Turk J Phys
30 (2006) , 345 – 348.

Casimir Energies for some Single Cavities

H. AHMEDOV[1] **and I. H. DURU**[2]
[1]*Feza Gürsey Enstitüsü, P.O. Box 6, 81220, Çengelköy, İstanbul-TURKEY*
[2]*İzmir Yüksek Teknoloji Enstitüsü, İzmir-TURKEY*

Received 18.07.2006

Abstract

Casimir energies are discussed for some cavities.

1. Introduction

If two parallel, perfectly conducting plates of "infinite" area are placed at a distance d from each other, there is an attractive force on per unit area of the plates given by

$$F = -\frac{\pi^2 \hbar c}{240 d^4}. \tag{1}$$

The above force, which is due to the vacuum fluctuations of electromagnetic field is first predicted in 1948 by Casimir [1]. The coefficient $\pi^2 \hbar c$ (with $\hbar$ and c being the Planck constant and the speed of light) is 1.3×10^{-27} Nm^2. Casimir energy calculations have been performed for several other two body geometries, like two spheres. A sphere and a plane i.e.; and they have been observed experimentally [2]. Although the existence of such an attraction between the parallel plates was observed in 1958 [3]; its quantitative confirmation had to wait until 2001 [4]. In this recent experiment the force is measured for plates of size 1.2×1.2mm^2 in 0.5–3.0μm range distant apart. The agreement with (1) is 15 percent. The force is measured indirectly by the measurement of the piezo electric current created by the attraction of the plates. Note that for a single cavity geometry no measurement has been performed.

The force (1), in the early years of its theoretical discovery gave a hope for an electron model . Electrons should be described as charged spherical shell; and, the attractive force in the shell should balance the repulsion of the Coulomb interaction. But the results of calculations [5] were surprising, for the Casimir energy for the spherical cavity with perfectly conducting walls is positive. In $\hbar = c = 1$ units it is given by

$$E_{sph} \cong \frac{0.046}{R}. \tag{2}$$

Derivative of the above expression with respect to the cavity radius R produces repulsive force

$$F = -\frac{\partial E_{sph}}{\partial R} = \frac{0.046}{R^2}. \tag{3}$$

The sign of the Casimir energy is still a mystery.

Magnitude of the Casimir energy is quite appreciable for structures of nanometer size. For example if we take $R = 10^{-7}$ cm, the energy given by (2) is [6]

$$E_{sph} \simeq 0.5 \times 10^6 \text{ cm}^{-1} \simeq 10\text{eV}. \tag{4}$$

(In $\hbar = c = 1$ units 1eV $\cong 0.5 \times 10^5$ cm^{-1}). In general for a cavity of any shape, the range of the absolute value of the energy differs between $E \simeq (10^{-2} - 10^{-1}) \times R^{-1}$ with R being the typical size of cavity. It is obvious that in the advancement of nanotechnology the Casimir force is an effect that should always be taken into the consideration. Talking about the magnitude of the Casimir energy we would like to emphasize that the effect must exist for any field whether it is massive or massless. However for massive fields the energy expressions are multiplied with an exponential factor e^{-ma}. Since for example for electron field that factor is $e^{-2.5\times10^3}$ we do not have to consider massive field Casimir energies. Similarly we do not have to take in account the spinor fields (at least in 3-dimensional laboratory geometries) for the only (almost) massless spinor fields are the neutrinos for which no boundary exists[1].

In the coming section we discuss the some old and recent results for the massless scalar fields for some 3-dimensional geometries. We must emphasize that an experimental investigation is not available for single cavities. Our basic motivation is the hope that rapid advancements in nanotechnologies may provide an experimental study for some of the geometries.

2. Casimir Energy Results for Some 3-dimensional Cavities

(i) Cube

The Casimir energy for massless scalar fields for spherical cavity is given by (2). Results for a cubical cavity with sides a is almost twice the value for the spherical cavity [5]

$$E_{cub} \simeq \frac{0.0916}{a}. \tag{5}$$

(ii) Rectangular Prism [5]

For a rectangular prism with square base of edges a and with height b the energy is

$$E_{rec} \simeq -\frac{0.013}{a} + \frac{(0.011)b}{a^2}, \text{ for } b > a \tag{6}$$

and

$$E_{rec} \simeq -\frac{0.013}{b} + \frac{(0.011)a}{b^2}, \text{ for } b < a \tag{7}$$

Inspecting the above formulae we see that the energy for $b > \frac{13a}{11} \simeq 1.2a$ or $b < \frac{11a}{13} \cong 0.8a$; otherwise it is negative. The force $F_b = -\frac{\partial E_{rec}}{\partial b}$ in the direction of the height of the prism is negative (attractive) for $b > a$ and positive (repulsive) for $b < a$.

The force $F_a = -\frac{\partial E_{rec}}{\partial a}$ on the walls perpendicular to the square bases on the hand is just opposite: that is, it is positive for $b > a$ and negative for $b < a$.

[1]Neutrinos may be relevant in some cosmological theories where the closed geometries are naturally constitute a confining region.

(iii) A Prism With Triangular Base [7]

For a prism of height b with equilateral triangular base of edges a the energy is

$$E_{tri} = \frac{1}{2}(E_{rec} - E_2), \tag{8}$$

where E_{rec} is same as (6) and (7); and E_2 is the energy for two dimensional rectangle with edges b and $\frac{a}{\sqrt{2}}$. Three cases are distinguished:

$$E_{tri} \simeq -\frac{0.053}{a} + \frac{(0.029)b}{a^2}, \text{ for } b > a, \tag{9}$$

and

$$E_{tri} \simeq \frac{1}{2}\left(-\frac{0.013}{b} + \frac{(0.011)a}{b^2} + \frac{0.093}{a} - \frac{(0.048)b}{a^2}\right), \text{ for } a > b > \frac{a}{\sqrt{2}}, \tag{10}$$

and

$$E_{tri} \simeq -\frac{0.039}{b} + \frac{(0.014)a}{b^2}, \text{ for } b < \frac{a}{\sqrt{2}}. \tag{11}$$

When we study the Casimir forces F_a and F_b we observe that for $b > \frac{a}{\sqrt{2}}$ we have $F_a > 0$ and $F_b < 0$; for $b < \frac{a}{\sqrt{2}}$ we have $F_a < 0$ and $F_b > 0$. Like in the case of the rectangular prism, too thin or too thick triangular prisms are not preferred.

(iv) A Conical Cavity [8]

For a conical cavity of height $h = \frac{2\sqrt{2}}{3}a$ and the opening angle $\beta = \arcsin\frac{1}{3}$ the energy is positive

$$E_{con} \simeq \frac{0.085}{a} > 0. \tag{12}$$

(v) A Pyramidal Cavity [9]

The energy in a pyramidal cavity with walls defined by the planes

$$z = x, y = 0, y = z, z = a = \text{ constant}, \tag{13}$$

is again positive and not much different from the previous example:

$$E_{pyr} \simeq \frac{0.069}{a} > 0. \tag{14}$$

3. Discussion

The results we have presented in this note do not provide an answer to our basic question: What the sign of Casimir energy depends on?

The difficulty is two fold. First of all we do not have any experiment performed for any single cavity. We hope that the advancements in nanoengineering provide new experimental results. Secondly our theoretical results are restricted, namely all of the calculations presented are performed for shape preserving geometries.

That is the cavities are either spherical or other one with constant angles, thus, we do not have any hint on the sensibility on the deformations. We have three definitely positive energy values. That is for the spherical, pyramidal and conical cavities. If we compare these geometries for equal volumes we have

$$E_{pyr} \simeq 0.51 E_{sph}, E_{con} \simeq 0.54 E_{sph} \tag{15}$$

We may think that the sphere which do not have any corner provides more reflections for the would be trapped field; that is the momentum space is larger in sphere.

Acknowledgments

The authors H. Ahmedov and I. H. Duru thanks the Turkish Academy of Sciences (TUBA) for its support.

References

[1] H. B. G. Casimir, *Proc. Kon. Nederl. Akad. Wet.,* **51**, (1948), 793.

[2] See for example the recent book by K. A. Milton "Physical Manifestations of Zero-Point Energy, The Casimir", World Sciebtific, Singapore (2001).

[3] M. J. Sparnaay, *Physica*, **24**, (1958), 751.

[4] G. Bressi, G. Carugno, R. Onofrio and G. Ruosa, *Phys. Rev. Lett.*, **88**, (2002), 041804.

[5] For the list of several calculations see for example the references in V. P. Mostepanenko and N. N. Trunov., *Sov. Phys. Usp.*, **31**, (1988), 965.

[6] I. H. Duru and M. Tomak, *Phys. Lett.,* **A176**, (1993), 265.

[7] H. Ahmedov and I. H. Duru, *Concepts Phys.* II, (2005), 137, special issue dedicated to A. O. Barut.

[8] H. Ahmedov and I. H. Duru, *J. Math. Phys.*, **46**, (2005), 022304.

[9] H. Ahmedov and I. H. Duru, *J. Math. Phys.*, **46**, (2005), 022303.