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The Investigation of Screening of Chromoelectric and Chromomagnetic Fields in Noncovariant Gauges

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Abstract

According to Thermal Quantum Chromodynamics (TQCD) hadron matter may exist in the form of quark-gluon plasma (QGP) at high temperatures and densities. The aim of the present work is to investigate the screening of chromoelectric and chromomagnetic fields in QGP in noncovariant gauges. In noncovariant gauges, gluon propagator has unphysical poles and singularities appear in the gluon self-energy diagram calculations. In this paper, we investigate gluon self-energy in lightcone and temporal gauges. In order to remove singularities, we use the Mandelstam-Leibbrandt prescription (ML prescription). We obtain that chromoelectric fields are screened, but chromomagnetic fields are not screened. In coordinate space, the screening of chromoelectric fields reduces the range of gauge interactions. In momentum space, it contributes to regulate the infrared behavior of the various n-point Green functions. Also, it is shown that the obtained results are gauge invariant.

Key Words: Thermal Quantum Chromodynamics, Quark-gluon Plasma, Lightcone Gauge, Temporal Gauge, Mandelstam-Leibbrandt Prescription, Gluon Screening Masses.

1. Introduction

According to TQCD hadron matter undergoes a phase transition at high temperatures and/or high density. Above critical temperatures, quarks and gluons are no longer confined. These extreme regimes are expected to be found in the interior of neutron stars or in heavy-ion collisions.

The quark-gluon plasma has long attracted interest as a new form of matter with unusual properties, such as deconfinement and Debye screening [1–3]. It is expected from asymptotic freedom that, at extremely high temperatures, the plasma may be described to a good approximation as a gas of weakly interacting quarks and gluons [5].

One of the main problems in TQCD is the severe infrared divergences which come from the massless gauge bosons having an infinite correlation length. The cure for these infrared divergences is the occurrence of electric m_{el}^2 and magnetic m_{mag}^2 screening masses which control the infrared behavior of the theory [6]. Therefore, it is very essential to calculate the so-called "gluon screening masses", which effect the infrared cutoff for chromoelectric and chromomagnetic fields [7, 8]. Much work has been devoted to this issue [9–12]. A further quantitative understanding of the screening effect is very important since it might be the physical mechanism responsible for the color deconfinement phase transition.

The infrared limit of thermal gluon self-energy is of great importance because this limit determines the screening masses of gluons. The new definition of the Debye mass proposed by Rebhan [10] defined at the pole of the longitudinal propagator

$$m_{el}^2 = \Pi_{44}\left(p_4 = 0,\ \mathbf{p}^2 \to -m_{el}^2\right), \tag{1.1}$$

is significantly improved over the old definition [1],

$$m_{el}^2 = \Pi_{44}\left(p_4 = 0,\, |\mathbf{p}| \to 0\right), \tag{1.2}$$

in the sense that it is self-consistent and also gauge and renormalization group invariant [10, 11]. In one-loop approximation, Eqs. (1.1) and (1.2) give the same results. Gluon magnetic screening mass is defined as [1, 2]

$$m_{mag}^2 = \frac{1}{2}\Pi_{ii}\left(p_4 = 0,\, |\mathbf{p}| \to 0\right). \tag{1.3}$$

The perturbative contributions to gluon screening masses were calculated long ago in covariant gauge [5]. Covariant gauge calculations have shown that electric mass is different from zero for Abelian and nonabelian theories. However, magnetic mass is exactly equal to zero within hot QED and seems for any Abelian theory. TQCD expects that $m_{mag}^2 \neq 0$ [7, 8], which is confirmed by self-consistent [13] and lattice TQCD results [14].

The scope of this paper is to investigate gluon self-energy based on the perturbative TQCD in temporal and lightcone gauges. The existence of residual symmetry in the temporal and lightcone gauges manifests itself in the appearance of unphysical poles. To handle these non-physical poles, we used ML prescription [15, 16] which can be obtained on the basis of consistent quantization [17]. The investigation of gluon self-energy in noncovariant gauges is very important for two reasons: firstly, to verify the gauge independence of screening masses of gluons and secondly to test ML prescription that is used in thermal field theories [18, 19]. We analyzed gluon screening masses in lightcone and temporal gauges.

2. The Investigation of Thermal Gluon Self-energy in Noncovariant Gauges

We shall consider TQCD, which is described by the Lagrangian

$$\begin{aligned} L = \quad & -\frac{1}{4}(F_{\mu\nu}^a)^2 + \frac{1}{2\alpha}(n_\mu A_\mu^a)^2 + \bar{c}^a n_\mu D_\mu^{ab} c^b \\ & + \sum_f \bar{\psi}_f \left[\gamma_\mu \left(\partial_\mu - ig\frac{\lambda^a}{2}A_\mu^a\right) + m_f\right]\psi_f \quad . \end{aligned} \tag{2.1}$$

Here, $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + igf^{abc}A_\mu^b A_\nu^c$ is the gluon field strength tensor; $D_\mu^{ab} = \partial_\mu \delta^{ab} + gf^{acb}A_\mu^c$ is the covariant derivative; c and $\bar{c}$ are ghost fields, which arise from consistent quantization when one eliminates the unphysical degrees of freedom.

The perturbative infrared catastrophe of a nonabelian gauge theory at high temperature is one of the most important unsolved problems in thermal field theory [6]. Infrared divergences become more and more severe the higher the loop order is. In this sense, we shall investigate the infrared behavior of gluon self-energy in lightcone and temporal gauges.

The noncovariant gauges are defined by $n_\mu A^a_\mu(x) = 0$, where $A^a_\mu(x)$, with $\mu = 0, 1, 2, 3$ and $a = 1, 2, \ldots N^2 - 1$, is a massless $SU(N)$Yang-Mills field, and n_μ is a constant four vector. Different forms of n_μ give rise to some particularly convenient axial type gauges, such as the planar gauge $\left(n^2 < 0\right)$, the lightcone gauge $\left(n^2 = 0\right)$, and the temporal gauge $\left(n^2 > 0,\ n_\mu = (0, 0, 0, 1)\right)$. In this work, we consider thermal gluon self-energy in lightcone and temporal gauges. The main advantage of these gauges is the decoupling of a ghost field from the gauge field.

The expression for gluon self-energy $\Pi_{\mu\nu}(p)$ is presented in noncovariant gauges by the three diagrams [3] shown in Figure 1:

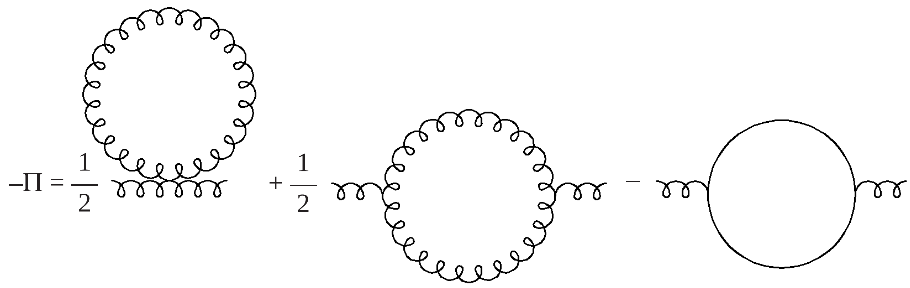

Figure 1. One-loop diagrams for gluon self-energy. Wavy and solid lines denote gluons and quarks, respectively.

Let us represent the contribution of the three diagrams as $\Pi^{(1)}_{\mu\nu}$, $\Pi^{(2)}_{\mu\nu}$ and $\Pi^{(3)}_{\mu\nu}$, respectively, i.e.

$$-\Pi_{\mu\nu} = \frac{1}{2}\Pi^{(1)}_{\mu\nu} + \frac{1}{2}\Pi^{(2)}_{\mu\nu} - \Pi^{(3)}_{\mu\nu}. \tag{2.2}$$

In order to calculate the above diagrams (Figure 1), we use the following standard TQCD Feynman rules:
(a) three gluon vertex:

$$\Gamma^{abc}_{\mu\nu\lambda}(k_1, k_2, k_3) = -igf^{abc}\Big\{\delta_{\mu\nu}\left(k_1 - k_2\right)_\lambda + \delta_{\nu\lambda}\left(k_2 - k_3\right)_\mu + \delta_{\lambda\mu}\left(k_3 - k_1\right)_\nu\Big\}. \tag{2.3}$$

(b) four gluon vertex:

$$\begin{aligned}\Gamma^{bcde}_{\mu\nu\rho\gamma}(k_1, k_2, k_3, k_4) = \quad & -g^2\left[f^{abc}f^{dea}\left(\delta_{\mu\rho}\delta_{\nu\gamma} - \delta_{\mu\gamma}\delta_{\nu\rho}\right)\right. \\ & \left.+f^{abd}f^{cea}\left(\delta_{\mu\nu}\delta_{\rho\gamma} - \delta_{\mu\gamma}\delta_{\nu\rho}\right) + f^{abe}f^{cda}\left(\delta_{\mu\nu}\delta_{\gamma\rho} - \delta_{\mu\rho}\delta_{\nu\gamma}\right)\right].\end{aligned} \tag{2.4}$$

(c) quark-gluon vertex:

$$\Gamma^{fij}_\mu = -ig\gamma_\mu\left(\frac{\lambda^f}{2}\right)_{ij}. \tag{2.5}$$

(d) quark propagator:

$$G(k) = \frac{-i\hat{k} + m}{k^2 + m^2}. \tag{2.6}$$

Here, the metric chosen is Euclidean: $k_4 = 2\pi nT$ for gluon fields and $k_4 = (2n + 1)\pi T$ for quark fields. All diagrams are calculated by using the above expressions for the free propagators and vertices. For the first contribution $\Pi^{(1)}_{\mu\nu}(p)$, the expression is written as

$$\Pi^{(1)}_{\mu\nu}(p) = \frac{1}{\beta^2}\sum_{k_4}\int\frac{d\mathbf{k}}{(2\pi)^3}D^{mn}_{\rho\sigma}(k)D^{ab}_{\alpha\beta}(p+k) \times\Gamma^{fma}_{\mu\rho\alpha}(p,k,-p-k)\;\Gamma^{hnb}_{\nu\sigma\beta}(-p,-k,p+k). \tag{2.7}$$

Similarly, the second contribution to gluon self-energy is

$$\Pi^{(2)}_{\mu\nu}(p) = \frac{1}{\beta}\sum_{k_4}\int\frac{d\mathbf{k}}{(2\pi)^3}D^{ab}_{\alpha\beta}(k)\Gamma^{fabh}_{\mu\alpha\beta\nu}(p,-k,k,-p). \tag{2.8}$$

3. Gluon Screening Masses in Lightcone and Temporal Gauges

Let us consider the gluon self-energy in lightcone gauge. In lightcone gauge, bare gluon propagator $D^{ab}_{\mu\nu}(p)$ is given by

$$D^{ab}_{\mu\nu}(k,\alpha=0) = \frac{\delta^{ab}}{k^2}\left[\delta_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{k\cdot n}\right]. \tag{3.1}$$

Using the standard free propagators and vertices, and the relation for antisymmetric structure constants $f^{fna}f^{hna} = N\delta^{fh}$, we obtain the infrared limit ($p_4 = 0$; $\mathbf{p}\to 0$) of $\Pi^{(1)}_{\mu\nu}$ and $\Pi^{(2)}_{\mu\nu}$ in the form

$$\Pi^{(1)}_{\mu\nu} = -2\frac{g^2N}{\beta}\delta^{fh}\sum_{k_4}\int\frac{d\mathbf{k}}{(2\pi)^3}\left[\frac{\delta_{\mu\nu}}{k^2} - \frac{4k_\mu k_\nu}{k^4} - \frac{k_\mu n_\nu + k_\nu n_\mu}{k^2\,(n\cdot k)} - \frac{n_\mu n_\nu}{(n\cdot k)^2}\right], \tag{3.2}$$

$$\Pi^{(2)}_{\mu\nu} = -2\frac{g^2N}{\beta}\delta^{fh}\sum_{k_4}\int\frac{d\mathbf{k}}{(2\pi)^3}\left[\frac{\delta_{\mu\nu}}{k^2} + \frac{k_\mu n_\nu + k_\nu n_\mu}{k^2\,(n\cdot k)}\right]. \tag{3.3}$$

The quark contribution to gluon self-energy in noncovariant gauges is the same as in covariant gauge. Taking into account the last two expressions in (2.2), we get for the gluon electric and magnetic masses:

$$m^2_{el} = \frac{g^2N}{\beta}\sum_{k_4}\int\frac{d\mathbf{k}}{(2\pi)^3}\left[-\frac{2}{k^2} + \frac{4\mathbf{k}^2}{k^4} - \frac{1}{(n\cdot k)^2}\right], \tag{3.4}$$

$$m^2_{mag} = \frac{g^2N}{2\beta}\sum_{k_4}\int\frac{d\mathbf{k}}{(2\pi)^3}\left[\frac{6}{k^2} - \frac{4\mathbf{k}^2}{k^4} - \frac{1}{(n\cdot k)^2}\right]. \tag{3.5}$$

The infrared limits of gluon self-energy can be calculated using the following sums:

$$\sum_{k_4}\frac{1}{\mathbf{k}^2 + k_4^2} = \frac{1}{2|\mathbf{k}|T}\coth\frac{|\mathbf{k}|}{2T}, \tag{3.6}$$

$$\sum_{k_4}\frac{1}{(\mathbf{k}^2 + k_4^2)^2} = \frac{1}{4|\mathbf{k}|^3T}\coth\frac{|\mathbf{k}|}{2T} + \frac{1}{8|\mathbf{k}|^2T^2}\frac{1}{\sinh^2\left(\frac{|\mathbf{k}|}{2T}\right)}. \tag{3.7}$$

Note that expressions (3.4) and (3.5) contain poles in the form of $1/(n\cdot k)$. In order to remove these singularities, we used the following ML prescription in Euclidean space:

$$\frac{1}{(n\cdot k)^2}=\lim_{\varepsilon\to 0}\left(\frac{\mathbf{k}\cdot\mathbf{n}+ik_4}{(\mathbf{k}\cdot\mathbf{n})^2+k_4^2+\varepsilon^2}\right)^2, \quad \varepsilon>0, \tag{3.8}$$

where we assume that $|\mathbf{n}|=1$. Also, $\Pi_{\mu\nu}$ contains ultraviolet divergences which are the same as at zero temperature, i.e. renormalization at zero temperature suffices to make the theory finite at non-zero temperature. Therefore $\Pi_{\mu\nu}$ can be divided into two parts [1, 2]:

$$\Pi_{\mu\nu}=\Pi_{\mu\nu\,mat}+\Pi_{\mu\nu\,vac}, \tag{3.9}$$

where $\Pi_{\mu\nu\,vac}=\lim\limits_{T\to 0}\Pi_{\mu\nu}$ is the vacuum part which can be subtracted from the gluon self-energy, since it is unobservable constant. The remaining matter part $\Pi_{\mu\nu\,mat}$ is completely free of ultraviolet divergences and needs no renormalization [1–3]. Hereafter we shall renormalize ultraviolet divergences by the subtracting vacuum part. Taking into account ML prescription in expressions (3.4) and (3.5), carrying out summations over k_4 and subtracting vacuum terms, we obtain

$$m_{el}^2=2g^2N\left(\frac{T^2}{\pi^2}\int_0^\infty\frac{x^2dx}{\sinh^2 x}+\frac{T}{2}\varepsilon^2 I_0\right), \tag{3.10}$$

$$m_{mag}^2=g^2N\left[\frac{T^2}{\pi^2}\int_0^\infty\left(\frac{4x}{e^{2x}-1}-\frac{x^2}{\sinh^2 x}\right)dx+\frac{T}{2}\varepsilon^2 I_0\right], \tag{3.11}$$

where

$$I_0=\int\frac{d\mathbf{k}}{(2\pi)^3}\left(\frac{1}{4\rho^3 T}\coth\frac{\rho}{2T}+\frac{1}{8\rho^2T^2}\frac{1}{\sinh^2(\rho/2T)}\right)$$

and $\rho^2=(\mathbf{n}\cdot\mathbf{k})^2+\varepsilon^2$.

Note that in expression (3.10) and (3.11) ε certainly regulates the infrared behavior and in the $\varepsilon\to 0$ limit, we obtain the expression of gluon screening masses as

$$m_{el}^2=\frac{1}{3}g^2NT^2, \tag{3.12}$$

$$m_{mag}^2=0, \tag{3.13}$$

which agrees with the Feynman gauge calculations [5]. Taking into account quarks contribution in (3.12), we can write gluons electric mass as $m_{el}^2=\left(N+\frac{N_f}{2}\right)\frac{g^2T^2}{3}$, where N_f is the number of thermally active flavors.

Let us look for gluon self-energy in temporal gauge. In this gauge the gluon propagator reads:

$$D_{ij}^{ab}(k)=\frac{\delta^{ab}}{k^2}\left(\delta_{ij}+\frac{k_ik_j}{k_4^2}\right), \quad D_{4i}=D_{i4}=D_{44}=0, \quad i,\,j=1,\,2,\,3. \tag{3.14}$$

Similar to the lightcone gauge calculations, using standard Feynman rules and after some algebraic transformations, the infrared limit ($p_4 = 0;\ \mathbf{p} \to 0$) of $\Pi^{(1)}_{\mu\nu}$ and $\Pi^{(2)}_{\mu\nu}$ can be written as

$$\begin{aligned}\Pi^{(1)}_{\mu\nu} = \; & \frac{g^2 N}{\beta}\delta^{fh}\sum_{k_4}\int\frac{d\mathbf{k}}{(2\pi)^3}\{D_{\mu\beta}(k)\,D_{\nu\lambda}(k)\,k_\beta k_\lambda + D_{\alpha\beta}(k)\,D_{\nu\mu}(k)\,k_\alpha k_\beta \\ & -2D_{\lambda\beta}(k)\;D_{\nu\lambda}(k)\,k_\mu k_\beta - 2D_{\mu\rho}(k)\,D_{\rho\lambda}(k)\,k_\lambda k_\nu - 2D_{\alpha\rho}(k)\,D_{\rho\mu}(k)\,k_\alpha k_\nu \\ & +4D_{\lambda\rho}(k)\,D_{\rho\lambda}(k)\,k_\mu k_\nu + D_{\mu\nu}(k)\,D_{\rho\lambda}(k)\,k_\lambda k_\rho + D_{\alpha\nu}(k)\,D_{\rho\mu}(k)\,k_\alpha k_\rho \\ & -2D_{\lambda\nu}(k)\;D_{\rho\lambda}(k)\,k_\mu k_\rho\}\ ,\end{aligned} \tag{3.15}$$

$$\Pi^{(2)}_{\mu\nu} = 2\frac{g^2 N}{\beta}\delta^{fh}\sum_{k_4}\int\frac{d\mathbf{k}}{(2\pi)^3}[D_{\mu\nu}(k) - \delta_{\mu\nu}TrD(k)] \quad . \tag{3.16}$$

Taking into account the gluon propagator in temporal gauge (3.14) and using the expressions of $\Pi^{(1)}_{\mu\nu}$ and $\Pi^{(2)}_{\mu\nu}$, we can write the gluons electric and magnetic masses:

$$m^2_{el} = \frac{g^2 N}{\beta}\sum_{k_4}\int\frac{d\mathbf{k}}{(2\pi)^3}\left[-\frac{2}{k^2} + \frac{4\mathbf{k}^2}{k^4} - \frac{1}{k_4^2}\right], \tag{3.17}$$

$$m^2_{mag} = \frac{g^2 N}{2\beta}\sum_{k_4}\int\frac{d\mathbf{k}}{(2\pi)^3}\left[\frac{6}{k^2} - \frac{4\mathbf{k}^2}{k^4} - \frac{1}{k_4^2}\right]. \tag{3.18}$$

Notice that the Eqs. (3.17) and (3.18) contains poles in the form of $1/k_4^2$. To handle singularities, we use the ML prescription for temporal gauge [14, 15]:

$$\frac{1}{k_4^2} = \lim_{\varepsilon\to 0}\left(\frac{k_4}{k_4^2+\varepsilon^2}\right)^2, \quad \varepsilon^2 > 0 \quad . \tag{3.19}$$

After carrying out summations, integrations and subtracting the vacuum contributions in (3.17) and (3.18), we get the same expressions for gluon screening masses in lightcone and temporal gauges.

The obtained expressions Eq. (3.12) and Eq. (3.13) means that chromoelectric fields are screened, but chromomagnetic fields are not screened in the lowest order of perturbation theory. Note that our results agree with the Feynman gauge calculations [5].

As a result, the ML prescription is successfully used for removing the unphysical poles in noncovariant gauges in thermal field theories. For the present, the infrared problem in nonabelian gauge theories is unsolved and open to discussions. In the future, one will be able to calculate the next to leading order correction to the electric and magnetic masses in noncovariant gauges. We hope that studies in this direction leads us much better understanding of QGP signals [20] and TQCD properties.

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