Turk J Phys
28 (2004) , 301 – 307.

Empirical Models for the Correlation of Monthly Average Daily Global Solar Radiation with Hours of Sunshine on a Horizontal Surface at Karachi, Pakistan

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Received 01.04.2004

Abstract

A new set of constants for Angstrom-type correlation of first and second order, to estimate monthly average daily global solar radiation, has been obtained employing sunshine hours data recorded at Karachi, Pakistan (Lat. 24° 54′ N, Long. 67° 08′ E). Least square regression is performed to derive these constants. The correlation equations developed are employed to calculate the monthly average daily global solar radiation. These results are then compared with various other existing correlations and the measured data. Excellent agreement has been found between the estimated and the measured values.

1. Introduction

The design of a solar energy conversion system requires precise knowledge regarding the availability of global solar radiation and its components at the location of interest. Since the solar radiation reaching the earth's surface depends upon climatic conditions of the place, a study of solar radiation under local climatic conditions is essential. In developing countries such as Pakistan, due to absence or malfunction of measuring instruments, reliable solar radiation data is not available. In Pakistan, Global Solar Radiation data on horizontal surface is recorded at only five stations: Karachi, Lahore, Quetta, Multan and Islamabad.

In the absence and scarcity of trustworthy solar radiation data, the need for an empirical model to predict and estimate global solar radiation seems inevitable. These models use climatological parameters of the location under study. Among all such parameters, sunshine hours are the most widely and commonly used. The models employing this common and important parameter are called sunshine-based models.

Sunshine-based models use only bright sunshine hours as input parameter while others use additional climatological data together with bright sunshine hours. In some of the models geographical and seasonal parameters are also taken into account to reflect the latitudinal and seasonal variation of the air mass.

The first empirical correlation using the idea of employing sunshine hours for the estimation of global solar radiation was proposed by Angstrom [1]. The Angstrom correlation was modified by Prescott and Page [2, 3]. Many researchers have employed hours of bright sunshine to estimate solar radiation [4–15]. Other workers, e.g. Reddy [16], Sayyigh [17], Glover and McCullouch [18], derived their equations by using sunshine duration, relative humidity, temperature and latitude of the locations under study. Reddy suggested the use of the number of rainy days, sunshine hours and a factor which depends on the geographical location of the place along with the latitude. Barbaro [19] related daily total solar radiation to the sunshine duration and the noon height of the sun on the 15^{th} of the given month.

The object of the work reported in this paper is the development of Angstrom-type polynomials of first and second order for the estimation of monthly average daily global solar radiation on horizontal surfaces at Karachi, Pakistan. The coefficients for developing these polynomials are derived by using least square regression analysis. These coefficients are generally valid for estimating the radiation in location of similar climate, latitude and altitude.

2. Methodology

The simplest model used to estimate monthly average daily solar radiation on horizontal surface is the well-known Angstrom equation

$$\overline{H}/\overline{H}_0 = a_1 + a_2 \left(\overline{n}/\overline{N}\right). \tag{1}$$

Values of $\overline{N}$ are computed from Cooper's formula [20]

$$\overline{N} = (2/15)\cos^{-1}\left(-\tan\phi\,\tan\delta\right). \tag{2}$$

$\overline{H}_0$ may be obtained by the Klein [21] relationship

$$\overline{H}_0 = \frac{24}{\pi} I_{sc}\left(1 + 0.33\cos\frac{360\overline{n}}{365}\right)\left(\cos\phi\,\cos\delta\,\sin\omega_s + \frac{2\pi\omega_s}{360}\sin\phi\,\sin\delta\right). \tag{3}$$

The sunset hour angle ω_s and the solar declination δ are defined by the relations

$$\omega_s = \cos^{-1}\left[\cos\left(-\tan\phi\,\tan\delta\right)\right] \tag{4}$$

$$\delta = 23.45\sin\left(360\frac{284 + d}{365}\right), \tag{5}$$

where d is the day of the year. Usually, the 15^{th} of each month is the day of the month on which the solar declination is calculated.

Glover and McCulloch [18] included latitude effect and presented the correlation

$$\overline{H}/\overline{H}_0 = 0.29\cos\phi + 0.52\left(\overline{n}/\overline{N}\right), \tag{6}$$

where ϕ is the latitude of the location under study. Rietveld [6] examined several published values of regression coefficient for Angstrom-type relations and suggested use of the following correlation:

$$\overline{H}/\overline{H}_0 = 0.18 + 0.62\left(\overline{n}/\overline{N}\right). \tag{7}$$

Using the data of bright sunshine hours and global radiation for 48 locations around the world, Bahel et al. [5] developed the relation

$$\overline{H}/\overline{H}_0 = 0.16 + 0.87\left(\overline{n}/\overline{N}\right) + 0.61\left(\overline{n}/\overline{N}\right)^2 + 0.349\left(\overline{n}/\overline{N}\right)^3. \tag{8}$$

Earlier, Ahmed et al. [22] estimated the global solar radiation on horizontal surface using a first order Angstrom-type correlation, whereas in the present study a second order polynomial of the form

$$\overline{H}/\overline{H}_0 = a_3 + a_4\left(\overline{n}/\overline{N}\right) + a_5\left(\overline{n}/\overline{N}\right)^2 \tag{9}$$

is developed. The regression constants $a_1,\ a_2,\ \ldots\ , a_5$ have been obtained from a general computer program specifically developed for the purpose of estimation. The program employees the Gaussian-Newton least squares technique. RMSE and MBE are calculated to assess the validity of estimation made through equations (1) and (9), and also to compare it with the model of Rietveld, Glover & McCulloch and Bahel et al.

3. Results and Discussions

Regression constants *ai* used in equation (1) and (9) have been computed using recent observations of sunshine hours and monthly average daily global radiation for Karachi [23], as given in Table 1.

Table 1. Input parameters for the estimation of monthly average daily global solar radiation at Karachi, Pakistan.

Month	$\overline{H}$	$\overline{H}_0$	$\overline{n}/\overline{N}$	$\overline{H}/\overline{H}_0$
Jan	15.89	23.98	0.805	0.662
Feb	18.09	28.18	0.776	0.642
Mar	21.16	32.96	0.762	0.642
Apr	22.57	37.07	0.738	0.608
May	23.56	39.29	0.743	0.599
Jun	22.73	39.94	0.595	0.569
Jul	19.21	39.49	0.381	0.486
Aug	17.91	37.78	0.390	0.474
Sep	19.84	34.33	0.602	0.578
Oct	19.35	29.50	0.818	0.656
Nov	16.68	24.92	0.837	0.669
Dec	15.04	22.72	0.830	0.662

All numerical values are in units of $MJm^{-2}d^{-1}$.

First- and second-order polynomials hence developed for Karachi are

$$\overline{H}/\overline{H}_0 = 0.324 + 0.405\left(\overline{n}/\overline{N}\right) \tag{10}$$

$$\overline{H}/\overline{H}_0 = 0.348 + 0.320\left(\overline{n}/\overline{N}\right) + 0.070\left(\overline{n}/\overline{N}\right)^2. \tag{11}$$

The third order polynomials were also developed, but did not further improve the present estimation (results not shown).

Figure 1 shows the variation of $\overline{n}/\overline{N}$ and $\overline{H}/\overline{H}_0$, the clearness index for Karachi. The dip in the months of June–August indicates poor sky conditions where $\overline{n}/\overline{N}$ goes as low as 0.391 and K_T values reaches minimum, i.e. 0.486 (for July) and 0.474 (for August). This is due to the fact that June–August corresponds to the monsoon season in this part of the world, with less sunshine hours and heavy overcast sky.

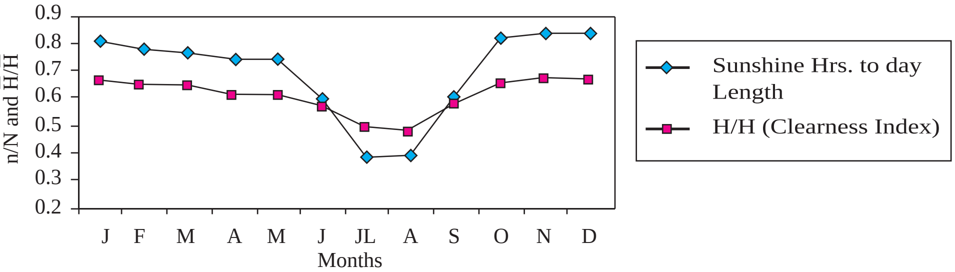

Figure 1. Variation of $\overline{n}/\overline{N}$ and $\overline{H}/\overline{H}_0$ (the clearness index) for Karachi, Pakistan.

Table 2. Estimation of monthly average daily global solar radiation from various models for Karachi, Pakistan.

Month	$H_{measured}$	Rietveld [6]	Glover and McCulloch [18]	Bahel et al. [5]	Equation 10	Equation 11
Jan	15.89	16.28	16.34	15.51	15.58	15.61
Feb	18.09	18.63	18.78	17.81	17.98	17.99
Mar	21.16	21.50	21.73	20.60	20.85	20.85
Apr	22.57	23.63	23.97	22.71	23.09	23.07
May	23.56	25.17	25.51	24.14	24.55	24.53
Jun	22.73	21.92	22.86	21.40	22.56	22.49
Jul	19.21	16.43	18.21	16.66	18.88	18.96
Aug	17.91	15.93	17.60	16.16	18.21	18.26
Sep	19.84	18.99	19.77	18.54	19.49	19.43
Oct	19.35	20.27	20.30	19.35	19.33	19.37
Nov	16.68	17.41	17.40	16.42	16.52	16.57
Dec	15.04	15.78	15.78	15.13	14.99	15.04

All numerical values are in units of $MJm^{-2}d^{-1}$.

The monthly average daily global solar radiation estimated through equations (10) and (11) for Karachi are given in Table 2, along with the measured values and the estimated values from the models of Rietveld [6], Glover and McCulloch [18] and Bahel et al. [5]. It is very encouraging to observe a very fine agreement between measured and estimated values obtained from our correlation.

Shown in Table 3 are the percentage errors, RMSE, MBE and Correlation Coefficient (CC) from the present work and these obtained from equations (6), (7) and (8). The percentage errors in Glover and McCulloch (eqn. (6)) ranges from −-0.57 to 8.28. Rietveld (eqn. (7)) gives a minimum of −-1.6 and maximum of 11.05, whereas for Bahel et al. (eqn. (8)) we get error as high as 13.2 percent.

In the present work, the percentage error for all the months are below 5.0%, which suggest that equations (10) and (11) can be used with confidence for Karachi, and also for other locations with similar climatological conditions. This statement is also supported by the high value of correlation coefficient.

The monthly average daily global solar radiation estimated by equations (10) and (11) are plotted with the measured data in Figure 2 and then compared with the other models in Figure 3.

The development of the Angstrom-type correlation of the first and second order will enable the solar energy research worker to use the estimated data with confidence because of its fine agreement with the observed data. These correlations will also be useful for the places with similar conditions and having no facilities of recording the global solar radiation data.

Table 3. Percentage Error, MBE, RMSE, Correlation Co-efficient, and Co-efficient of Determination.

Month	Rietveld [6]	Glover and McCulloch [18]	Bahel et al. [5]	Equation 10	Equation 11
Jan	–2.45	–2.83	2.39	1.90	1.76
Feb	–2.98	–3.81	1.54	0.60	0.55
Mar	–1.60	–2.70	2.64	1.46	1.46
Apr	–4.70	–6.20	–0.62	–2.30	–2.21
May	–6.83	–8.27	–2.46	–4.20	–4.10
Jun	3.56	–0.57	5.85	0.74	1.05
Jul	14.47	5.20	13.27	1.71	1.30
Aug	11.05	1.73	9.77	–1.67	–1.95
Sep	4.28	0.35	6.55	–1.76	2.06
Oct	–4.75	–4.90	zero	0.10	–0.10
Nov	–4.37	–4.31	1.55	0.96	0.66
Dec	–4.92	–4.92	–0.60	0.33	zero
MBE	–0.0066	0.5166	–0.633	——	0.01166
RMSE	1.266	0.9055	1.08	0.396	0.387
CC	0.913	0.972	0.952	0.992	0.974
CD	0.834	0.945	0.906	0.984	0.949

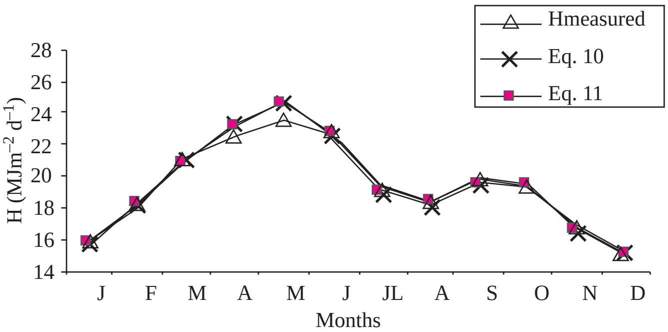

Figure 2. Estimated value of monthly average daily global solar radiation from equations (10) and (11), and comparison with measured data.

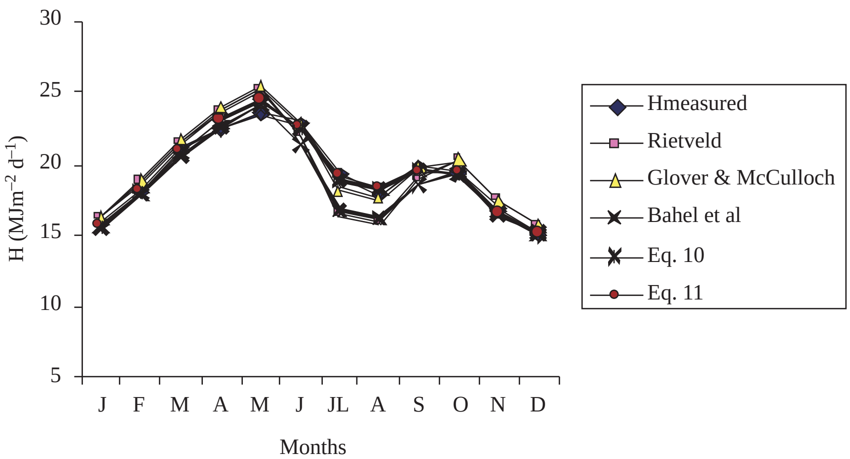

Figure 3. Comparison of the estimated value of monthly average daily global solar radiation from various models, with the measured data.

Nomenclature

a_1, a_2, . . . ,a_5 = Angstrom Regression Coefficients
$\overline{H}$ = Monthly average daily global Solar Radiation on horizontal surface
$\overline{H}_0$ = Extra Terrestrial Solar Radiation
$\overline{n}$ = Monthly Average Daily sunshine hours
N = Monthly Average Day Length
Φ = Latitude of the Place
Isc = Solar constant
δ = Solar Declination
ω_s= Sunset hour angle

Acknowledgement

The authors are thankful to the Pakistan Meteorological Department, Computerized Data Processing Center (CDPC), University Road, Karachi, for providing the requisite data.

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