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Single Mode Optical Radiation Distribution and Reflectivity Calculations in Novel-Hot Electron Light Emission and Lasing In Semiconductor Heterostructures VCSELs

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Abstract

In this work, we calculate the power reflectivity in vertical cavity surface emitting lasers (VCSELs) using a new method. In VCSELs, the stop band of the reflectivity spectrum should exhibit a dip at the lasing wavelength, which is a condition for lasing. This current approximation method gives a simple analytical expression to find the power reflectivity as a function of wavelength in the vicinity of lasing wavelength, $\lambda_\circ$. The proposed method can generally be applied to semiconductor VCSEL systems for a given lasing wavelength.

1. Introduction

Various light emitting devices have been developed with the advanced crystal-growth technologies that have been developed in the last two decades. This progress has led us to design and produce light emitting semiconductor devices based upon transport parallel to the quantum well layers, as suggested by N. Balkan et al. [1–3]. Although emission bandwidth of a typical light emitting diode (LED) is 20–40 nm, emission bandwidth of VCSEL devices is 10–20 nm, depending on wavelength. Lasing wavelength is determined by the discrete energy levels of carriers confined in the quantum well/s placed in the active region of devices. Inversion population is obtained through either vertical transport of carriers along the growth direction or in-plane transport of carriers injected from the adjacent n-p doped regions by tunneling and thermionic emission. Quantum well/s is/are placed near the standing wave maximum of the radiation electric field to increase the stimulated emission in the oscillating field. Light emission is, unlike conventional ones, at right angles to the in-plane direction in vertical cavity surface emitting lasers (VCSELs).

VCSELs are often different from conventional laser diodes in that the beam travels at right angles to the active region instead of emitting parallel to the active layer. A VCSEL has top and bottom Distributed Bragg Reflectors (DBRs) which consist of alternating quarter wavelength layers having higher and lower refractive indices. Here, a basic problem associated with the VCSEL design would be indicated by the following property: if the layers of the Bragg reflectors have optical thickness nd equal to one-quarter the lasing wavelength, and if the optical thickness of the active layer is an integer multiple of half of the lasing wavelength, then the stop band will not exhibit a dip at the lasing wavelength.

Two different methods can be used to obtain the dip in the stop band: one is to change slightly the width of Bragg layers; and the other is to change the width of the active layer. One of the earliest approaches to the calculation of reflectivity was shown by T. Sale [4], who calculated the phase in terms of the rate of change of the refractive index with wavelength ($dn/d\lambda$). This would have led to an analytical approach if he had realized that ($dn/d\lambda$) is negligibly small in the vicinity of the lasing wavelength. In this study, neglecting ($dn/d\lambda$) in the vicinity of lasing wavelength results in a new approximate method for a single mode TE or TM to obtain the dip in the center of the stop band [4–5] of the power reflectivity.

Theoretical description as well as the method of calculation is given in Section 2. The results are compared to those obtained by an exact method (transfer matrix method) in the vicinity of the lasing wavelength.

2. Theory

Electric and magnetic field components of radiation at each interface (parallel to the interface) can be related as follows [5]:

$$\begin{vmatrix} E_i(0) \\ \mu_\circ H_i(0) \end{vmatrix} = \begin{vmatrix} Cos\,\delta_i & (i/n_i)\,Sin\,\delta_i \\ in_i\,Sin\,\delta_i & Cos\,\delta_i \end{vmatrix} \cdot \begin{vmatrix} E_i(d_i) \\ \mu_\circ H_i(d_i) \end{vmatrix} \tag{1}$$

Expecting continuity of the fields at each interface (of thickness d_i) for two different consecutive materials, the matrix

$$M = \prod_{i=0}^{N-1} M_i(d_i) M_{i+1}(d_{i+1}) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \tag{2}$$

relates the output components of the fields to the input values for multiple layers. In equation 1, $\delta_i = (2\pi/\lambda)N_i d_i$ is optical phase; in eq. 2, $N_i = n_1$ - ik is the complex refractive index and d_i is the i^{th} layer thickness. The phase of the reflected field from the top surface is the same as that of the incident field when n_i ¡ n_{i+1}. For a given pair of N_i (where lossless refractive indexes are n_1 and n_2, and optical phases are δ_1 and δ_2, respectively) with the substrate refractive index n_s, in the absence of absorption, the reflectivity at the Bragg condition is given by setting $\delta_1 = \delta_2 = \pi/2$:

$$r = \frac{n_\circ - n_s(n_1/n_2)^{2N}}{n_\circ + n_s(n_1/n_2)^{2N}}, \tag{3}$$

where N is the number of periodic pairs. Using Kramers-Kröning relations, loss parameters k_i can be shown to be very small in comparison with known real refractive indices n_i which depends on the wavelength [6]. Therefore, in the lossless case, in the vicinity of the Bragg wavelength $\lambda_\circ$ (*i.e.* $\lambda_\circ \pm \delta\lambda$), reflectivity can be obtained for N on a substrate as,

$$r = \tanh\left[-.5Ln \frac{i\frac{\pi}{2}\left(-\frac{\delta\lambda}{\lambda_0}(n_1+n_2)\sum_{i=1}^{N/2}\left(\gamma^{2i-1}+\frac{1}{\gamma^{2i-1}}\right)\right)+\frac{1}{\gamma^N}n_s}{\gamma^N + n_s\left[i\frac{\pi}{2}\left(-\frac{\delta\lambda}{\lambda_0}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\sum_{i=1}^{N/2}\left(\gamma^{2i-1}+\frac{1}{\gamma^{2i-1}}\right)\right)\right]}\right], \tag{4}$$

where $n_2$and n_1 the refractive indices of the periodic layers, $\gamma = n_2/n_2$, and n_s is refractive index of substrate material. This equation does not depend on the sequence of the layers which may have high-low or low-high index order. In this approximation, the change of refractive indices in the vicinity of $\lambda_\circ$is neglected (*i.e.* it is assumed $dn/d\lambda|_{\lambda=\lambda_o} \cong .001nm^{-1}$). Reflectivity r in eq. 4, approaches the true value for $\delta\lambda \leq 10nm$. Equation 4 also gives eq. (3) when $\delta\lambda \to 0$, as expected. In this interval, the reflectivity of the full lasing structure can be easily examined with the approximation given above, instead of eq. (2), since eq. (2) does not yield an analytical equation for r in the same interval. Similarly, the approximation of McLeod [5] does not generally give correct results in the vicinity of the lasing wavelength. Figure 1 shows the exact power reflectivity obtained from eqs. (1–3) by using transfer matrix method (solid line) and approximate solution in the neighborhood of the lasing wavelength as given in eq. 4 (dashed line) for single DBRs with $N = 20$.

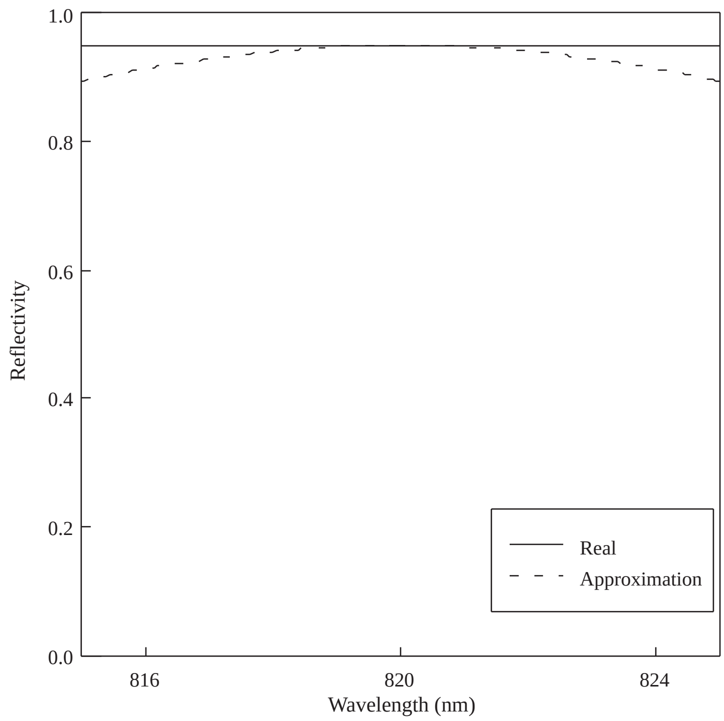

Figure 1. Comparison of power reflectivity for single DBRs (for 20 periods) calculated using exact matrix transfer method (solid curve) with calculated using a simple approximation (dashed curve) as a function of $\delta\lambda$ in eq. 4.

An example of the numerical calculation consists of n_1(AlAs) and n_2 ($Ga_{.7}Al_{.3}As$) Bragg pairs while n_3(GaAs) is chosen as the substrate. Optical thickness of the active region placed between the upper and lower DBRs must be an integer multiple of the lasing wavelength $\lambda_\circ/2$ for a lasing structure (Fabry-Perot). Optical phase of the active layer is given by $\delta_{iac} = 2\pi/[\lambda_\circ(1 + \delta\lambda/\lambda_\circ)]p\lambda_\circ/2$ in the vicinity of the lasing wavelength. For $\delta\lambda << \lambda_\circ$, the optical phase ($n_{ac}d$) of the active region takes the form

$$\delta_{iac} = [\pi(1 - \delta\lambda/\lambda_\circ)]p, \tag{5}$$

where p is a positive integer, and thus $Cos\,\delta_{ac} \cong \pm 1$ and $Sin\,\delta_{ac} \cong \pi\delta\lambda/\lambda_\circ$. Phases of the Bragg layers corresponding to optical thickness near the lasing wavelength are

$$\delta_{n1} = \delta_{n2} = 2\pi/[\lambda_\circ(1 + \delta\lambda/\lambda_\circ)](2p + 1)\lambda_\circ/4 = \pi/2(1 - \delta\lambda/\lambda_\circ)(2p + 1)$$

and

$\text{Cos}[\pi(p + 1/2)(1 - \delta\lambda/\lambda_\circ)] \cong \pm\pi(p + 1/2)\delta\lambda/\lambda_\circ$

by means of $\mathrm{Sin}[\pi(p+1/2)(1-\delta\lambda/\lambda_\circ)] \cong \pm 1$ for small p (namely $p = 0$, 1). If the structure has even (odd) pairs of quarter wave layers for both upper and lower DBRs, and the active region has an optical width of $\lambda_\circ/2$, in the lossless case, reflectivity can be approximated to

$$r = \tanh\left[-1/2\ln\frac{-in_3\pi\delta\lambda/\lambda_\circ u \mp v/\gamma^N \mp f^N bw - f^N f^M bz}{-\gamma^N u \mp i/n_3\pi\delta\lambda/\lambda_\circ v \mp f^N as - f^N f^M at}\right], \tag{6}$$

where $\mp$ is (—-) for even and (+) for odd number of layers; $a = 1/n_1 + 1/n_2$; $b = n_2 + n_2$; N and M are periodicity numbers for upper and lower DBRs, respectively; and other variables in eq. 6 are as follows

$$f^{N(M)} = \left\{\begin{array}{l} -i\pi/2\delta\lambda/\lambda_\circ \sum\limits_{n=0}^{N(M)/2-1} (\gamma^{2n+1} + 1/\gamma^{2n+1}), N(M) is even \\ i\pi/2\delta\lambda/\lambda_\circ \sum\limits_{n=0}^{(N(M)-1)/2} (\gamma^{2n} + 1/\gamma^{2n}) - 1, N(M) is odd \end{array}\right\}$$

where

$$u = \gamma^M \pm f^M an_s, \quad v = f^M b \pm n_s/\gamma^M,$$

$$w = \gamma^M + i/n_3\pi\delta\lambda/\lambda_\circ n_s/\gamma^M, \quad z = i/n_3\pi\delta\lambda/\lambda_\circ b + an_s,$$

$$s = in_3\pi\delta\lambda/\lambda_\circ\gamma^M + n_s/\gamma^M, \quad t = b + in_3\pi\delta\lambda/\lambda_\circ an_s,$$

while $n_3$and n_s are active and substrate refractive indices, respectively. Comparison of the power reflectivity for exact calculation (solid curve) and our approximation (dashed curve) is presented in Figure 2 for 18 periods of upper DBRs and 28 periods of lower DBRs placed on top of the substrate. The active region between upper and lower DBRs has an optical thickness of $\lambda/4$.

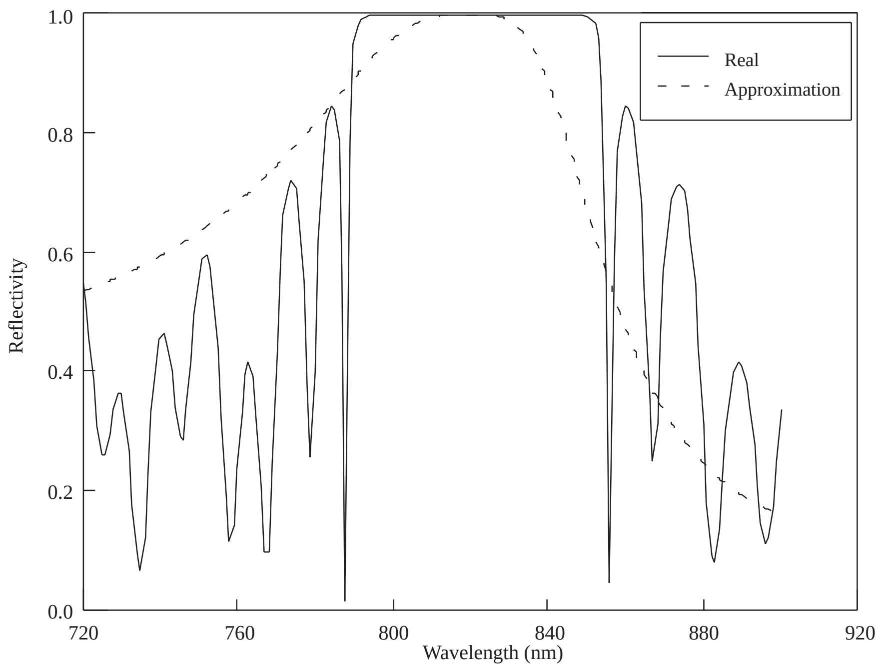

Figure 2. Power reflectivity versus wavelength for VCSEL; dashed curve is obtained from approximated calculation eq. 6 and solid curve shows the power reflectivity from the exact calculations using eqs. (1–3).

The approximate reflectivity shown in Figure 2 is the same as that obtained from the exact calculation in the vicinity of $\lambda_\circ$. On the other hand, reflectivity spectrum obtained with approximation shows that smoothness near the lasing wavelength coincides with the exact calculation of reflectivity, and its width is a parameter to find the lasing windows. During lasing, power reflectivity decreases at the lasing wavelength because the reflected part of the oscillation field provide the feedback needed for stimulated emission. For large number of pairs $N(M)$, a DBR has a wavelength region called "stop band" in which the reflectivity is close to unity, whose center is at the wavelength, $\lambda_\circ$. Thus, a "dip" at the center of the stop band has a power reflectivity which is smaller than the reflectivity in the rest of the stop band. In order to obtain this dip, two different methods can be used: (a) width of the Bragg reflectors can be taken slightly different from $\lambda_\circ/4n$; or; (b) width of the active region may be chosen slightly different from $\lambda_\circ/2n$ for a single mode. We preferred to apply the second option for the approximate calculation of reflectivity to prevent non in-phase radiation in the cavity. If the refractive index of active region is n_3, the optical phase δ_{ac}in the vicinity of lasing wavelength is

$$\delta_{ac} = \pi(1 - \delta\lambda/\lambda_\circ)(p + 2n_3 dl/\lambda_\circ), \tag{7}$$

where p is any integer and indicates the multiple of $\lambda_\circ/2$. The last term in eq. 7 can be rewritten as $1/2 + \xi\lambda/\lambda_\circ$, since $\xi\lambda = 2n_3 dl - \lambda_\circ/2$. For a single mode ($p = 1$), the optical phase of active region can be approximated as $\delta_{ac} \cong 3\pi/2 - \pi((p + 1/2)\delta\lambda/\lambda_\circ - \xi\lambda/\lambda_\circ)$. In this case, for the reflectivity of the structure, we obtain the relation for the even (odd) number of Bragg pairs, (N and$M$ are number of pairs in the top and the bottom Bragg reflectors, respectively) as

$$r = \tanh\left[-.5\, ln \frac{-f^N f^M bu \mp f^M/\gamma^N v \mp f^N bw - z/\gamma^N}{-f^N f^M av \mp f^N az \mp f^M \gamma^N u - \gamma^N w}\right] \tag{8}$$

where,$\mp$ is $(-)$ for even and $(+)$ for odd number of layers,

$$a = 1/n_1 + 1/n_2,$$

$$b = n_1 + n_2,$$

$$u = i/n_3 b + a\kappa n_s,$$

$$v = \kappa b + i n_3 a n_s,$$

$$w = \kappa\gamma^M + (i/n_3) n_s/\gamma^M,$$

$$z = i n_3 \gamma^M + \kappa n_s/\gamma^M,$$

$$\kappa = \pi((p + 1/2)\delta\lambda/\lambda_\circ - \delta\xi/\lambda_\circ),\ p$$

indicates the integer multiple of $\lambda_\circ/2$, and n_3 is the index of refraction of the active region. Equations (6) and (8) can also be simplified for small N and M because the summation given in $f^{N(M)}$ is also small and thus higher terms of $\delta\xi$ in the matrix products are ignored.Reflectivity spectrum given in Figure 3 is the same as in Figure 2 (power reflectivity between 700 nm and 900 nm is also illustrated in Figure 4); but now, it has a window at 820 nm, as expected. Numerical interpretation using eq. (8) shows that for dl = -53 nm, power reflectivity is the same as Figure 2, but dips at the lasing wavelength of 820 nm.

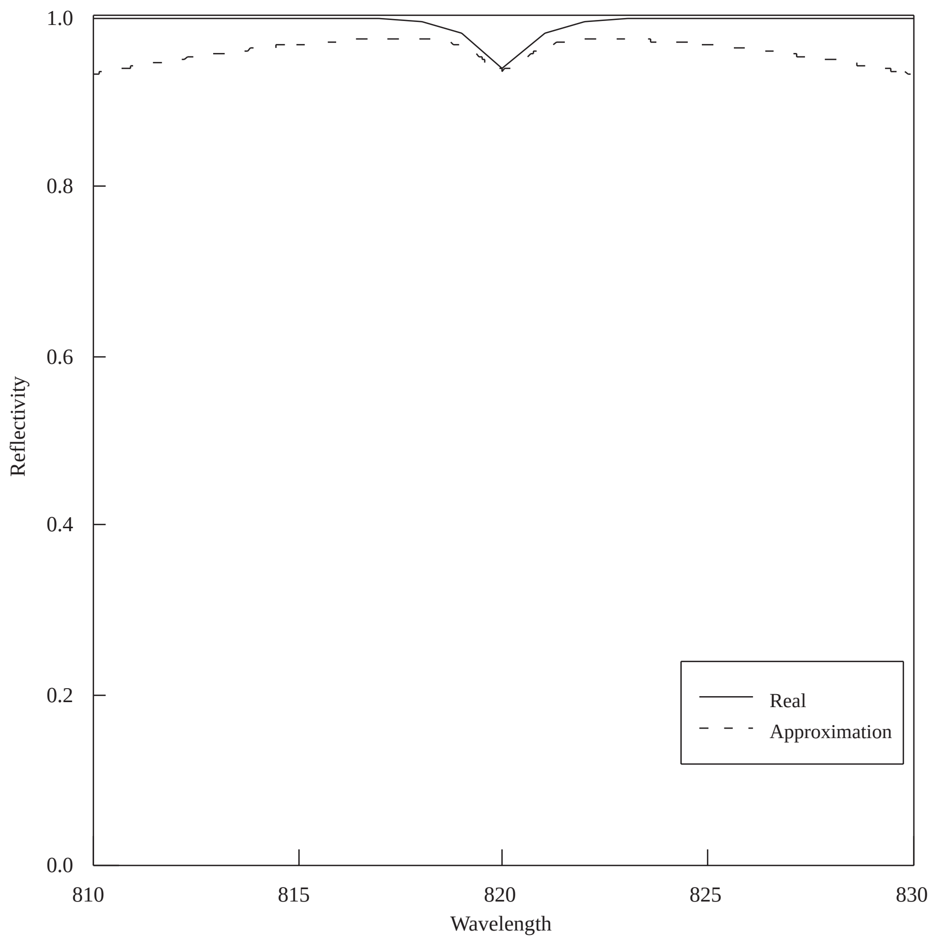

Figure 3. Power reflectivity of the single mode laser structure. Solid curve shows the exact calculation by using eqs. (1–3) and dashed line represents the approximation by using eq. (8).

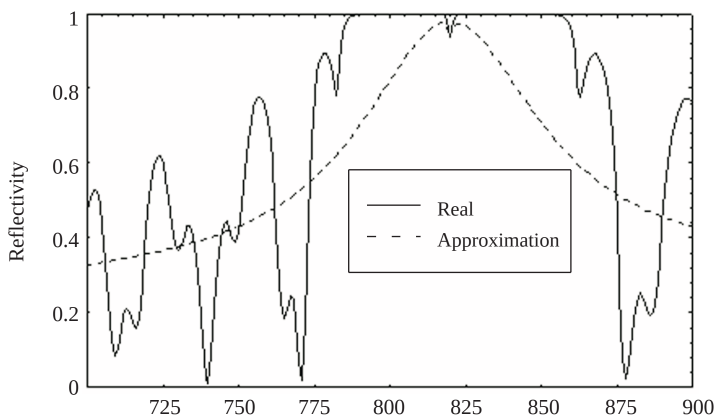

Figure 4. Power reflectivity to show between 700 nm and 900 nm for exact calculation and present approximation method. Dashed curve shows present approximation method and solid curve exact calculation obtained from transfer matrix method.

3. Conclusions

We have investigated a simple approximation to find the reflectivity and its phase relation around the neighborhood of the lasing wavelength. In modeling of a VCSEL, this method is considered to be very useful because of its simplicity. The light emitting dip can be easily obtained using the equations given above by just varying the active layer thickness with a small of amount dl. This calculation technique is found to be a very useful method to find the lasing dip with typical parameters of layers such as the periodicity number and the width of Bragg layers.

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