# Cyclotron Resonant Heating and Acceleration of protons, O VI \& Mg X ions in the North Polar Coronal Hole 

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#### Abstract

Ultraviolet Coronograph Spectrometer aboard Solar and Heliospheric Observatory (SOHO) has revealed the existence of temperature anisotropy of many minor ion species that populate the north solar polar coronal hole. In this study, we examined the propagation characteristics of ion-cyclotron waves which resonate with protons, O VI and Mg X ions. In our empirical model, radial variation of the magnetic field, electron and ion number densities, perpendicular and parallel temperature of protons, O VI and Mg X ions are adopted from the measurements of various instruments aboard SOHO. Since most of the measurements of SOHO instruments begin at $1.5 R\left(=r / R_{\odot}\right)$ where the collisionless plasma properties dominate towards the outer regions, we assumed bi-Maxwellian velocity distribution functions for protons, O VI and Mg X ions. Dispersion relation for the left circularly polarized ion cyclotron waves which are assumed to be generated at the bottom of the coronal hole is deduced. We do not consider the in situ generation of these waves. We found that ion cyclotron waves with frequency band $2.5 \mathrm{kHz}-10$ kHz come into resonance with O VI ions between $1.5 R-3.0 R$. We also solved the dispersion relations for protons and Mg X ions in the same distance range and have shown that the waves of $2.5 \mathrm{kHz}-10$ kHz frequency band preferentially heat the O VI ions. O VI ions come into resonance with ion cyclotron waves of certain frequency long before the protons and the Mg X ions. We do not know the evolution of the power spectrum of these waves. Therefore it is hard to tell if, after having been depleted partly (if not, totally) by O VI ions, any power is left over from ion cyclotron waves for either protons or $M g X$ ions. But if cyclotron resonance process is to be favored for the perpendicular heating, then we may hypothesize that ion cyclotron waves, after having heated the $O V I$ ions first, have enough power left to heat protons and $M g X$ ions in the same distance range.


Key Words: Sun: solar corona: ion-cyclotron waves, heating

## 1. Introduction

### 1.1. Line Width Measurements

Mg X and O VI lines obtained by SOHO/UVCS have revealed that the north solar polar coronal hole plasma changes its property from collisional to collisionless in the radial distance range $1.75 R-2.1 R$ ([1]; [2]). Using the SOHO/UVCS and SUMER data base, Doyle et al. [3] calculated the collisional/collisionless border as $1.5 R$. This means that in the relevant distance range, plasma transport processes should be treated not in
the classical Coulomb context. Vocks [4] and Vocks and Marsch [5] have recently treated the same problem by considering both, wave-particle interactions and Coulomb collisions. However, we shall not follow this highly sophisticated model, and assume that the coronal hole plasma in the distance range $1.5 R-3.0 R$ is collisionless. Extraordinarily large line widths observed by Kohl et al. [6] beyond $2.0 R_{\odot}$ could be partially due to ion-cyclotron resonance process (see, e.g., [7]). While the effective temperature of O VI ions may be as high as $10^{8} \mathrm{~K}$ at about $2.0 R$, the temperature anisotropy with $T_{\perp} \sim 10^{2} \times T_{\|}$is inferred from Doppler dimming analysis ([8]). A collisionless and a low $\beta=N_{e} k_{B} T_{e} /\left(B^{2} / 2 \mu_{o}\right)$ (where the symbols have their usual meanings) plasma is expected to display temperature anisotropy. SOHO/UVCS measured the line of sight velocities of plasma particles in the north solar polar coronal hole. These velocities translate into the perpendicular (to the local magnetic field) temperatures ([9]; [7]]. The result is, e.g., $T_{\perp}^{O V I}>T_{\perp}^{H e^{++}}>$ $T_{\perp}^{p}>T_{\perp}^{e}$ where superscripts, $e$ and $p$ refer to electrons and protons, respectively. Besides, perpendicular temperatures of every species are greater than the parallel ones, e.g., $T_{\perp}^{O V I}>T_{\|}^{O V I}$. The fine structure of the coronal hole displays denser and cooler plumes and less dense and hotter interplume lanes([10]). Images taken by SOHO/SUMER O VI ( $\lambda 1032$ ) clearly show the lanes and interplume lanes. Line widths obtained from interplume lanes are wider than those obtained from plumes ([10]). This situation implies that perpendicular heating is more effective in interplume lanes than other areas. Outflow velocities in the solar wind of O VI ions heated in interplume lanes will be greater. Since the ions with greater perpendicular velocities feel the greater mirror force, $F_{m f}(R)=-\mu \partial B / \partial R$, where $\mu(R)=(1 / 2) m_{i} v_{\perp}^{2}(R) / B(R)$ is the magnetic moment, the origin of the fast solar wind is identified as the interplume lanes ([11]; [12]; [13]; [10]).

Outflow velocities of O VI ions measured by using the Doppler dimming diagnostic is of the order of $v_{1 / e} \sim 130-230 \mathrm{kms}^{-1}([8])$. The relation between the ion temperature and the effective temperature is given by $T_{\text {eff }}=\left(m_{i} / 2 k_{B}\right) v_{1 / e}^{2}=T_{i}+\left(m_{i} / 2 k_{B}\right) \xi^{2}$, where $v_{1 / e}$ is the most probable speed along the line of sight and $\xi$ is the most probable speed of the turbulent velocity space which is assumed to show isotropic and gaussian distribution. The effective temperature of O VI ion at $3.0 R$ is found to be, $T_{\text {Leff }}^{O V I} \sim 2 \times 10^{8} \mathrm{~K}$ ([1]). Figure 2 of Antonucci et al. [14] shows the effective temperature profile of O VI ions in the distance range of $1.5 R-3.0 R$. The contribution to the line of sight temperature may come from non-thermal processes the most favourable of which is ion-cyclotron resonance one. The motivation given to this idea come from not only $\mathrm{SOHO} / \mathrm{UVCS}$ data but also from earlier data acquired by Helios $1 \& 2$ ([9]). Reduced velocity distribution function contour maps of protons in the solar wind shows an anisotropy with $T_{\perp}^{P}>T_{\|}^{P}$ (Figure 8.19a of [9]) and the magnetic moment variation of $\alpha$ particles with radial distance (Figure 8.25 of [9]) also give rise to the idea that the source of perpendicular heating is highly probably ion cyclotron resonance process ([25]).

The dispersion relation which we are going to present in the next section contains plasma frequency of O VI ions. It is a function of particle number density. The particle number density of O VI ions in the transition region from the chromosphere to the corona is taken as $N_{O V I}=10^{-3} N_{p}$ ([10]; [4]). Raymond et al. [15] gives $N_{O V I}=6.8 \times 10^{-5} N_{p}$ at the center of the coronal streamer. Assuming that these abundances are also characteristic for the coronal hole, we shall use both the values of $N_{O V I}$ given by Wilhelm et al. [10] and Raymod et al. [15] and compare the results in our figures below. On the other hand, coronal hole plasma is regarded as electrically neutral; this means that, $N_{e}=N_{p}=N([16] ;[7] ;[17])$. The Solar wind consists of electrons and protons with an admixture of a few percent helium and heavy but much less abundant ions in different ionization states. Therefore it may be approximated as an electron - proton fluid. Of course, together with minor ion species it is treated under the assumption of quasi-neutrality. Hollweg ([11], [12]) reports the electron number density distribution measured by Feldman et al. [18] as

$$
\begin{equation*}
N_{e}=\frac{3.2 \times 10^{8}}{R^{15.6}}+\frac{2.5 \times 10^{6}}{R^{3.76}}+\frac{1.4 \times 10^{5}}{R^{2}} \mathrm{~cm}^{-3} \tag{1}
\end{equation*}
$$



Figure 1. The variation of the wavenumbers with respect to radial distance from the Sun. Waves are assumed to be launched from the coronal base. In the distance range, $1.5 R-3.0 R$ the waves with frequency range $2.5 \mathrm{kHz} \leq \omega \leq$ 10 kHz resonate with O VI ions. Thick $(+) \mathrm{s}$ are for $N_{O V I}=10^{-3} N_{p}$ and thin dots are for $N_{O V I}=6.8 \times 10^{-5} N_{p}$, the latter of which apparently confines the resonance region to a much narrower R range. The locations where the wavenumbers (ks) go infinity are ion-cyclotron resonance regions. $k$ graphs are for waves with frequencies (a) 2.5 kHz , (b) 5 kHz , (c) 7.5 kHz , and (d) 10 kHz .

### 1.2. Polar Coronal Hole Magnetic Field

We use the magnetic field model given by Hollweg [11].

$$
\begin{equation*}
B=1.5\left(f_{\max }-1\right) R^{-3.5}+1.5 R^{-2} G \tag{2}
\end{equation*}
$$

with $f_{\max }=9$. The range of validity of this model seems to be $1-10 R$, which covers the range we are interested in. Ion - cyclotron frequency, $\Omega_{i c}=q_{i} B / m_{i}$ is a function of the magnetic field intensity. We
use Eq. (2) wherever $\Omega_{i c}$ appears in the dispersion relation of the ion cyclotron waves, and the magnetic moment of $O V I, M g X$ ions and protons.


Figure 2. The same as Figure 1 but for Mg X ions.

### 1.3. Ion - cyclotron waves

In the present investigation we are not touching upon the wave generation mechanisms. We assume that ion cyclotron waves are generated by some mechanism at the coronal base and propagate along the open magnetic field lines into the corona. But we should mention in passing that, especially magnetic reconnection is believed to provide the necessary wave flux needed for heating ([16]). These authors claim that the scale lengths of reconnection processes fall into the range $10^{5}-10^{7} \mathrm{~m}$. From the dispersion relation of Alfven waves one can deduce that, $\omega_{k}=2 \pi V_{A} / \lambda$ should be in a frequency range of $1-10^{2} H z$. However, we show later that the frequency range of ion cyclotron waves that come into resonance with O VI ions is $2.5 \mathrm{kHz}-$ 10 kHz .


Figure 3. The same as Figure 1 but for protons.

The information about the amplitude of the ion cyclotron waves is deduced from the non-thermal velocity, $\xi$. The relation between the wave amplitude $\left\langle\delta v^{2}\right\rangle$ and the non-thermal velocity is given by Banerjee et al [19] as $\xi^{2}=(1 / 2)\left\langle\delta v^{2}\right\rangle$. Observations give the wave amplitude value $\left\langle\delta v^{2}\right\rangle=2 \times\left(43.9 \mathrm{~km} \mathrm{~s}^{-1}\right)^{2}$ at heights corresponding to 120 " off the solar limb. Wave flux density of Alfven and ion cyclotron waves are given by the relation, $F_{w}=\sqrt{\rho / 4 \pi}\left\langle\delta v^{2}\right\rangle B$. Using the wave amplitude value given above, $B=5 G$ and $N_{e}=4.8 \times 10^{13} \mathrm{~m}^{-3}$ at $r=1.25 R$, Banerjee et al. [19] find the wave flux density as $F_{w}=4.9 \times 10^{5} \mathrm{erg} \mathrm{cm}^{-2} \mathrm{~s}^{-1}$. This value is high enough to replace the coronal energy lost by optically thin emission and heat conduction to the transition region below ([32]). Power spectra of the ion - cyclotron waves in coronal hole are not known. High frequency waves are generated either directly or by turbulent cascade from low to high frequencies ([20]).

### 1.4. Wave-Particle Interaction

The most efficient way of transferring wave energy into the internal energy of particles is ion - cyclotron resonance process. Kohl et al [6] show that the temperature anisotropy of O VI ions in coronal hole is considerably high $\left(T_{\perp} / T_{\|} \sim 10-100\right)$ and most importantly, whatever the heating process, it favours O VI ions among many other minor ion species. Cranmer [21] examines the dissipation of ion cyclotron resonant Alfven waves in the extended corona in detail. He considers the wave damping arising from more than 2000 minor ion species. One has to refer to this paper for the recent O VI and $M g X$ results and earlier work on ion - cyclotron resonance process. In the next section we present our model and its results.

## 2. The Model

We start with the wave equation derived for the cold plasma. In this study we make plane wave assumption, i.e., space - time variation of the perturbed quantities is given as, $\exp [i(\omega t-\mathbf{k} \cdot \mathbf{r})]$ where $\omega$ is the wave frequency, $t$ is time, $\mathbf{k}$ is the wave vector and it is assumed that $k_{\perp}=0$ and $\mathbf{r}$ is distance from the source. The wave equation, in terms of Fourier components is written as below:

$$
\begin{equation*}
\mathbf{k} \times(\mathbf{k} \times \mathbf{E})+\frac{\omega^{2}}{c^{2}} \kappa \cdot \mathbf{E}=0 \tag{3}
\end{equation*}
$$

where $\kappa$ is the dielectric tensor which is to be derived from the Vlasov equation, $\mathbf{E}$ is the electric vector of the wave. Linearized Vlasov equation may describe the collisionless plasma like the coronal one in time intervals shorter than the inter - particle collision time ([22]),

$$
\begin{equation*}
\frac{\partial f_{1}}{\partial t}+\mathbf{v} \cdot \frac{\partial f_{1}}{\partial \mathbf{r}}+\frac{q_{i}}{m_{i}}\left(\mathbf{v} \times \mathbf{B}_{\mathbf{o}}\right) \cdot \frac{\partial f_{1}}{\partial \mathbf{v}}=\frac{d f_{1}}{d t}=-\frac{q_{i}}{m_{i}}\left(\mathbf{E}_{\mathbf{1}}+\mathbf{v} \times \mathbf{B}_{\mathbf{1}}\right) \cdot \frac{\partial f_{o}}{\partial \mathbf{v}} \tag{4}
\end{equation*}
$$

where $f_{o}$ is the unperturbed velocity distribution function, $f_{1}$ is the perturbed part of the distribution function, $\mathbf{B}_{o}$ is the magnetic field intensity of the polar coronal hole given by the Eq. (2), $\mathbf{E}_{1}$ and $\mathbf{B}_{1}$ are the electric and the magnetic fields of the ion - cyclotron wave. The time derivative is taken along the unperturbed trajectories in phase - space of O VI ions. Fourier amplitude of the perturbed velocity distribution function which will interact with the left circularly polarized ion - cyclotron waves is given as (ibid)

$$
\begin{equation*}
f_{L}=-\frac{q_{i}}{m_{i}}\left[\left(1-\frac{v_{\|} k}{\omega}\right) \frac{\partial f_{0}}{\partial v_{\perp}}+\frac{k v_{\perp}}{\omega} \frac{\partial f_{0}}{\partial v_{\|}}\right] \frac{E_{x} \exp \left(-i \theta_{0}\right)}{\omega-k v_{\|}-\Omega_{i c}}, \tag{5}
\end{equation*}
$$

where $\theta_{o}$ is the initial phase of the wave. The perturbed part of the velocity distribution function produces a current; $J_{x}$ component of this current is given by (ibid),

$$
\begin{equation*}
J_{x}=\frac{q_{i}^{2} \pi}{\omega m_{i}} i E_{x} \int \frac{\left(\omega-k v_{\|}\right)\left(\partial f_{0} / \partial v_{\perp}\right)+k v_{\perp}\left(\partial f_{0} / \partial v_{\|}\right)}{\omega-k v_{\|}-\Omega_{i c}} v_{\perp}^{2} d v_{\perp} d v_{\|} \tag{6}
\end{equation*}
$$

Following the same procedure, we obtain the $J_{y}$ component of the current density. It is found that, $J_{x} / E_{x}=$ $J_{y} / E_{y}=\mathbf{J} / \mathbf{E}$. Plasma dielectric tensor is found from the relation, $\mathbf{J}+i \omega \varepsilon_{0} \mathbf{E}=i \omega \varepsilon_{0} \kappa \cdot \mathbf{E}$, where $\varepsilon_{0}$ is the electric permittivity of the vacuum. Thus, we obtain the relation between the electric field, current density and the dielectric tensor of the left circularly polarized ion - cyclotron waves as below:

$$
\begin{equation*}
\kappa_{L}=1+\frac{J / E}{i \omega \varepsilon_{0}}=1+\frac{q_{i}^{2} \pi}{m \omega^{2} \varepsilon_{0}} \int_{-\infty}^{+\infty} d v_{\|} \int_{0}^{\infty} \frac{\left(\omega-k v_{\|}\right)\left(\partial f_{0} / \partial v_{\perp}\right)+k v_{\perp}\left(\partial f_{0} / \partial v_{\|}\right)}{\omega-k v_{\|}-\Omega_{i c}} v_{\perp}^{2} d v_{\perp} \tag{7}
\end{equation*}
$$

The dispersion relation for the left circularly polarized ion cyclotron waves is obtained by using the equation, $\kappa_{L}=n^{2}=c^{2} k^{2} / \omega^{2}$, where $n$ is the refractive index of the medium for the waves in consideration.

In the present investigation we use a single fluid approximation. Otherwise, one derives dispersion relations for each plasma species and take the sum total of them by putting summation sign before the integral sign in Eq. (7).

Now, we have to make an assumption on the form of the velocity distribution function. Since Helios 1 \& 2 data revealed that electrons, protons and minor ion species in the fast solar wind exhibit a bi - Maxwellian distribution ([9]), we may assume that this form of distribution is acquired at the top of the coronal base where collisional/collisionless boundary is located. The unperturbed velocity distribution function is assumed to be of the form,

$$
\begin{equation*}
f_{0}=N_{O V I} \alpha_{\perp}^{2} \alpha_{\|} \pi^{-3 / 2} \exp \left[-\left(\alpha_{\perp}^{2} v_{\perp}^{2}+\alpha_{\|}^{2} v_{\|}^{2}\right)\right] \tag{8}
\end{equation*}
$$

where $\alpha_{\perp}=\left(2 k_{B} T_{\perp i} / m_{i}\right)^{-1 / 2}$ and $\alpha_{\|}=\left(2 k_{B} T_{\| i} / m_{i}\right)^{-1 / 2}$ are the inverse of most probable speeds in the perpendicular and parallel direction to the local magnetic field, respectively. $k_{B}$ is Boltzmann constant, $N_{O V I}$ is the particle number density for $O V I$ ions. $\partial f_{0} / \partial v_{\perp}$ and $\partial f_{0} / \partial v_{\|}$derivatives that appear in Eq. (7) are to be derived from Eq. (8). If we make these substitutions into Eq. (7) we get the contribution of the principal integral to the dispersion relation, which is given by the Eq. (9) below:

$$
\begin{equation*}
k^{2}\left(c^{2}-\frac{\omega^{2}}{k^{2}}\right)+\frac{\omega_{i p}^{2}}{\omega-\Omega_{i c}}\left[\omega-\frac{k}{\alpha_{\|} \pi^{1 / 2}} P+\frac{k^{2}}{2 \alpha_{\|}^{2}\left(\omega-\Omega_{i c}\right)} P\right]=0 \tag{9}
\end{equation*}
$$

where $\omega_{i p}=\left(N_{O V I} q_{i}^{2} / \varepsilon_{0} m_{i}\right)^{1 / 2}$ is the plasma frequency for O VI ions (e.g., [23]), and we put, $P=$ $\left[\omega /\left(\omega-\Omega_{i c}\right)\right]+\left(T_{\perp} / T_{\|}\right)-1$, for brevity. The residual contribution to the integral is obtained as below:

$$
\begin{equation*}
R=-i \pi^{1 / 2} \alpha_{\|} \frac{\omega_{i p}^{2}}{k \omega^{2}}\left[\Omega_{i c}\left(1-\frac{T_{\perp}}{T_{\|}}\right)+\omega\right] \exp \left[-\alpha_{\|}^{2}\left(\frac{\omega-\Omega_{i c}}{k}\right)^{2}\right] \tag{10}
\end{equation*}
$$

The sum total of the Eqns. (9) \& (10) gives the general dispersion relation for the left circularly polarized ion cyclotron waves. We arrange the dispersion relation (9) \& (10) in accordance with the descending powers of wave number $k$. The result is,

$$
\begin{equation*}
k^{2}\left[\frac{\omega_{i p}^{2}}{2 \omega^{2} \alpha_{\|}^{2}\left(\omega-\Omega_{i c}\right)^{2}} P+\frac{c^{2}}{\omega^{2}}\right]-\frac{k}{\alpha_{\|} \pi^{1 / 2}}\left(\frac{\omega_{i p}^{2}}{\omega^{2}\left(\omega-\Omega_{i c}\right)}\right) P+\frac{\omega_{i p}^{2}}{\omega\left(\omega-\Omega_{i c}\right)}-1+R=0 \tag{11}
\end{equation*}
$$

The solution of Eq. (11) gives the variation of wavenumber $k$ with distance $R$. We solve the equation (11) for real k and complex $\omega$, the sign of the imaginary part of which determines whether the waves are damped or grow. For ionic plasma frequencies we use both $N_{O V I}=10^{-3} N_{p}([10] ;[4])$ and $N_{O V I}=6.8 \times 10^{-5} N_{p}$ ([15]) values separately and compare the results in Figs. 1, $2 \& 3$ below. For proton plasma frequency we take $N_{e}=N_{p}$. Ion - cyclotron frequencies for protons, O VI and Mg X ions, $\Omega_{i c}$ are calculated by using the magnetic field intensity given by Eq. (2). For $T_{\perp}$ and $T_{\|}$we refer to the profiles given by Cranmer et al. [24] and Kohl et al. [2]. Figune 1, 2 and 3 show the variation of wave number with distance for O VI, Mg X ions and for protons, respectively.

The locations where the wavenumber k goes to infinity are the ones where ion - cyclotron resonance processes take place. Since the power spectrum of ion-cyclotron waves in the north polar coronal hole is not yet known, we restrict our investigation to the frequency band of waves which are likely to get into resonance with the O VI ions. That is to say, in the distance range $1.5 R-3.0 R$, we observe that waves with frequency band in the range $2.5 \mathrm{kHz}<\omega<10 \mathrm{kHz}$ resonate with O VI ions. Assuming that no waves are generated with frequencies outside of this range, we also solved the dispersion relation for Mg X ions and protons to see if the latter two also are "fed" by the same waves. Comparison of these three figures show that, for instance, 10 kHz waves first resonate with O VI ions at $R=1.55-1.70$; and then, if any power is left, they resonate with Mg X ions at $R=1.6-1.72$ and much further with protons at $R=2.25$. At the lower limit of the frequency band, ion - cyclotron waves with a frequency of 2.5 kHz resonate with O VI ions at about
$R=2.42 ; 2.5 \mathrm{kHz}$ waves resonate with Mg X ions at distances greater than $2.57 \mathrm{R} ; 2.5 \mathrm{kHz}$ waves do not resonate with protons, in the distance range we are considering. Since the reliable data of SOHO/UVCS ceases at about $R=3.0$ we do not investigate the wave damping and resonance processes beyond that limit.

The final dispersion relation is given as the sum total of Eq. (9) and (10). In order to obtain the wave damping rate we rearrange this sum and set,

$$
\begin{equation*}
\kappa_{0}=1-\frac{\omega_{i p}^{2}}{\omega^{2}\left(\omega-\Omega_{i c}\right)}\left[\omega-\frac{k}{\alpha_{\|} \pi^{1 / 2}} P+\frac{k^{2}}{2 \alpha_{\|}^{2}\left(\omega-\Omega_{i c}\right)} P\right] \tag{12}
\end{equation*}
$$

and

$$
\begin{equation*}
\delta \kappa=\frac{\alpha_{\|} \omega_{i p}^{2} \pi^{1 / 2}}{\omega^{2} k}\left[\Omega_{i c}\left(1-\frac{T_{\perp}}{T_{\|}}\right)+\omega\right] \exp \left[-\alpha_{\|}^{2}\left(\frac{\omega-\Omega_{i c}}{k}\right)^{2}\right] \tag{13}
\end{equation*}
$$

so that the final dispersion relation becomes,

$$
\begin{equation*}
\kappa_{0}+i \delta \kappa-\frac{c^{2} k^{2}}{\omega^{2}}=0 \tag{14}
\end{equation*}
$$

Since we are interested in the solution of the dispersion relation in the neighborhood of $\omega_{\text {res }}$ (resonance frequency), we take the derivative of Eq. (14) at $\omega_{\text {res }}$ (see, e.g., Schmidt, 1979) :

$$
\begin{equation*}
\left.\frac{\partial \kappa_{0}}{\partial \omega}\right|_{\omega_{r e s}}+\left.i \frac{\delta \kappa}{\delta \omega}\right|_{\omega_{r e s}}+\frac{2 c^{2} k^{2}}{\omega_{r e s}^{3}}=0 \tag{15}
\end{equation*}
$$

If we substitute Eq. (13) into Eq. (15), we get

$$
\begin{gather*}
-i \frac{\alpha_{\|} \omega_{i p}^{2} \pi^{1 / 2}}{\omega^{2} k}\left[\Omega_{i c}\left(1-\frac{T_{\perp}}{T_{\|}}\right)+\omega\right] \exp \left[-\alpha_{\|}^{2}\left(\frac{\omega-\Omega_{i c}}{k}\right)^{2}\right] \\
=\left[\left.\frac{\partial \kappa_{0}}{\partial \omega}\right|_{\omega_{r e s}}+\frac{2 c^{2} k^{2}}{\omega_{r e s}^{3}}\right] \delta \omega \tag{16}
\end{gather*}
$$

For a real $\omega_{\text {res }}$, Eq. (14) can be written as $\kappa_{o}\left(\omega_{r e s}\right)-c^{2} k^{2} / \omega_{r e s}^{2}=0$. Now, from Eq. (16) we can deduce the damping rate, $\gamma=\operatorname{Im} \delta \omega$, as below (see, e.g., Schmidt, 1979):

$$
\begin{equation*}
\gamma=\operatorname{Im} \delta \omega=\frac{\alpha_{\|} \omega_{i p}^{2} \pi^{1 / 2}}{\omega^{2} k}\left[\Omega_{i c}\left(\frac{T_{\perp}}{T_{\|}}-1\right)-\omega\right] \exp \left[-\alpha_{\|}^{2}\left(\frac{\omega-\Omega_{i c}}{k}\right)^{2}\right]\left[\omega_{r e s} \frac{\partial \kappa_{0}}{\partial \omega}+2 \kappa_{0}\right]^{-1} \tag{17}
\end{equation*}
$$

where,

$$
\begin{equation*}
\omega_{r e s} \frac{\partial \kappa_{0}}{\partial \omega}+2 \kappa_{0}=2-\frac{\omega_{i p}^{2}}{\omega\left(\omega-\Omega_{i c}\right)^{2}}\left[\frac{k \Omega_{i c}}{\alpha_{\|} \pi^{1 / 2}\left(\omega-\Omega_{i c}\right)}+\frac{k P}{\alpha_{\|} \pi^{1 / 2}}-\frac{k^{2} P}{\alpha_{\|}^{2}\left(\omega-\Omega_{i c}\right)}-\Omega_{i c}-\frac{k^{2} \Omega_{i c}}{2 \alpha_{\|}^{2}\left(\omega-\Omega_{i c}\right)^{2}}\right] \tag{18}
\end{equation*}
$$

Substitution of Eq. (18) into (17) gives the damping rate of the resonant waves. The damping rate, $\gamma$, turns out to be positive in the distance range we are interested in. The factor multiplying the first bracketed terms in $\gamma$ is of the order of $10^{9}$; the first bracketed terms give a value of $\sim 10^{5}$; the exponential term is very small, and the inverse of the last bracketed factor gives values in the range $10^{-6}-10^{-1}$. The negligibly small value of $\gamma$, outside resonance region indicate that the ion cyclotron waves propagate without damping. But in the close vicinity of $\omega_{\text {res }}$, the resonance frequency, $\gamma$ suddenly increases to large values.

We also investigated the consequences of perpendicular heating of protons and O VI and Mg X ions. If these species are to be accelerated in the fast solar wind then the mirror force, $F_{m f}(R)=-\mu(R) \partial B / \partial R$ should be greater than the gravitational force exerted, for instance on O VI ions, $F_{g}(R)=G M A m_{p} / r^{2}$ ([20]); where the symbols have their usual meanings. The result is shown in Figure 4.


Figure 4. The variations of the mirror force $\left(F_{m}\right)$ and gravitational force $\left(F_{g}\right)$ for O VI $(a), \mathrm{Mg} \mathrm{X}(b)$ and protons (c) with respect to the radial distance. Since the mirror force is much greater than the gravitational one, all three species are accelerated in solar wind by cyclotron resonance process. For $v_{\perp}$ appearing in the magnetic moment, $\mu$, in the mirror force, we used $v_{1 / e}$ as given in [8]

## 3. Summary and Conclusion

In the present investigation, we used SOHO data on electron number density distribution, perpendicular and parallel temperatures of protons, O VI and Mg X ions. In order to solve the dispersion relation for the left circularly polarized ion - cyclotron waves derived from linearized Vlasov equation, we have to make use of two different number densities of protons, O VI and Mg X ions. Again, the radial dependence of the magnetic field is also assumed to be of the form given by Eq. (2). An important constituent of our model is the velocity distribution function. We adopted a bi - Maxwellian form, partly because Helios $1 \& 2$ data implies the existence of such distribution and partly because in collisionless plasma we thought it would be the most appropriate one. Then, we deduced the dispersion relation for the left circularly polarized ion cyclotron waves. When we substitute all the measured values of the coronal hole plasma and the modelled parameters, we get the wave number profile with respect to the radial distance. Magnetic field given by Eq. (2) and ion number densities are not-well-defined parameters of the model. Any uncertainty in these parameters shows itself in the frequency range of the resonant ion-cyclotron waves. For instance when we
take $f_{\max }=88$, by which Eq. (2) gives magnetic field intensity as $130 \mathrm{G}([25])$ the frequency range of resonant waves becomes $10 \mathrm{kHz}-60 \mathrm{kHz}$. As is well known, waves resonate with the relevant species where $k$, the wave number, and $n=c k / \omega$, the refractive index of the medium goes to infinity. This is where the linear approximation breaks down. In this sense, our results point only to the locations where the waves of particular frequencies resonate with the relevant species. Whatever happens to the wave after the resonance region cannot be predicted by the linear theory.

Radii of ion cyclotron resonance for monochromatic waves were computed previously in the literature (see, e.g., [27]; [21]; [26] and [28]), but not in such exhaustive detail as in our investigation. One may argue that the resonance radii are kind of trivial to compute - just set the wave frequency in Eq. (2) equal to the radially dependent ion cyclotron frequency and solve for the radius - and a full dispersion analysis is kind of 'overkill' just to compute these radii. But, only after having derived the dispersion relation and solved for the real wave number and complex frequency one can say as to whether the waves in frequency band 2.5 $\mathrm{kHz}-10 \mathrm{kHz}$ are evanescent or damping or unstable. Eq. (2) gives us the radially dependent ion cyclotron frequency. Ions may encounter waves with the same frequency as their cyclotron frequency at a certain radii, but Eq. (2) does not tell us if these waves can perpendicularly heat ions. Therefore, knowing from observations that ions are perpendicularly heated and speculating that heating is caused by cyclotron waves through cyclotron resonance process and claiming that one can determine the resonance radii with the help of Eq. (2) is an a posteriori argument from which we refrained ourselves.

Our major interest is O VI ion. The solution of the dispersion relation shows that waves in frequency range of $2.5 \mathrm{kHz}-10 \mathrm{kHz}$ that are assumed to be produced at the base of the corona, can resonate with O VI ions in the distance range of $1.5 R-3.0 R$. This is considerably narrow a frequency band. We looked if any other ion species could be fed by the same waves. Protons and Mg X ions are the only ones the temperature profiles of which we could find in the literature. These two species can also extract energy from the waves in frequency band $2.5 \mathrm{kHz}-10 \mathrm{kHz}$. But, if we assume that no other waves are produced outside of this frequency range, then we can say that O VI ions are preferentially heated in the coronal hole. Because, the first encounter of these waves that are assumed to be produced at the coronal base happens to be with O VI ions. And then, if any power is left over, the same waves can feed energy to Mg X ions and much later to protons. Indeed, the comparison of temperature profiles of O VI ions with the protons show that the former is heated preferentially and much effectively than the latter two. Cranmer et al. [29] argue that natural decay of waves can hardly explain the strong extended heating observed spectroscopically. In their view, the wave spectrum must be maintained in a steep power law. This implies that waves heating ions above $\sim 1.5 R_{\odot}$ "do not originate at the solar surface, but are generated in the wind by, e.g., MHD turbulent cascade".

Damping rate of the resonant waves show a steep increase towards the resonance region. What happens to the waves after resonance cannot be predicted by our model. Because we work with a linearized Vlasov equation and the resonance is a nonlinear process. One should bear in mind, however, that the source of the ion-cyclotron waves is still unclear. Axford \& McKenzie [30] suggested that reconnection events can launch the high frequency waves directly; these waves, Hollweg [31] points out must be in kHz range if they are to resonate with the coronal protons. It is encouraging to see that our result also points to the kHz range waves that resonate with O VI and Mg X ions and protons. The possible detection of these waves by interplanetary scintillation technique would further confirm the existence of such waves of solar origin.

An important consequence of perpendicular heating is acceleration along the open magnetic field lines. Increase in the perpendicular temperature and magnetic moment causes the increase in mirror force; then particles overcome the gravitational potential and are accelerated to greater speeds. Again the two different number densities we adopted from the literature for the ion species O VI and Mg X , give two slightly different values for the mirror force; the difference is only one order of magnitude. In either case, the mirror force felt by O VI ions is five to six orders of magnitude greater than the gravitational force; about five orders of magnitude for Mg X ions and about an order of magnitude for protons. Therefore, we may conclude that
ion - cyclotron resonance process seems to be capable of accounting for both heating and acceleration of minor species in north solar polar coronal hole.

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