

Variation of Chemical Potential Oscillations of a 2DEG in a Quantum Well Under a Magnetic Field for Multiple Sub-Band Occupation as Function of Temperature and Level-Broadening

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Abstract

We study the variation of chemical potential in a two dimensional electron gas in a uniform magnetic field as a function of temperature and level-broadening parameter. Schrödinger and Poisson equations are solved self-consistently for a two dimensional electron gas formed in a $\text{Ga}_x\text{Al}_{1-x}\text{As}/\text{GaAs}$ quantum well, to numerically determine the potential profile, empty or occupied energy sub-bands, the wave-functions and electron concentrations corresponding to each sub-band at various temperatures. Then assuming that the electron concentration is not altered, the variation of chemical potential with respect to both temperature and energy level-broadening parameter is studied under a uniform and constant magnetic field. Gaussian, exponential and lorentzian forms of broadening are assumed and the results are compared. Increasing temperature or level-broadening have been found to have qualitatively the same effect on chemical potential oscillations, although the underlying processes are different. It is found that, as the number of occupied sub-bands increases, the shape of oscillations increases in complexity. The results can be directly tested by experimental studies.

Key Words: Chemical potential, level-broadening, quantum well, Poisson-Schrödinger solution.

1. Introduction

The effects of a uniform magnetic field on a two dimensional electron gas (2DEG) have been extensively studied since the discovery of the formation of new energy levels by Landau [1] long ago. Much of the work in this field concentrated on Shubnikov-de Haas (SdH) oscillations [2] and integer [3] and fractional [4, 5] quantum Hall effects.

The oscillations of chemical potential for a 2DEG formed at a $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ heterojunction in a uniform magnetic field has been studied earlier by Zawadski and Lassnig [6]. In their numerical study, only the occupation of the first sub-band (electric quantum limit) is assumed and the chemical potential, specific heat and magneto-thermal oscillations of electron gas is obtained as a function of applied magnetic field, where a Gaussian form of broadening of Landau levels is assumed. In a later paper Sanchez-Dehesa, et al. [7] studied the chemical potential and sub-band energy variations in a 2DEG under a constant magnetic field formed in a $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ quantum well in the zero-temperature limit. They determined the conduction energy band profile, sub-band energy levels and the electron population in each sub-band self consistently in

the Hartree approximation. The parameters and doping concentrations in the studied geometry are chosen in a such way to guarantee the occupation of up to three sub-bands by electrons. They found that the chemical potential and sub-band levels, as well as electron concentrations, are affected by the magnetic field. The energy difference between sub-bands vary with magnetic field and this variation is attributed to the oscillations of electron charge density in the quantum well. Because of crossover of the sub-bands and the presence of electrons beyond the first sub-band, the oscillatory behavior of the chemical potential becomes complicated. Numerous other theoretical and experimental studies [8]–[17] are devoted to various properties (magnetization, heat capacity, magnetoresistance, spin effects) of a 2DEG under a uniform or varying applied magnetic field.

The above mentioned properties are rarely studied when sub-bands beyond the first band are occupied in the 2DEG at finite temperatures. Accordingly, in this study, we consider a 2DEG formed in a $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ quantum well and investigate the variation of chemical potential of the system under a constant and uniform magnetic field for the cases when only one sub-band, two sub-bands and three sub-bands are occupied by electrons. The conduction energy band profile, the energy sub-bands and electron concentration in each sub-band are determined from the self-consistent numerical solution of Poisson and Schrödinger equations in the absence of a magnetic field and at a series of particular temperatures. Then the variation of chemical potential as a function of applied magnetic field is determined at various temperatures for various forms of level-broadening, assuming that the electron concentration in the well is not altered by the magnetic field. The effect of broadening is included using Gaussian, lorentzian and exponential types of level-broadening.

The paper is organized as follows: In section 2, a short theoretical review of the subject is given with emphasis on the type of density of states function used. In section 3 the main results of the study and their discussions are presented. This section is devoted to the study of chemical potential oscillations when only the first sub-band, two sub-bands and three sub-bands are occupied. A summary and conclusions are presented in section 4.

2. Theory

The organizational geometry of a typical $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ quantum well and doping profiles used in this study is shown in Figure 1. The growth axis is chosen to be the z -axis. The Schrödinger equation in the effective mass and Hartree approximations for a single electron is given by [18]

$$-\frac{\hbar^2}{2} \frac{d}{dz} \frac{1}{m^*(z)} \frac{d\psi}{dz} + V(z)\psi(z) = E\psi(z), \quad (1)$$

where $V(z)$ is the self-consistently calculated potential energy which includes contributions arising both from dopant charges and electrons localized in the quantum well; $m^*(z)$ is the effective mass; \hbar is Planck's constant divided by 2π ; and E is the energy eigenvalue. The Poisson equation for the quantum well under consideration is

$$\frac{d}{dz} \left[\epsilon(z) \frac{d\phi}{dz} \right] = -e[N_D(z) - N_A(z) - n(z)], \quad (2)$$

where ϕ is the electrostatic potential; e is the electronic charge; ϵ is the dielectric constant of the material; N_D and N_A are the donor and acceptor concentrations, respectively; and $n(z)$ is the free electron concentration localized in the well. The potential energy $V(z)$ is related to the electrostatic potential $\phi(z)$ as

$$V(z) = -e\phi(z) + \Delta E_c(z), \quad (3)$$

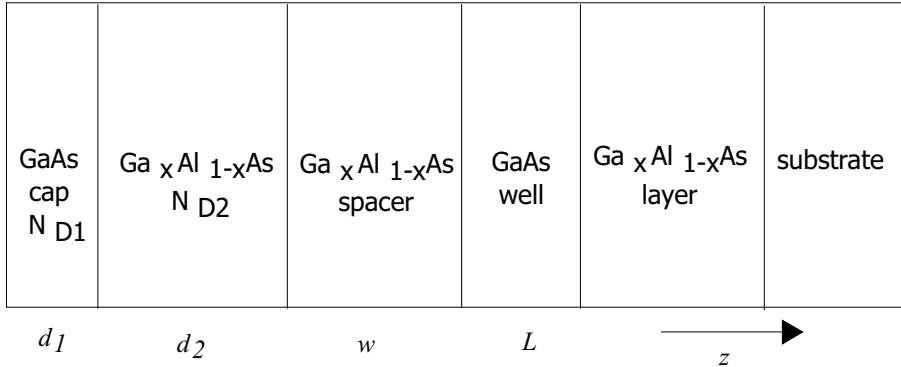


Figure 1. The organizational geometry of a typical sample used in the simulations.

where $\Delta E_c(z)$ represents conduction band discontinuities at the $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ hetero interfaces. The electron concentration $n(z)$ in the quantum well is related to the eigenfunctions $\psi_i(z)$ as

$$n(z) = \sum_{i=1}^M \psi_i^*(z) \psi_i(z) n_i, \quad (4)$$

where M is the number of occupied sub-bands, n_i is the electron concentration in each sub-band given by

$$n_i = \frac{m^*}{\pi \hbar^2} \int_{E_i}^{\infty} \frac{1}{1 + e^{(E-\mu)/k_B T}} dE, \quad (5)$$

where E_i are the energy eigenvalues (sub-band energies), μ is the chemical potential, k_B Boltzmann's constant and T is the absolute temperature. Equations (1) and (2) are discretized using finite difference approximations for derivatives [19, 20] and an iterative method is used to solve them self consistently. A first guess for $V(z)$ is used to find the eigenfunctions ψ_i and the energy eigenvalues E_i from (1) and (2). Then the electron concentration $n(z)$ and the electron density n_i in each level are found through Equations (4) and (5), respectively. Next, a new electrostatic potential $\phi(z)$ is determined from the solution of Poisson equation (2) using the computed value of $n(z)$ and the doping profiles. Finally, a new potential $V(z)$ is obtained from Equation (3). This procedure is repeated until the potential does not vary beyond a predetermined error tolerance. The notion of position-dependent energy levels is used to accelerate the convergence of the solutions [21]. The Fermi integrals appearing in Equation (5) are solved numerically [22, 23] without using a Boltzmann approximation. This is necessary because the two dimensional electron gas is highly degenerate for all cases considered. Since we are looking for bound state solutions, both the wave function and its derivative must vanish at $\pm\infty$ and this is checked throughout the numerical calculations.

Since the focus in this paper is on a comparative study of chemical potential oscillations of a 2DEG in a uniform magnetic field for various forms of level-broadening, and for the cases when the number of occupied sub-bands goes from one to three sub-bands, no further details of the self-consistent solution of Poisson/Schrödinger equation will be given here. A typical conduction band energy profile and the first three eigenfunctions for a quantum well that corresponds to the geometry shown in Figure 1 is depicted in Figure 2, when the temperature is $T = 4.2$ K. It is assumed that a potential of 0.7 V is present at the GaAs cap to account for the surface charges present there [24]. Only the first sub-band is occupied for the case shown in the figure. When a magnetic field is applied to the hetero structure new levels, known as Landau levels, become attached to each energy sub-band E_i . The magnetic field is assumed to be normal to the hetero interface, that is with direction $(0, 0, B)$. Since only the field B will have a physical significance in the end, the results will be independent of the choice of vector potential A . Thus we can choose a vector

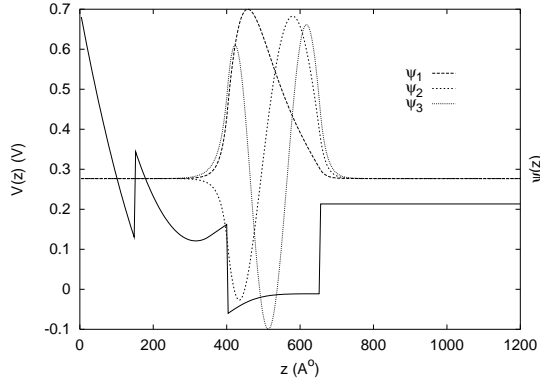


Figure 2. The conduction band profile (solid line) and the first three wave functions for a quantum well corresponding to the geometry of a sample shown in Figure 1, for the case when only one sub-band is occupied. The parameters used are as follows (see Figure 1): $d_1 = 152 \text{ \AA}$, $d_2 = 200 \text{ \AA}$, $N_{D1} = 1 \times 10^{18} \text{ cm}^{-3}$, $N_{D2} = 1.2 \times 10^{18} \text{ cm}^{-3}$, $w = 52 \text{ \AA}$ and $L = 252 \text{ \AA}$, $\epsilon_{\text{GaAs}} = 12.9$, $\epsilon_{\text{GaAlAs}} = 13.1$, $m^* = 0.069 m_0$. The density of electrons in the first sub-band is $n_s = 4.6 \times 10^{11} \text{ cm}^{-2}$ and the first three energy eigenvalues (not shown) with respect to well minima are 44.2, 65.2 and 97.0 meV, respectively. The right hand side axis for wave functions is in arbitrary units.

potential of the form $(0, Bx, 0)$. Then the Hamiltonian for a single electron in the presence of the magnetic field becomes

$$H = \frac{1}{2m^*} \left[\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial y} - eBx \right)^2 + \frac{\partial^2}{\partial z^2} \right] + V(z), \quad (6)$$

where $V(z)$ is the confinement potential shown in Figure 2. The potential $V(z)$ consists of the barrier potentials of the well and the electrostatic potential of the impurities and electrons confined to the quantum well. The eigenfunctions of the above Hamiltonian are given by

$$\psi(r) = \frac{1}{\sqrt{L_y}} e^{ik_y y} u(x, z), \quad (7)$$

where L_y is the sample dimension in the y -direction and $u(x, z)$ is a function of x and z only. The energy eigenvalues corresponding to $\psi(r)$ are given by [18]

$$E_{in\sigma_z} = E_i + \left(n + \frac{1}{2} \right) \hbar w_c + g^* \mu_B \sigma_z, \quad (8)$$

where $n=0, 1, 2, \dots$ is a positive integer, $w_c = eB/m^*$ is the cyclotron frequency, g^* is the Lande g factor of the material, μ_B is the Bohr magneton and σ_z is the spin projection in the z -direction which can take only the values $\pm 1/2$. E_i are the energy levels (sub-bands) in the quantum well in the absence of a magnetic field and the second term in above equation represents the Landau levels attached to i th sub-band due to the presence of a magnetic field. The third term in (8), the spin-splitting term, is neglected everywhere in the rest of this study. Thus we are left with only E_{in} in (8), which represents the energy of the n th Landau level attached to the i th sub-band.

To calculate the density of electrons at each Landau level we need to know the density of states function for electrons. The density of states can be written as a sum of the density of states corresponding to each sub-band i as

$$D(E) = \sum_i^M D_i(E), \quad (9)$$

where M is the number of sub-bands. The density of states for each sub-band per unit area is given by

$$D_i(E) = \frac{1}{2\pi\lambda^2} \sum_n \delta(E - E_{in}), \quad (10)$$

where $\lambda = \sqrt{\frac{\hbar}{eB}}$ is known as the magnetic length and E_{in} is the energy of n th Landau level attached to the i th sub-band given in (8). The delta-like density of states in Equation (10) is deformed due to scattering of electrons by impurities and phonons. As a result the density of states becomes broadened. A derivation of the level broadening Γ is given by Ando and Uemura [25] where they found a lorentzian type of broadening which is proportional to the mobility μ_m as $\Gamma \propto \mu_m^{1/2}$. However, other forms of broadening, such as Gaussian broadening, are also suggested [2, 26] and are widely used. In this study, in addition to Gaussian and lorentzian types of broadening, we use an exponential form of broadening without any theoretical justification for it and compare the results corresponding to the three types of broadening. In some studies [12] a constant background density of states is added to the zero field density of states to make the experimental observations and theoretical predictions agree. This application is not considered in this study.

The gaussian and lorentzian forms of broadening are given by [2, 26]

$$D_i(E) = \frac{1}{2\pi\lambda^2} \sum_n \frac{1}{\sqrt{2\pi}\Gamma_G} e^{-(E-E_{in})/2\Gamma_G^2} \quad (11)$$

$$D_i(E) = \frac{1}{2\pi\lambda^2} \sum_n \frac{1}{\pi} \frac{\Gamma_L}{(E - E_{in})^2 + \Gamma_L^2}, \quad (12)$$

respectively. The exponential form of broadening considered in this study is assumed to be of the form

$$D_i(E) = \frac{1}{2\pi\lambda^2} \sum_n \frac{1}{2\Gamma_E} e^{-|E-E_{in}|/\Gamma_E}, \quad (13)$$

where the Γ 's in Eqs. (11)–(13) are the level broadening parameters for the corresponding forms of level broadening. Γ 's are assumed to be constants for all temperature and magnetic field values considered in this study. The total density of two dimensional electrons is given by

$$n = \int_0^\infty D(E)f(E)dE = \sum_i \int_0^\infty D_i(E)f(E)dE, \quad (14)$$

where $f(E) = [1 + \exp((E - \mu)/k_B T)]^{-1}$ is the Fermi-Dirac function and $D_i(E)$ is the density of states function appropriate for the type of broadening used. The system under consideration can be investigated for the following cases: (i) electron concentration is constant and the chemical potential varies to adjust itself as magnetic field changes, (ii) the chemical potential remains fixed while the electron concentration varies to adopt to the existing conditions as magnetic field changes, and (iii) both the electron concentration and chemical potential varies as a function of magnetic field. We assume that the electron density remains constant for field values considered and thus consider only case (i) above. For this case, for a given value of magnetic field, temperature and electron density, the chemical potential μ is the only unknown in Equation (14). Determination of chemical potential simply turns into a root finding problem. Equation (14) is solved for $\mu(B)$ using the bisection method [19] and observed to converge fairly rapidly.

To facilitate the comparison for the three types of broadening models, the level-broadening parameter values for the lorentzian and exponential broadening are chosen in such a way that the widths of corresponding density of states functions become equal to the gaussian broadening at half maximum value. For example, the gaussian and lorentzian forms of broadening become equal at half maximum when the broadening parameters are related as $\Gamma_L = \sqrt{2 \ln 2} \Gamma_G = 1.2 \Gamma_G$. A similar calculation can also be made for the

Table 1. Material parameters, the first three sub-band energies E_i and the corresponding electron concentrations n_{si} calculated at $T = 4.2$ K when one, two and three sub-bands are occupied by electrons in the quantum well.

occupied subbands	1 sub-band	2 sub-bands	3 sub-bands
d_1 (Å°)	152	152	152
d_2 (Å°)	200	152	190
w (Å°)	52	52	12
L (Å°)	252	252	452
N_{D1} (10^{18} cm $^{-3}$)	1	1	0.5
N_{D2} (10^{18} cm $^{-3}$)	1.2	2.4	2.5
n_{si} (10^{11} cm $^{-2}$)	4.57	9.68	15.2
	–	1.11	3.02
	–	–	0.46
E_1 (meV)	44.2	71.1	90.7
E_2 (meV)	65.2	100.5	132.5
E_3 (meV)	97.0	129.6	141.3

case of gaussian and exponential forms of broadening. The level broadening parameters for lorentzian and exponential broadenings are therefore related to the gaussian one via

$$\begin{aligned}\Gamma_L &= 1.2\Gamma_G \\ \Gamma_E &= 1.7\Gamma_G,\end{aligned}\tag{15}$$

respectively, throughout this study.

3. Results and Discussion

The parameters of the hetero junction for the cases when only one, two and three sub-bands are occupied are shown in Table 1 together with the calculated first three eigenvalues (sub-band energies) and their corresponding electron concentrations. Energy eigenvalues and electron concentration values given in the table are the values calculated at $T = 4.2$ K. There are minor changes in electron concentrations at higher temperatures. These do not create any substantial variation in sub-band energies. For example, when the temperature is increased from 4.2 to 30 K for the case of single band occupancy, the number of electrons increases (by much less than one percent) and the second sub-band is also populated by an insignificant (less than two percent of total) number of electrons. The dielectric constant of GaAs and GaAlAs are assumed to be 12.9 and 13.1, respectively. The effective mass is taken as $m^* = 0.069m_0$ everywhere, where m_0 is the free electron mass. The Al concentration is always taken to be $x = 0.3$.

There are two basic parameters of the present problem. One is the broadening parameter and the other is the temperature. The broadening parameter values for gaussian type of broadening are taken to be $\Gamma_G = 0.2, 0.5, 1$ and 2 meV throughout in this paper. Lorentzian and exponential forms of level broadening parameters are related to the gaussian broadening parameter as given in Equation (15) above. The temperature values considered for all types of broadening are: 4.2, 10, 20 and 30 K.

First we consider the effect of the level-broadening parameters on the variation of chemical potential at a fixed temperature. Figures 3(a-c) show the variation in chemical potential as a function of applied magnetic field for Landau level-broadening parameter values $\Gamma_G = 0.2, 0.5, 1.0,$ and 2.0 meV at $T = 4.2$ K when the gaussian form of level-broadening is in effect for the three cases of band occupancy. Figure 3a depicts the situation when only one sub-band is occupied. The energy of the Landau levels ('Landau fan') attached to the first sub-band are also shown in the figure. The amplitude of oscillations in chemical potential decreases as Γ_G increases. At absolute zero, the chemical potential (and the Fermi level) is expected to be pinned to the last occupied Landau level by electrons. As the magnetic field increases the electron occupation capacity of each level increases [27] and therefore electrons are transferred from high energy Landau levels to low energy ones. At a critical value of magnetic field, the last (upper most) partially occupied level becomes completely depopulated and thus the chemical potential abruptly drops to one Landau energy level below. This behavior can be seen in Figure 3a when the temperature is low enough ($T=4.2$ K) and level-broadening is small. As the level-broadening increases, the density of states corresponding to different Landau levels start overlapping and therefore the distinction between the levels become less pronounced. This results in a decrease in the sharpness of the oscillations in $\mu(B)$, as seen in Figure 3a. Although not shown here, the behavior for lorentzian and exponential forms of level-broadening cases for the corresponding parameter values are also qualitatively similar to the gaussian broadening case shown in Figure 3a.

When the number of occupied sub-bands increases, the behavior of chemical potential under a constant magnetic field becomes more complicated. For the case when two sub-bands are occupied, there are a number of Landau levels attached to each sub-band. The chemical potential now can make transitions between Landau levels attached to different sub-bands. Figure 3b shows the variation of chemical potential as a function of magnetic field at $T = 4.2$ K for level parameter values $\Gamma_G = 0.2, 0.5, 1.0$ and 2.0 meV, where gaussian form of broadening is assumed. At low magnetic fields Landau levels attached to different sub-bands are intermixed. Therefore $\mu(B)$ oscillates wildly between Landau levels attached to the first and second sub-bands. As the magnetic field increases the density of Landau levels decreases and $\mu(B)$ begins varying more smoothly. As the magnetic field increases the value of μ drops to the energy levels attached to the first sub-band also as seen in the figure. Because of these transitions between Landau levels attached to different sub-bands, the chemical potential varies at different rates (has different slopes) between two critical magnetic field values as seen in Figure 3b. For this case also increasing the level-broadening parameter results in the decrease of amplitude of oscillations of chemical potential.

For the case when three sub-bands are occupied, the behavior of chemical potential as a function of B is qualitatively similar to the case of two sub-band occupation. Its variation with B at $T = 4.2$ K for the level-broadening parameter values $\Gamma_G = 0.2, 0.5, 1.0$ and 2.0 meV for the case of gaussian broadening is shown in Figure 3c. At low field values the oscillations take place between Landau levels attached to the different sub-bands. As the magnetic field increases, it begins oscillating between the first Landau levels attached to the second and third sub-bands, respectively. Eventually all electrons collapse to the first Landau level attached to the first sub-band at high magnetic fields, as seen in the figure.

When the number of sub-bands occupied by electrons increases, the variation in μ , especially at low field values, becomes spike-like. For the case of single band occupancy, the chemical potential varies smoothly between two critical B values at low fields and small broadening parameters following closely one of the Landau levels attached to the first band. For the case of two or three sub-band occupancy, there are many closely spaced crossing Landau levels attached to different sub-bands, especially at low fields. Thus the chemical potential may assume any appropriate one of these values when the temperature is low and the broadening parameter is small (see Figures 3b and 3c). As the magnetic field increases, the variation in μ becomes smoother for 2- and 3-band occupancy. But it varies at different rates because it follows different Landau levels attached to different sub-bands.

The effect of increasing the temperature on the oscillations of μ is shown in Figures 4(a-c). When temperature increases, the amplitude of oscillations and their sharpness decreases for all

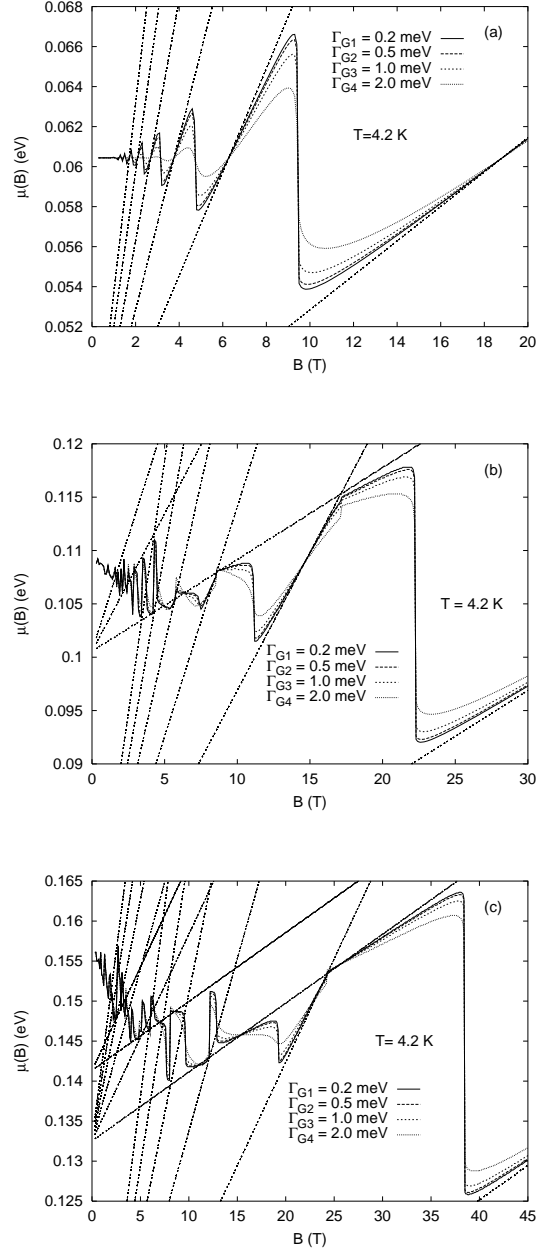


Figure 3. Oscillations in μ as a function of B for the level-broadening parameters $\Gamma_G = 0.2, 0.5, 1.0$ and 2.0 meV for the gaussian type broadening at $T = 4.2$ K when only one sub-band (a), two sub-bands (b) and three sub-bands (c) are occupied in quantum well in the absence of magnetic field. The parameters of the hetero junctions, calculated electron concentrations and sub-band energies for each case are given in Table 1. The energy of Landau levels E_{in} ('Landau fan') attached to each sub-band are also shown for each case (straight lines).

level-broadening parameter values considered. Figure 4 displays the situation for the same parameter values as in Figure 3 but at $T = 30$ K. For single band occupancy shown in Figure 4a, the variation in μ becomes almost identical for all broadening parameter values considered in this study. The same situation is observed for the cases when two and three sub-bands are occupied as well as depicted in Figures 4b and 4c, respectively. Although not shown here, the situation for lorentzian and exponential forms of broadening are also qualitatively identical to the case shown in Figures 4(a-c) for gaussian form of broadening. Therefore one can conclude that different level-broadening parameters have no considerable effect on μ at high temperatures for the range of level-broadening parameter values considered in this study.

One important effect of increasing the temperature is that the variation of chemical potential between consecutive Landau levels now becomes smoother, i.e. the negative slope of μ at critical magnetic field values, where it goes to a lower Landau energy level, becomes smaller as the temperature increases. It increases abruptly and then decreases slowly compared to the case of low temperatures (compare Figures 3 and 4).

Next we consider the effect of temperature on the behavior of μ at a fixed level-broadening parameter value. We first give the results for the temperature values $T = 4.2, 10, 20$ and 30 K when the gaussian level-broadening parameter is $\Gamma_G = 0.2$ meV. The temperature dependence of chemical potential when only one sub-band is occupied is shown in Figure 5a. When the temperature increases, higher Landau levels previously unoccupied at $T = 0$ also become populated. The low lying levels are no longer completely filled, they are partially occupied and the variation in chemical potential becomes smoother compared to low temperatures. Therefore the resulting behavior of chemical potential when temperature increases is very similar to that of the case obtained when the level-broadening parameter is increased at a fixed low temperature. Thus, although for different underlying reasons, note the similarity between Figures 3a and 5a. The results for the case of two and three sub-band occupancy are presented in Figures 5b and 5c, respectively. For two sub-band occupancy also note the similarity between Figures 3b and 5b, and for three sub-band occupancy that of 3c and 5c. The similarity between Figures 3 and 5 can be explained as follows: The total number of electrons depends on the product of density of states and Fermi-Dirac functions. Fermi-Dirac function smoothes out at about $E = \mu$ as the temperature increases. Similarly, the density of states functions used in this study broaden as the broadening parameter increases. Thus their product tends to smooth out when either of them is increased and as a result their values corresponding to neighboring Landau levels close to the chemical potential begin to overlap. This leads to blurred chemical potential oscillations with smaller amplitudes.

The variation of chemical potential with temperature for lorentzian and exponential forms of level-broadening qualitatively follow the same behavior as in gaussian broadening shown in Figures 5(a-c). If the broadening parameter is further increased, the results are still qualitatively similar to the case shown in Figure 5.

Finally we compare the three forms of level-broadening. The variation of chemical potential as a function of magnetic field is shown in Figures 6(a-c) at $T = 4.2$ K for three related values of level-broadening parameters for gaussian, lorentzian and exponential forms of broadening. The broadening parameters for lorentzian (Γ_L) and exponential (Γ_E) forms of broadening are related to the level-broadening parameter Γ_G for the gaussian case, as given in Equation (15). Figure 6a depicts the situation when $\Gamma_G = 0.2$ meV at $T = 4.2$ K. The variation in μ for the corresponding parameters when two and three sub-bands are occupied are shown in Figures 6b and 6c, respectively. For small values of broadening parameter the variation of chemical potential for each broadening form are close to each other. The distinction between different broadening forms at a given temperature become more pronounced as the level-broadening increases as shown in Figures 7(a-c). Figure 7 is the same as Figure 6 except that $\Gamma_G = 0.5$ meV.

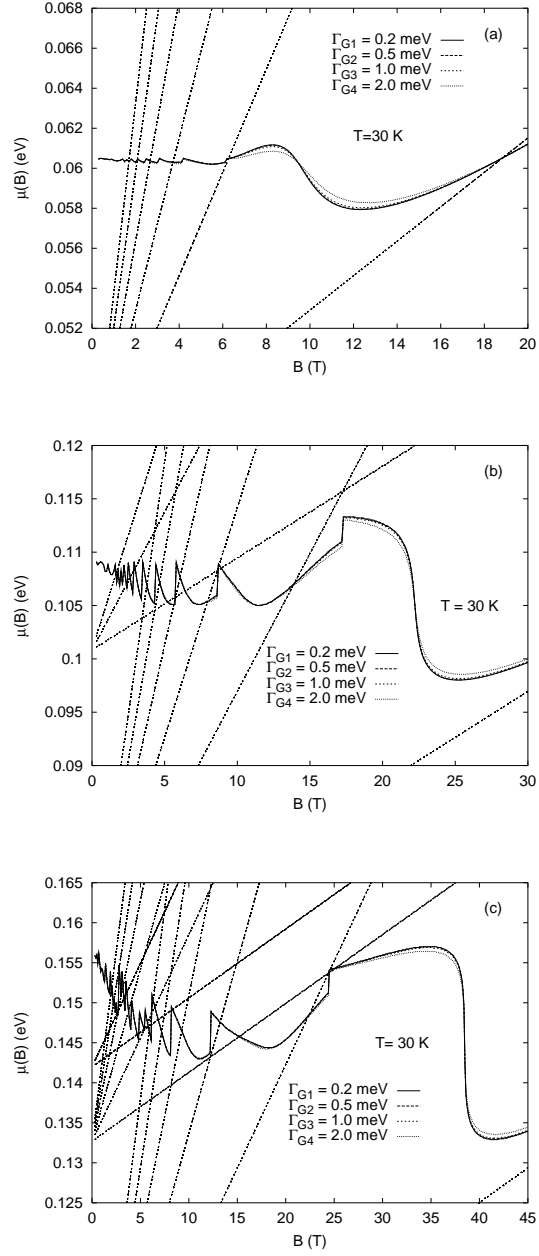


Figure 4. The same as Figure 3 but at $T = 30$ K. Note that the amplitude of the oscillations of μ decreases and different level-broadening parameters give very close results for each case as temperature increases.

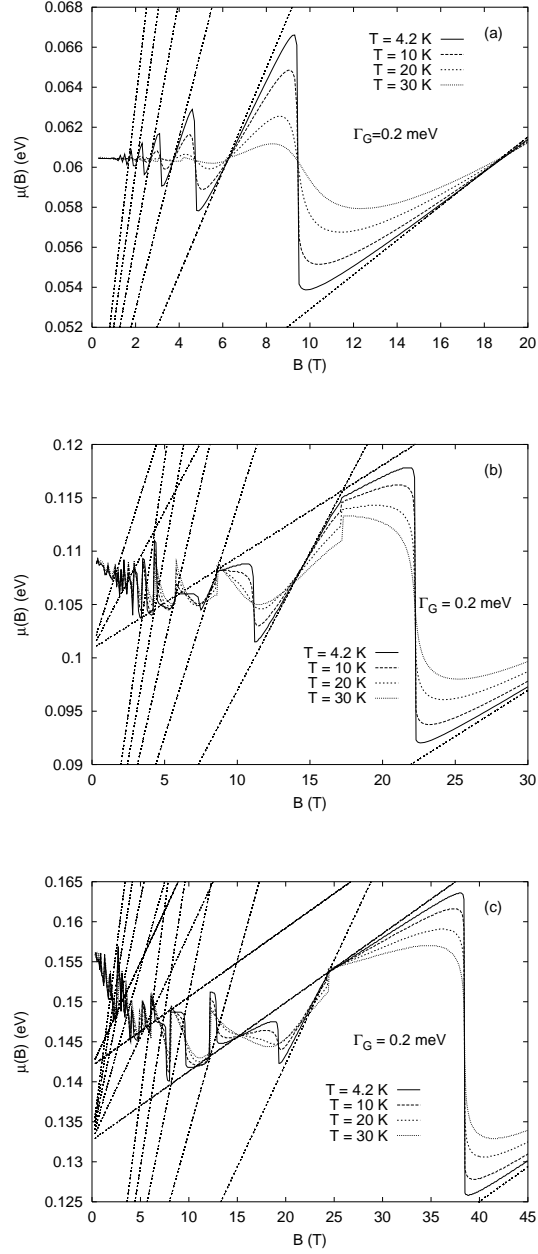


Figure 5. Oscillations in μ as a function of B for temperature values $T = 4.2, 10, 20$ and 30 K for the gaussian type of broadening when the level-broadening parameter is $\Gamma_G = 0.2$ meV when only one sub-band (a), two sub-bands (b) and three sub-bands (c) are occupied in the quantum well when no magnetic field is present. Note the similarity between Figures 3(a–c) and 5(a–c) in that order. The parameters of the hetero junctions, calculated electron concentrations and sub-band energies for each case are given in Table 1.

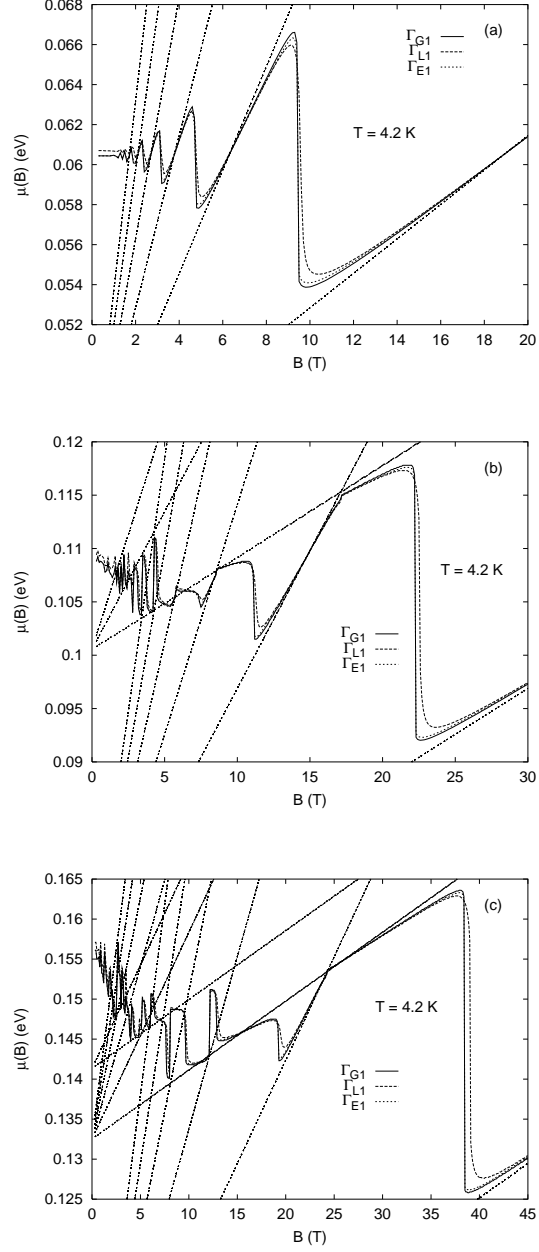


Figure 6. Comparison of gaussian, lorentzian and exponential forms of level-broadening at $T = 4.2$ K when only one sub-band (a), two sub-bands (b) and three sub-bands (c) are occupied. The level-broadening parameter for the gaussian case is $\Gamma_{G1} = 0.2$ meV and the level-broadening parameter values for the lorentzian (Γ_{L1}) and exponential (Γ_{E1}) forms of broadening are related to Γ_{G1} as given in equation (15), namely $\Gamma_{L1} = 1.2\Gamma_{G1}$ and $\Gamma_{E1} = 1.7\Gamma_{G1}$.

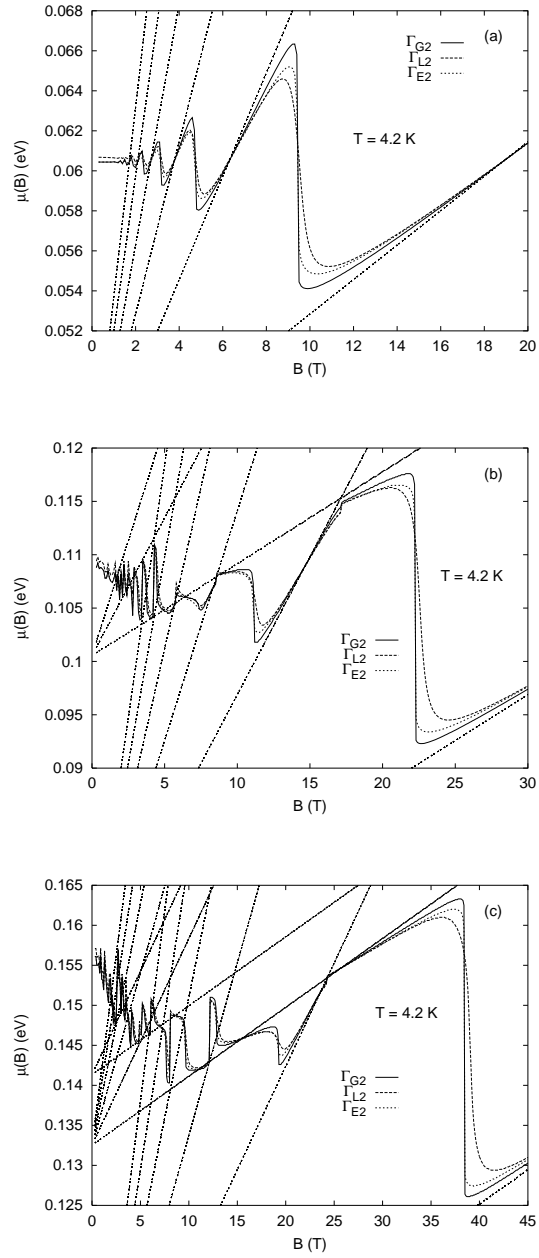


Figure 7. The same as Figure 6, but for the case when level-broadening parameter for the gaussian form is $\Gamma_{G2} = 0.5$ meV.

4. Conclusions

In this paper, we have investigated the effect of temperature and level-broadening parameter on the oscillations of chemical potential of a 2DEG under a constant magnetic field for three different forms of level broadening: gaussian, lorentzian and exponential. To make the comparison easier, the level-broadening parameter values are chosen in such a way that the widths of corresponding density of states functions become equal at half maximum value. The comparisons are also made with respect to the number of levels occupied by electrons in the GaAs/GaAlAs quantum wells.

Although for different underlying reasons, it is observed that increasing level-broadening at a fixed temperature or increasing the temperature at a given value of level-broadening parameter have qualitatively the same effect on the chemical potential variations for all cases considered. This is a result of overlapping of the product $D(E)f(E)$ for Landau levels in the close proximity of the chemical potential.

When the level-broadening parameters are small, the three forms of broadening gives almost identical results, especially at low magnetic fields for chemical potential oscillations for the Γ values considered.

Chemical potential oscillations have the sharpest variations for gaussian form of level-broadening at any given temperature or level-broadening parameter. Next comes exponential and lorentzian forms of broadening.

Field-dependent level-broadening [8] is also considered, but this subject is left out not to complicate the comparisons. Magnetization of the 2DEG have also been considered but the results are not presented here.

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