulomb energy term in (25) is not negligible.

In the limit of negligible self-inductance of the loop, the quantization condition (14) fixes the sum of the phases to the external flux, $\chi_1 + \chi_2 \equiv \phi = 2\pi\Phi/\Phi_0$. This leaves only one independent phase combination, the superconducting phase of the island, $\theta = (\chi_1 - \chi_2)/2$.

Keeping for simplicity only the first two harmonics in the current-phase relation of the junctions,

$$I_i = I_i^{(I)} \sin \chi_1 - I_i^{(II)} \sin 2\chi \equiv \\ \equiv I_{0i} \left[\sin \chi_i \sin \gamma_i - \sin 2\chi_i \cos \gamma_i \right], \tag{28}$$

we find for the Josephson potential of the qubit the expression (Fig. 12)

$$U_J(\theta, \phi) = -(I_{01}/2e) \left[f(\phi/2 + \theta, \gamma_1) + \eta f(\phi_2 - \theta, \gamma_2) \right]. \tag{29}$$

Here $f(\varphi, \gamma) = \cos(\varphi)\sin(\gamma) - (1/2)cos(2\varphi)\cos(\gamma)$, and $\eta = I_{02}/I_{01}$. Parameter $\gamma \in [0, \pi/2]$ provides a convenient parametrization for the current-phase dependence in a DD junction.

In the absence of the external flux ($\phi = 0$) the qubit potential $U_J(\theta, 0)$ has two degenerate minima. Moreover, if the junctions only differ in the amplitude of critical current, but have the same γ, there is no spontaneous current in the loop in either minimum, which means that the qubit is decoupled from the

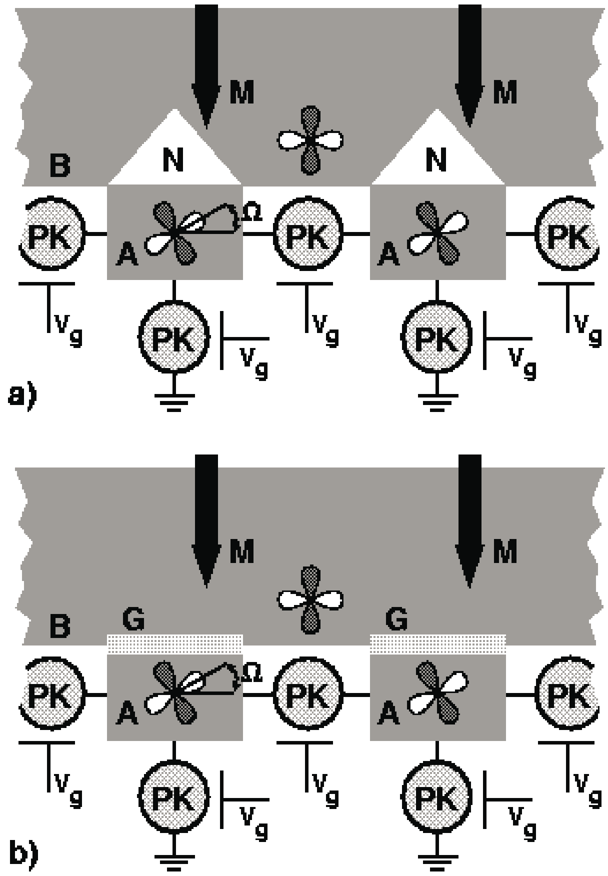

Figure 10. A qubit based on an SD (a) or grain boundary DD (b) Josephson junction[24, 34]. Parity keys (superconducting single electron transistors) are used to connect the nearest neighbours. The critical current through the parity key (a small superconducting island) can be tuned by changing the gate voltage, V_g [35]. M are magnetic force microscope tips, intended to read out spontaneous fluxes. An alternative readout scheme relies on direct detection of the phase on the island, which affects the critical current between the bus and the ground [36].

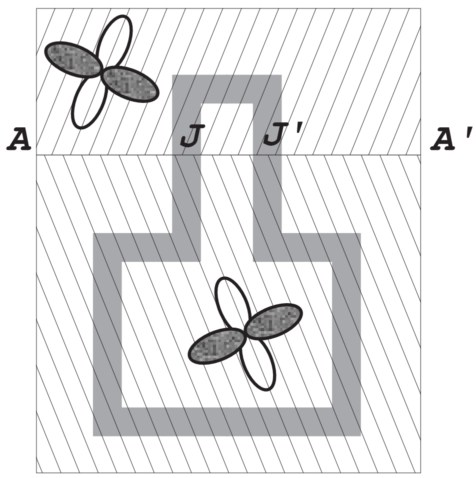

Figure 11. A silent d-wave qubit[38]. The structure is formed of a high-T$_c$ film around the grain boundary AA' and contains two grain boundary junction (J,J').

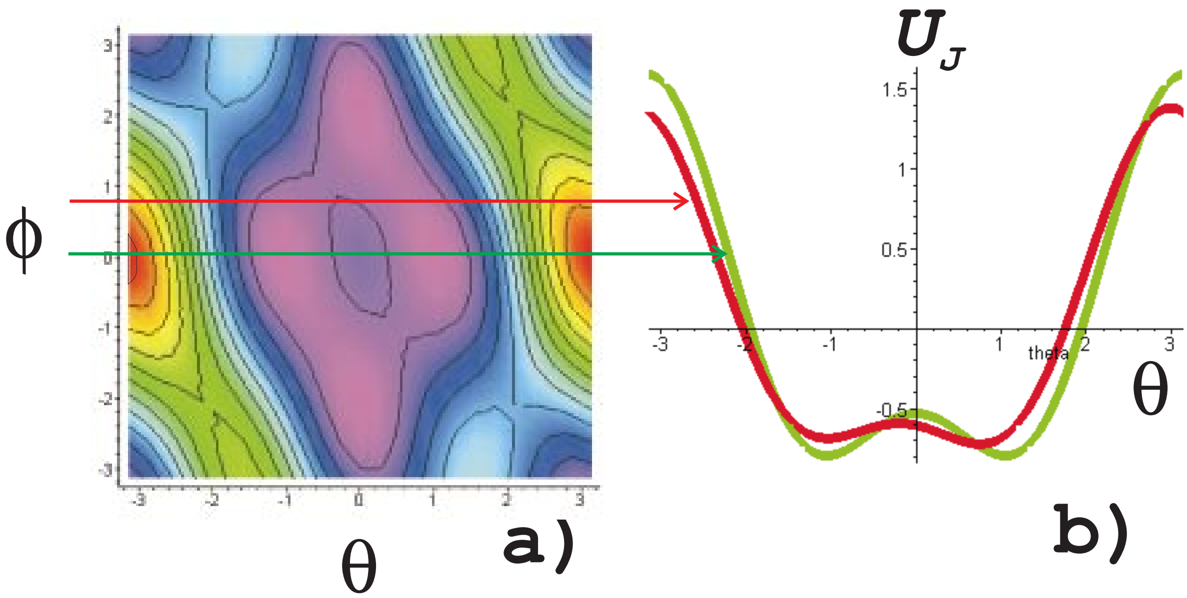

Figure 12. The Josephson energy profile for the silent qubit[38] as a function of the island phase, θ; $\gamma_1 = \gamma_2 = \pi/4$; $\eta = 0.5$. In the limit of negligible self-inductance the potential depends on the external magnetic flux, $\Phi_{\rm ext} = \Phi_0 \times (\phi/2\pi)$ as on parameter (a). In the absence of external flux the potential has two degenerate minima; degeneracy is lifted by finite external flux, which also affects the height of the potential barrier(b).

external magnetic fields (if we disregard spontaneous currents in the junctions themselves, which can be very small [21, 22, 23]). This justifies the moniker "silent".

Both the barrier and the bias between the wells can be controlled by the external flux. It is noteworthy that the corrections are of at least second order in ϕ, which drastically reduces the influence of fluctuations in the external circuitry[38]. The mechanism of noise reduction is similar to that of the "quantronium" qubit[39], but the "sweet spot" (an extremal point on the energy surface) appears already on the classical level, and at zero external field.

Finally, let us briefly mention two more proposals.

A "no tunneling" design[40] combines the ideas of CBJJ (current-based Josephson junction) qubits [42, 41] and the intrinsic bistability of d-wave junctions. In a single bistable Josephson junction, the potential barrier between the degenerate levels corresponding to phase difference $\pm\chi_0$ is too high to allow tunneling. The transitions between the states are realized through Rabi transitions via an auxiliary energy level, situated above the top of the barrier. Rabi transitions are induced by applying an external high-frequency field.

"Dot/antidot" proposals are based on the spontaneous flux (less than a flux quantum) generated in a high-T$_c$ island or around a hole in bulk high-T$_c$ due to the presence of a subdominant order parameter[23, 36] (see the end of Section 1.3), but so far lack experimental support.

2.3. Fabrication and experiment

Due to fabrication difficulties, as well as expected problems with decoherence from e.g. nodal quasiparticles (see Section 2.4), experimental research on d-wave qubits is not as far advanced as on the devices with conventional superconductors. The experimental confirmation of quantum behaviour in these systems is still missing. Nevertheless several recent successes should be noted.

Arrays of half-flux quanta were realized and manipulated (classically) in YBCO-Nb zigzag junctions [37]. Each facet of the junction effectively constitutes a π-ring, which supports spontaneous flux of $\pm\Phi_0$. Interaction between the fluxes is mainly due to the superconducting connection, which leads to robust

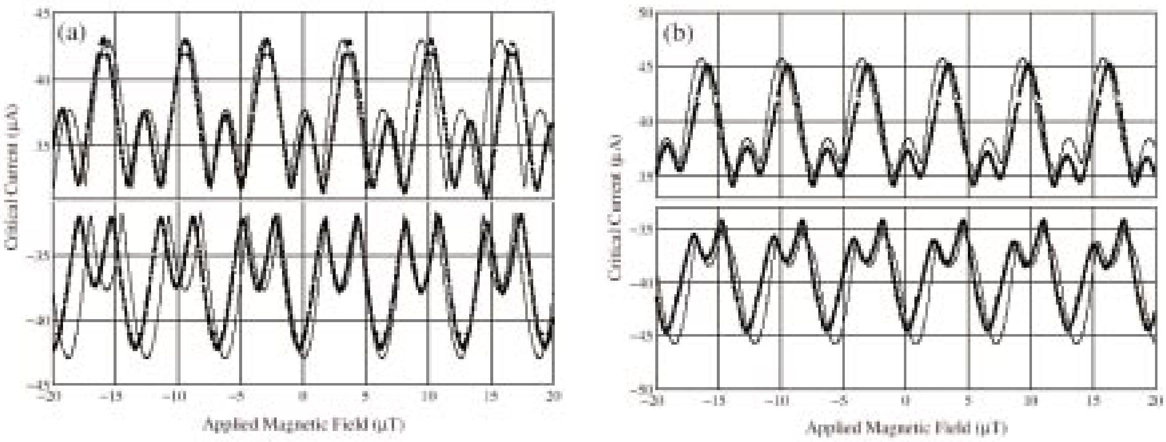

Figure 13. Critical currents of two nominally identical dc SQUIDs, based on YBCO grain boundary junctions, as a function of applied magnetic field[44]. Dots: experiment. Thin line: theoretical fit. Fitting parameters: (a) $I_1^{(I)} = 9\,\mu A$; $I_1^{(II)} = 3.7\,\mu A$; $I_2^{(I)} = 0.3\,\mu A$; $I_2^{(II)} = 22.7\,\mu A$; (b) $I_1^{(I)} = 7.8\,\mu A$; $I_1^{(II)} = 5.3\,\mu A$; $I_2^{(I)} = 3\,\mu A$; $I_2^{(II)} = 4.3\,\mu A$.

antiferromagnetic ordering. In its absence, a weaker, magnetic interaction establishes ferromagnetic flux ordering. The authors consider the possibility of using their structures in qubit design.

Good quality submicron grain boundary YBCO junctions were fabricated and bistable energy vs. phase dependence was demonstrated [26, 43]. Dc SQUIDs YBCO ($15\times15\,\mu$m^2 square loops with nominally $2\,\mu$m wide grain boundary junctions) were fabricated and tested, and their classical behaviour is very well described by the existing theory [44] (Fig. 13). Like in (28), only two harmonics of current-phase dependence were considered. From fitting the experimental data, we had to conclude that the junctions in the same SQUID have not only different critical current amplitudes, but different ratios of first to second harmonic ($\gamma_1 \neq \gamma_2$), probably due to the variation in the grain boundary properties over a distance of $\sim 15\,\mu$m. This is one reason for putting the junctions in the silent qubit, Fig. 11, close to each other, as it was done when fabricating its prototype[38].

2.4. Decoherence in d-wave qubits

Decoherence is the major concern for any qubit realization, especially for solid state qubits, due to abundance of low-energy degrees of freedom. In superconductors, this problem is mitigated by the exclusion of quasiparticle excitations due to the superconducting gap. This explains also why the very fact of existence of gapless excitations in high-T$_c$ superconductors long served as a deterrent against serious search for macroscopic quantum coherence in these systems. An additional source of trouble may be zero-energy states in DD junctions.

Nevertheless, recent theoretical analysis of DD junctions [47, 46, 45], all using quasiclassical Eilenberger equations, shows that the detrimental role of nodal quasiparticles and ZES could have been exaggerated.

Before turning to these results, let us first do a simple estimate of dissipation due to nodal quasiparticles in bulk d-wave superconductors.

Consider, for example, a three-junction qubit with d-wave phase shifters. The $|0\rangle$ and $|1\rangle$ states support, respectively, clockwise and counterclockwise persistent currents around the loop, with superfluid velocity $\mathbf{v}_s$. Tunnelling between these states leads to nonzero average $\langle \dot{\mathbf{v}}_s^2 \rangle$ in the bulk of the superconducting loop.

Time-dependent superfluid velocity produces a local electric field

$$\mathbf{E} = -\frac{1}{c}\dot{\mathbf{A}} = \frac{m}{e}\dot{\mathbf{v}}_s, \tag{30}$$

and quasiparticle current $\mathbf{j}_{qp} = \sigma \mathbf{E}$. The resulting average energy dissipation rate per unit volume is

$$\dot{\mathcal{E}} = \sigma E^2 \approx m\tau_{qp}\langle \bar{n}(v_s)\dot{v}_s^2\rangle. \tag{31}$$

Here τ_{qp} is the quasiparticle lifetime, and

$$\bar{n}(v_s) = \int_0^\infty d\epsilon \bar{N}(\epsilon)\left[n_F(\epsilon - p_F v_s) + n_F(\epsilon + p_F v_s)\right] \tag{32}$$

is the effective quasiparticle density. The angle-averaged density of states inside the d-wave gap is [48]

$$\bar{N}(\epsilon) \approx N(0)\frac{2\epsilon}{\mu\Delta_0}, \tag{33}$$

where $\mu = \frac{1}{\Delta_0}\frac{d|\Delta(\theta)|}{d\theta}$, and Δ_0 is the maximal value of the superconducting order parameter. Substituting (33) in (32), we obtain

$$\bar{n}(v_s) \approx N(0)\frac{2}{\mu\Delta_0}\left(-T^2\right)\left(\mathrm{Li}_2\left(-e^{-\frac{p_F v_s}{T}}\right) + \mathrm{Li}_2\left(-e^{\frac{p_F v_s}{T}}\right)\right), \tag{34}$$

where $\mathrm{Li}_2(z) = \int_z^0 dt \frac{\ln(1-t)}{t}$ is the dilogarithm. Expanding for small $p_F v_s \ll T$, and taking into account that $\mathrm{Li}_2(-1) = -\pi^2/12$, $\mathrm{Li}_2'(-1) = \ln 2$, and $\mathrm{Li}_2''(-1) = -1/2 + \ln 2$, we obtain

$$\bar{n}(v_s) \approx \frac{N(0)}{\mu\Delta_0}\left(\frac{\pi^2 T^2}{3} + (p_F v_s)^2\right). \tag{35}$$

The two terms in parentheses correspond to thermal activation of quasiparticles and their "Cherenkov" generation by current-carrying state. Note that finite quasiparticle density does not lead by itself to any dissipation.

In the opposite limit ($T \ll p_F v_s$) only the "Cherenkov" contribution remains,

$$\bar{n}(v_s) \approx \frac{N(0)}{\mu\Delta_0}(p_F v_s)^2 \tag{36}$$

(since $\mathrm{Li}_2(z) \sim -(1/2)(\ln z)^2$ for large negative z, and $\sim z$ for small z).

The energy dissipation rate provides the upper limit τ_ϵ for the decoherence time (since dissipation is sufficient, but not necessary condition for decoherence). Denoting by I_c the amplitude of the persistent current in the loop, by L the inductance of the loop, and by Ω the effective volume of d-wave superconductor, where it flows, we can write

$$\tau_\epsilon^{-1} = \frac{2\dot{\mathcal{E}}\Omega}{LI_c^2} \approx \frac{2m\tau_{qp}N(0)\Omega\left(\frac{\pi^2 T^2}{3}\langle\dot{v}_s^2\rangle + p_F^2\langle v_s^2\dot{v}_s^2\rangle\right)}{\mu\Delta_0 LI_c^2}. \tag{37}$$

Note that the thermal contribution to τ_ϵ^{-1} is independent on the absolute value of the supercurrent in the loop ($\propto v_s$), while the other term scales as I_c^2. Both contributions are proportional to Ω and (via $\dot{v}_s$) to ω_t, the characteristic frequency of current oscillations (i.e. tunneling rate between clockwise and counterclockwise current states).

It follows from the above analysis that the intrinsic decoherence in a *d*-wave superconductor due to nodal quasiparticles can be minimized by decreasing the amplitude of the supercurrent through it, and the volume of the material where *time-dependent* supercurrents flow. From this point of view the designs with phase shifters, as well as the Newns-Tsuei and "no tunneling" designs are at a disadvantage (the latter, because the microwave field necessary to produce Rabi transitions will affect the whole sample).

Now let us estimate dissipation in a DD junction. First, following [19, 24], consider a DND model with ideally transmissive ND boundaries. Due to tunneling, the phase will fluctuate, creating a finite voltage on the junction, $V = (1/2e)\dot{\chi}$, and normal current $I_n = GV$. The corresponding dissipative function and decay decrement are

$$\mathcal{F} = \frac{1}{2}\dot{\mathcal{E}} = \frac{1}{2}GV^2 = \frac{G\dot{\chi}^2}{2}\left(\frac{1}{2e}\right)^2; \tag{38}$$

$$\gamma = \frac{2}{M_Q\dot{\chi}}\frac{\partial\mathcal{F}}{\partial\dot{\chi}} = \frac{G}{4e^2M_Q} = \frac{4N_\perp E_Q}{\pi}. \tag{39}$$

Here $E_Q = e^2/2C$, $M_Q = C/16e^2 = 1/32E_Q$, $N_\perp$ are the Coulomb energy, effective "mass" and number of quantum channels in the junction respectively. The latter is related to the critical Josephson current I_0 and spacing between Andreev levels in the normal part of the system $\bar{\epsilon} = v_F/2L$ via

$$I_0 = N_\perp e\bar{\epsilon}. \tag{40}$$

We require, that $\gamma/\omega_0 \ll 1$, where $\omega_0 = \sqrt{32N_\perp E_Q\bar{\epsilon}}/\pi$ is the frequency of small phase oscillations near a local minimum. This means,

$$N_\perp \ll \frac{\bar{\epsilon}}{E_Q}. \tag{41}$$

The above condition allows a straightforward physical interpretation. In the absence of thermal excitations, the only dissipation mechanism in the normal part of the system is through the transitions between Andreev levels, induced by fluctuation voltage. These transitions become possible, if $\bar{\epsilon} < 2e\bar{V} \sim \sqrt{\bar{\dot{\chi}^2}} \sim \omega_0$, which brings us back to (41). Another interpretation of this criterion arises if we rewrite it as $\omega_0^{-1} \gg (v_F/L)^{-1}$ [24]. On the right-hand side we see time for a quasiparticle to traverse the normal part of the junction. If it exceeds the period of phase oscillations (on the left-hand side), Andreev levels simply don't have time to form. Since they provide the only mechanism for coherent transport through the system, the latter is impossible, unless our "no dissipation" criterion holds.

For the thickness of the normal layer $L \sim 1000$ Å and $v_F \sim 10^7$ cm/s this criterion limits $\omega_0 < 10^{12}$ s^{-1}, which is comfortable three orders of magnitude above the usually obtained tunneling splitting in such qubits (~ 1 GHz) and can be accommodated in the above designs. Nevertheless, while presenting a useful qualitative picture, the DND model is not adequate for the task of extracting quantitative predictions. For example, the coherence length l_T in the normal metal can be very large, while in the high-T$_c$ compound it is short. Therefore the estimates for crucial parameters (like $\bar{\epsilon}$) based on the assumption $l_T \gg L$ can be wrong. Moreover, the assumption of ideal ND boundaries is not realistic.

A calculation[32], which used a model of a DD junction interacting with a bosonic thermal bath, gave an optimistic estimate for the quality of the tricrystal qubit $Q > 10^8$.

The role of size quantization of quasiparticles in small DD and SND structures was suggested in [24, 33]. The importance of the effect is that it would exponentially suppress the quasiparticle density and therefore the dissipation below temperature of the quantization gap, estimated as $1 - 10$ K. Recently this problem was investigated[45] for a finite width DD junction. Contrary to the expectations, the size quantization as such turned out to be effectively absent on the scale exceeding ξ_0 (that is, practically irrelevant). On the other hand, the finite transverse size of the system imposed an effective band structure. Namely, due to the direction dependence of the order parameter, a quasiparticle travelling along the trajectory, bouncing between the sides of the junction, goes through a periodic 1D off-diagonal potential. The influence of this band structure on dissipation in the system is not straightforward. Ironically, from the practical point of view this is a moot point, since the decoherence time from the quasiparticles in the junction, estimated in [45, 47], already corresponds to the quality factor $\tau_\varphi/\tau_g \sim 10^6$, which exceeds by two orders of magnitude the theoretical threshold allowing to run a quantum computer indefinitely.

The expression for the decoherence time obtained in [45, 47],

$$\tau_\varphi = \frac{4e}{\delta\phi_2 I(\Delta_t/e)}, \tag{42}$$

where $\delta\phi$ is the difference between equilibrium phases in degenerate minima of the junction (i.e. $\delta\phi = 2\chi_0$ in other notation), contains the expression for quasiparticle current in the junction at finite voltage Δ_t/e (where

Δ_t is the tunneling rate between the minima). This agrees with our back-of-the-envelope analysis: phase tunneling leads to finite voltage in the system through the second Josephson relation and with finite voltage comes quasiparticle current and decoherence. The aforementioned quality factor is defined as $Q = \tau_\varphi \Delta_t / 2\hbar$, that is, we compare the decoherence time with the tunneling time. This is a usual optimistic estimate, since it will certainly take several tunneling cycles to perform a quantum gate operation.

It turns out that a much bigger threat is posed by the contribution from zero energy bound states, which can be at least two orders of magnitude larger. We can see this qualitatively from (42): a large density of quasiparticle states close to zero energy (i.e., on Fermi level) means that even small voltages create large quasiparticle currents, which sit in the denominator of the expression for τ_φ. Fortunately, this contribution is suppressed in the case of ZES splitting, and such splitting is always present due to, e.g., finite equilibrium phase difference across the junction[45].

A similar picture follows from the analysis presented in [46]. A specific question addressed there is especially important: it is known that RC-constant measured in DD junctions is consistently 1 ps over a wide range of junction sizes [43], and it is tempting to accept this value as the dissipation rate in the system. It would be a death knell for any quantum computing application of high-T$_c$ structures, and nearly that for any hope to see there some quantum effects. Nevertheless, it is not quite that bad. Indeed, we saw that ZES play a major role in dissipation in a DD junction, but are sensitive to phase differences across it. Measurements of the RC constant are done in the resistive regime, when a finite voltage exists across the junction, so that the phase difference grows monotonously in time, forcing ZES to approach the Fermi surface repeatedly. Therefore τ_{RC} reflects some averaged dissipation rate. On the other hand, in a free junction with not too high a tunneling rate phase differences obviously tend to oscillate around χ_0 or $-\chi_0$, its equilibrium values, and do not spend much time near zero or π; therefore ZES are usually shifted from the Fermi level, and their contribution to dissipation is suppressed.[3]

This qualitative picture is confirmed by a detailed calculation (Fig. 14). The decoherence time is related to the phase-dependent conductance via

$$\tau_\varphi = \frac{1}{\alpha F(\chi_0)^2 \delta E} \tanh \frac{\delta E}{2T}. \tag{44}$$

Here α is the dissipation coefficient, δE is interlevel spacing in the well, and

$$G(\chi) = 4e^2 \alpha \left(\partial_\chi F(\chi) \right)^2. \tag{45}$$

For a realistic choice of parameters Eq.(44) gives a conservative estimate $\tau_\varphi = 1 - 100$ ns, and quality factor $Q \sim 1 - 100$. This is, of course, too little for quantum computing, but quite enough for observation of quantum tunneling and coherence in such junctions.

3. Conclusions

Since the d-wave character of superconducting pairing in high-T$_c$ cuprates was established, our understanding of high-T$_c$ Josephson structures and ability to fabricate them progressed significantly. Now it enables us to ask the question in the title of this paper: Can high-T$_c$ cuprates play a role in quantum computing?- and to tentatively answer: Yes.

We have seen that submicron junctions of sufficiently high quality are now fabricated. Several designs, which take advantage of the intrinsic bistability of d-wave structures, were developed, fabricated, and tested in classical regime. If the decoherence time turns out to be large enough, their better scalability can be a decisive advantage for quantum computing applications.

This is, of course, a big if. Still, theoretical estimates tend to be rather optimistic. Even though they widely differ, all of them predict τ_φ long enough to encourage experimental search for quantum coherence.

[3] A useful model Hamiltonian for this system can be written as

$$H = \frac{\hat{Q}^2}{2C} + U(\chi) + \frac{1}{2}\left[\hat{Q}, V(\chi)\right]_+ \sum_\alpha c_\alpha x_\alpha + \sum_\alpha \left[\frac{\hat{p}_\alpha^2}{2m_\alpha} + \frac{m_\alpha \omega_\alpha^2 x_\alpha^2}{2} \right] \tag{43}$$

. Here $U(\chi)$ is the two-well potential, and $V(\chi)$ is centered near $\chi = 0$.

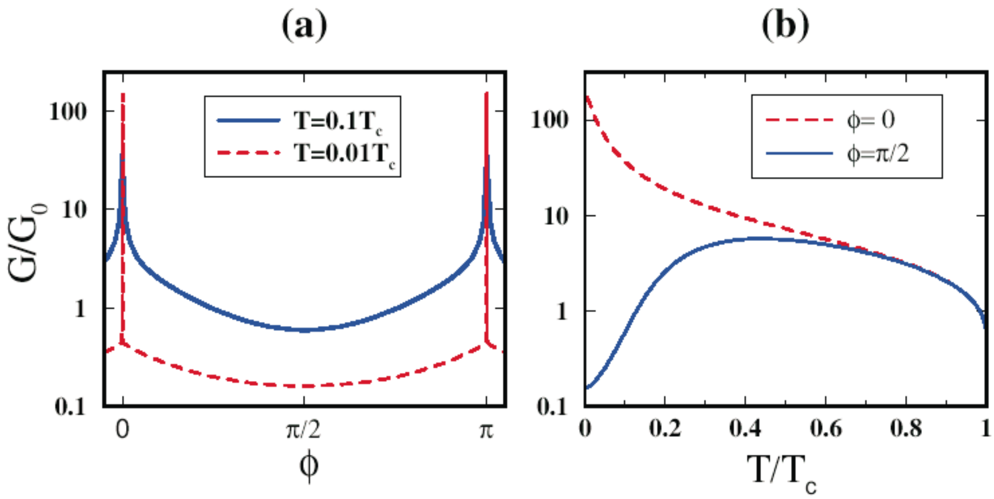

Figure 14. Phase (a) and temperature (b) dependence of the normal conductance of a DD junction [46].

There are several compelling reasons to do that. First, it would be a spectacular result. Second, it would clarify why different models give different answers. Third, it could indeed lead to practical application of high-T$_c$ devices in quantum computing.

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