

# IBM-2 Calculations of Selected Even-Even Ruthenium Nuclei

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## Abstract

In this study, we have employed the Interacting Boson Model-2 (IBM-2) to determine the most appropriate Hamiltonian for the study of ruthenium nuclei in the region  $A \cong 100$ . Using the best fit values of parameters to construct the Hamiltonian, we have estimated energy levels and multipole mixing ratios ( $\delta(E2/M1)$ ) for some doubly-even Ru nuclei. The results are compared with previous experimental and theoretical data and it is observed that they are in good agreement.

**Key Words:** Ruthenium, electromagnetic transition, multipolarity, Interacting Boson Model-2 (IBM-2), deformation parameters.

## 1. Introduction

The mass region  $A \cong 100$  is a new region of neutron-excess nuclei. In view of the growing this kind of theoretical interest, the Interacting Boson Model (IBM) is one of those attempts that has been successful in describing the low-lying nuclear collective motion in medium and heavy mass nuclei [1, 2].

The neutron rich even even Ru isotopes are very important for understanding the gradual change from spherical to a deformed state via transitional phase [3]. The purpose of this paper is to investigate properties of some even-even ruthenium nuclei that is around the mass region  $A \cong 100$ .

The outline of the remaining part of this paper is as follows. Starting from an approximate IBM-2 formulation for the Hamiltonian in section 2, we give a review of theoretical background of the study. The previous experimental and theoretical [4–8] data are compared with estimated values and the general features of Ru isotopes in the range  $A=102-110$  are reviewed in section 3. There are three tables in this section: Table 1 gives the best fitted parameters used in the present work while Table 2 contains the comparison of estimated and experimental energy levels for  $^{102-110}\text{Ru}$ . Table 3 shows a comparison of estimated and experimental  $\delta(E2/M1)$  multipole mixing ratios of some transitions in  $^{102-110}\text{Ru}$  isotopes. The final section contains some concluding remarks.

**Table 1.** The best fit values of the Hamiltonian parameters for  $^{102-110}\text{Ru}$ .

$\frac{A}{Z}X_N$	$N_\pi$	$N_\nu$	$N$	$\varepsilon$	$\kappa$	$\chi_\nu$	$\chi_\pi$	$C_{L\nu}$ (L=0,2,4)	$C_{L\pi}$ (L=0,2,4)
$^{102}_{44}\text{Ru}_{58}$	3	4	7	0.740	-0.080	-1.00	0.30	-0.85, 0.0, 0.0	-0.3, 0.0, 0.0
$^{104}_{44}\text{Ru}_{60}$	3	5	8	0.650	-0.080	-0.90	0.30	-0.15, 0.0, 0.0	0.0, 0.0, 0.0
$^{106}_{44}\text{Ru}_{62}$	3	6	9	0.600	-0.090	-0.70	0.30	0.10, 0.0, 0.0	0.0, 0.0, 0.0
$^{108}_{44}\text{Ru}_{64}$	3	7	10	0.580	-0.080	-0.50	0.30	0.20, 0.0, 0.0	0.0, 0.0, 0.0
$^{110}_{44}\text{Ru}_{66}$	3	8	11	0.570	-0.080	-0.30	0.30	0.25, 0.0, 0.0	0.0, 0.0, 0.0

**Table 2.** The comparison of estimated and experimental energy levels for  $^{102-110}\text{Ru}$ .

Isotope	Spin Parity ( $I^\pi$ )	This Work (MeV)	Experiment [5]
$^{102}_{44}\text{Ru}_{58}$	$2_1^+$	0.481	0.475
	$4_1^+$	1.082	1.106
	$6_1^+$	1.791	1.873
	$8_1^+$	2.599	2.704
	$10_1^+$	3.496	3.432
	$12_1^+$	4.474	4.054
	$14_1^+$	5.523	4.803
	$2_2^+$	1.101	1.103
	$3_1^+$	1.802	1.522
	$4_2^+$	1.806	1.603
	$0_2^+$	0.948	0.944
	$2_3^+$	1.550	1.581
	$4_3^+$	2.234	1.799
$^{104}_{44}\text{Ru}_{60}$	$2_1^+$	0.377	0.358
	$4_1^+$	0.862	0.889
	$6_1^+$	1.443	1.556 [6]
	$8_1^+$	2.115	-
	$10_1^+$	2.870	-
	$12_1^+$	3.723	-
	$14_1^+$	4.624	-
	$2_2^+$	0.884	0.893
	$3_1^+$	1.456	1.242
	$4_2^+$	1.465	-
	$0_2^+$	0.975	0.988
	$2_3^+$	1.532	1.515
	$4_3^+$	2.125	-

**Table 2.** Continued.

Isotope	Spin Parity ( $I^\pi$ )	This Work (MeV)	Experiment [5]
$^{106}_{44}\text{Ru}_{62}$	$2_1^+$	0.298	0.270
	$4_1^+$	0.703	0.715
	$6_1^+$	1.205	1.296
	$8_1^+$	1.801	1.973
	$10_1^+$	2.475	-
	$12_1^+$	3.286	-
	$14_1^+$	4.097	-
	$2_2^+$	0.722	0.792
	$3_1^+$	1.213	1.092
	$4_2^+$	1.223	-
	$0_2^+$	0.990	0.991
	$2_3^+$	1.490	1.392
	$4_3^+$	1.808	-
$^{108}_{44}\text{Ru}_{64}$	$2_1^+$	0.286	0.242
	$4_1^+$	0.663	0.665
	$6_1^+$	1.120	1.219
	$8_1^+$	1.660	-
	$10_1^+$	2.260	-
	$12_1^+$	3.006	-
	$14_1^+$	3.712	-
	$2_2^+$	0.666	0.708
	$3_1^+$	1.118	0.975
	$4_2^+$	1.122	1.183
	$0_2^+$	0.973	0.976
	$2_3^+$	1.447	1.249[6]
	$4_3^+$	1.655	-
$^{110}_{44}\text{Ru}_{66}$	$2_1^+$	0.264	0.241
	$4_1^+$	0.613	0.663
	$6_1^+$	1.036	1.239
	$8_1^+$	1.543	1.948
	$10_1^+$	2.096	-
	$12_1^+$	2.823	-
	$14_1^+$	3.472	-
	$2_2^+$	0.613	0.613
	$3_1^+$	1.033	0.860
	$4_2^+$	1.034	-
	$0_2^+$	0.982	-
	$2_3^+$	1.427	1.396
	$4_3^+$	1.537	-

**Table 3.** The comparison of estimated multipole mixing ratios of some transitions for  $^{102-110}\text{Ru}$  . isotopes.

Isotope	Transition $I_i^+ \rightarrow I_f^+$	Energy (MeV)	$\delta(E2/M1)$ ( $eb/\mu_N$ )		
			Experimental	This Work	Theoretical
$^{102}_{44}\text{Ru}_{58}$	$2_2^+ \rightarrow 2_1^+$	0.586	-60 <sup>(a,b)</sup> -5.5 <sup>(c)</sup> 5 (+12,-3) <sup>(c)</sup>	3.26	-16 <sup>(a)</sup>
	$2_3^+ \rightarrow 2_1^+$	0.121	0.25 <sup>(a,b)</sup> 0.28 <sup>(c)</sup>	0.13	0.11 <sup>(a)</sup>
	$2_3^+ \rightarrow 2_2^+$	0.535	-	2.97	-
	$2_4^+ \rightarrow 2_2^+$	1.211	-	0.09	-
	$2_4^+ \rightarrow 2_3^+$	0.676	-	4.22	-
	$3_1^+ \rightarrow 2_2^+$	0.671	-7.2 <sup>(a)</sup>	7.33	-0.6 <sup>(a)</sup>
	$3_1^+ \rightarrow 2_3^+$	0.136	-	0.57	-
	$3_2^+ \rightarrow 2_4^+$	0.741	-	8.15	-
	$4_2^+ \rightarrow 4_1^+$	0.687	-	1.57	-
	$4_3^+ \rightarrow 4_1^+$	1.220	-	0.11	-
	$4_3^+ \rightarrow 4_2^+$	0.533	-	1.01	-
	$4_4^+ \rightarrow 4_2^+$	0.763	-	2.38	-
$^{104}_{44}\text{Ru}_{60}$	$2_2^+ \rightarrow 2_1^+$	0.507	-8.5 (+1.5,-2.5) <sup>(c)</sup> -36 (+14,-54) <sup>(c)</sup>	6.10	-10 <sup>(a)</sup>
	$2_3^+ \rightarrow 2_1^+$	1.155	-	0.27	-
	$2_4^+ \rightarrow 2_2^+$	1.250	-	3.11	-
	$2_4^+ \rightarrow 2_3^+$	0.602	-	1.42	-
	$4_2^+ \rightarrow 4_1^+$	0.603	-	2.65	-
	$4_3^+ \rightarrow 4_1^+$	1.263	-	2.42	-
	$4_3^+ \rightarrow 4_2^+$	0.660	-	4.78	-
	$4_4^+ \rightarrow 4_1^+$	1.306	-	0.18	-
	$4_4^+ \rightarrow 4_2^+$	0.703	-	4.99	-
$^{106}_{44}\text{Ru}_{62}$	$2_2^+ \rightarrow 2_1^+$	0.424	7.1 (+1.6,-1.1) <sup>(a,b)</sup>	1.87	-9 <sup>(a)</sup>
	$2_3^+ \rightarrow 2_1^+$	1.192	0.24 (+0.13,-0.12) <sup>(a,b)</sup>	0.31	0.43 <sup>(a)</sup>
	$2_3^+ \rightarrow 2_2^+$	0.768	-	3.99	-
	$4_2^+ \rightarrow 4_1^+$	0.520	-	1.09	-
	$4_3^+ \rightarrow 4_2^+$	0.585	-	1.39	-
	$4_4^+ \rightarrow 4_1^+$	1.379	-	0.24	-
	$4_4^+ \rightarrow 4_2^+$	0.859	-	2.54	-
	$4_4^+ \rightarrow 4_3^+$	0.274	-	0.19	-

**Table 3.** Continued.

Isotope	Transition $I_i^+ \rightarrow I_f^+$	Energy (MeV)	$\delta(E2/M1)$ ( $eb/\mu_N$ )		
			Experimental	This Work	Theoretical
$^{108}_{44}\text{Ru}_{64}$	$2_2^+ \rightarrow 2_1^+$	0.380	4.3 (+0.9,-0.6) <sup>(a)</sup>	0.88	-7 <sup>(a)</sup>
$^{108}_{44}\text{Ru}_{64}$	$2_3^+ \rightarrow 2_1^+$	1.161	0.9 (+0.7,-0.5) <sup>(a,b)</sup>	0.29	0.6 <sup>(a)</sup>
	$2_3^+ \rightarrow 2_2^+$	0.781	-	1.69	-
	$2_4^+ \rightarrow 2_3^+$	0.213	-	0.08	-
	$3_1^+ \rightarrow 2_1^+$	0.832	-3.0 (+0.7,-1.4) <sup>(a)</sup>	7.20	-4.6 <sup>(a)</sup>
	$3_1^+ \rightarrow 2_2^+$	0.452	-	1.00	-
	$4_2^+ \rightarrow 4_1^+$	0.459	-	0.55	-
	$4_3^+ \rightarrow 4_1^+$	0.992	-	3.32	-
	$4_3^+ \rightarrow 4_2^+$	0.533	-	0.65	-
	$4_4^+ \rightarrow 4_1^+$	1.346	-	0.22	-
	$4_4^+ \rightarrow 4_2^+$	0.887	-	1.04	-
	$4_4^+ \rightarrow 4_3^+$	0.354	-	0.16	-
$^{110}_{44}\text{Ru}_{66}$	$2_2^+ \rightarrow 2_1^+$	0.348	-	3.58	-
$^{110}_{44}\text{Ru}_{66}$	$2_3^+ \rightarrow 2_1^+$	1.163	-	0.98	-
	$2_3^+ \rightarrow 2_2^+$	0.815	-	8.78	-
	$3_1^+ \rightarrow 2_2^+$	0.421	-	4.07	-
	$4_2^+ \rightarrow 4_1^+$	0.421	-	2.27	-
	$4_3^+ \rightarrow 4_2^+$	0.503	-	2.69	-
	$4_4^+ \rightarrow 4_1^+$	1.358	-	0.73	-
	$4_4^+ \rightarrow 4_2^+$	0.937	-	5.03	-
	$4_4^+ \rightarrow 4_3^+$	0.3618	-	2.58	-

<sup>(a)</sup> Ref.[4], <sup>(b)</sup> Ref.[7], <sup>(c)</sup> Ref.[8]

## 2. Theoretical Framework

The Ru isotopes, in particular, have recently been investigated within the IBA-1 model by Frank [9, 10]. The “geometrical” General Collective Model (GCM) has also been recently applied to the Ru isotopes by Troltenier et al. [11] and the isotopes were found to exhibit spherical structure with a tendency to triaxiality. It is proposed the change in structure is related to exceptionally strong neutron-proton interaction. It is also suggested that the neutron-proton effective interactions have a deformation-producing tendency, while the neutron-neutron and proton-proton interactions are of spheriphying nature [12, 13].

While defining such nuclei in a geometric description [14], these phenomena will have a standard description that is given in terms of nuclear triaxiality [15] going from rigid triaxial shapes to softer potential energy surfaces. In the first version of the interacting boson model (IBM-1) [16], no distinction is made between proton and neutron variables while describing triaxiality explicitly. This can be done by introducing the cubic terms in the boson operators [17, 18]. This is in contrast to the recent work of Dieperink and Bijker [19, 20] who showed that triaxiality also occurs in particular dynamic symmetries of IBM-2 that does distinguish between protons and neutrons. Within the region of medium-heavy and heavy nuclei, a large number of nuclei exhibit properties that are neither close to anharmonic quadrupole vibrational spectra nor to deformed rotors [21].

According to A. Arima et al. [22] and F. Iachello [23] IBM Hamiltonian takes different forms depending on the applicable regions (SU(5), SU(3), O(6)) of the traditional IBA triangle. The Hamiltonian that we consider is in the form [17],

$$H = H_{sd} + \Sigma \theta_L [d^+ d^+ d^+]^{(L)} [d^\sim d^\sim d^\sim]^{(L)}, \quad (1)$$

where  $H_{sd}$  is the standard Hamiltonian of the IBM [15, 16],

$$H_{sd} = \epsilon_d \eta_d + \kappa \mathbf{Q} \cdot \mathbf{Q} + \kappa' \mathbf{L} \cdot \mathbf{L} + \kappa \mathbf{P}^+ \cdot \mathbf{P} + q_3 \mathbf{T}_3 \cdot \mathbf{T}_3 + q_4 \mathbf{T}_4 \cdot \mathbf{T}_4 \quad (2)$$

In the Hamiltonian,  $\epsilon_d \eta_d$  and  $\mathbf{P}^+ \cdot \mathbf{P}$  terms produce the characteristics of U(5) and O(6) structures, respectively. So the Hamiltonian is a mixture of the U(5) and SO(6) chains, but not diagonal in any of the IBM chains. In the IBA-2 model, the degrees of freedom of the neutrons and protons are explicitly taken into account. Thus the Hamiltonian [24, 25] can be written as

$$H = \epsilon_\nu n_{d\nu} + \epsilon_\pi n_{d\pi} + \kappa \mathbf{Q}_\pi \cdot \mathbf{Q}_\nu + V_{\pi\pi} + V_{\nu\nu} + M_{\pi\nu}, \quad (3)$$

where  $n_{d\nu(\pi)}$  is the neutron (proton) d-boson number operator.

$$\begin{aligned} n_{d\rho} &= d^+ d^\sim, \rho = \nu, \pi \\ d_{\rho m}^\sim &= (-1)^m d_{\rho -m} \end{aligned} \quad (4)$$

where  $s_\rho^+, d_{\rho m}^+$  and  $s_\rho, d_{\rho m}$  represent the s- and d-boson creation and annihilation operators. The rest of the operators in equation (3) are defined as

$$\begin{aligned} Q_\rho &= (s_\rho^+ d_\rho^\sim + d_\rho^+ s_\rho) + \chi_\rho (d_\rho^+ d_\rho^\sim) \\ V_{\rho\rho} &= \sum_{L=0,2,4} C_{L\rho} \left( (d_\rho^+ d_\rho^+)^{(L)} (d_\rho^+ d_\rho^\sim)^{(L)} \right)^{(0)}; \rho = \nu, \pi \end{aligned} \quad (5)$$

and

$$M_{\pi\nu}; \sum_{L=1,3} \xi_L (d_\nu^+ d_\pi^+)^{(L)} (d_\nu d_\pi)^{(L)} + \xi_2 (s_\nu d_\pi^\sim - s_\pi d_\nu^\sim)^{(2)} \cdot (s_\nu^+ d_\pi^+ - s_\pi^+ d_\nu^+)^{(2)}. \quad (6)$$

In the present case,  $M_{\pi\nu}$  affects only the position of the non-fully symmetric states relative to the symmetric states. For this reason  $M_{\pi\nu}$  is often referred to as the Majorana force [25].

The rule of choice for the total angular momentum is given as

$$|J_i - J_f| \leq L\gamma \leq |J_i + J_f| \quad (7)$$

The mixing ratio E2/M1, where  $T(E2; J_i \rightarrow J_f)$  is the number of E2 transitions per second and  $T(M1; J_i \rightarrow J_f)$  is the number of M1 transitions per second, is given by

$$\delta(E2/M1; J_i \rightarrow J_f) = \frac{\sqrt{T(E2; J_i \rightarrow J_f)}}{\sqrt{T(M1; J_i \rightarrow J_f)}}. \quad (8)$$

The ratio of  $\delta(E2/M1)$  can be written in terms of matrix elements as follows

$$\delta(E2/M1) = 0.836 E\gamma (MeV) \frac{\langle J_f || M(E2) || J_i \rangle}{\langle J_f || M(M1) || J_i \rangle}. \quad (9)$$

The electric quadrupole (E2) transitions are one of the important factors within the collective nuclear structure. In IBM, the general linear E2 operator with  $L = 2$  is given by

$$\begin{aligned} T(E2) &= \alpha_2(s^+d + d^+s) + \beta_2[d^+d]_2 \\ &= \alpha_2(s^+d + d^+s) + \xi[d^+d]_2. \end{aligned} \quad (10)$$

In this form,  $\alpha_2$ ,  $\beta_2$  and  $\chi$  are free parameters.  $B(E2; J_i \rightarrow J_f)$  is given in the following formulation:

$$\begin{aligned} B(E2; J_i \rightarrow J_f) &= \sum_{mM'} | \langle J_f M' | T(E2) m | J_i M \rangle |^2 \\ B(E2; J_i \rightarrow J_f) &= \frac{1}{2J_i + 1} | \langle J_f || T(E2) || J_i \rangle |^2. \end{aligned} \quad (11)$$

### 3. Computational Details

The parameters  $\varepsilon$ ,  $\kappa$  and  $C_{L\rho}$  are free parameters that have been determined so as to reproduce as closely as possible the excitation-energy of all positive parity levels for which a clear indication of the spin value exists, following the same procedure described in [4]. The value of  $\chi_\pi$  has been kept fixed along the isotopic chain as suggested by microscopic considerations which predict that this parameter depends only on the proton number [26]. This parameter is extremely important because it is closely related to the nuclear shape (prolate or oblate) [27]. The full sets of adopted parameters are reported in Table 1. Altogether, six parameters are appearing in the Hamiltonian. The  $^{102-110}\text{Ru}$  isotopes have  $N_\pi = 3$  (relative to  $Z = 50$ ) and  $N_\nu$  varies from 4 to 8 (relative to  $N = 50$ ), while the parameters  $\varepsilon$ ,  $\chi_\rho$  and  $\kappa$ , as well as  $C_{L\rho}$ , with  $L = 0, 2, 4$  were treated as free parameters and their values were estimated by fitting to the measured level energies. This procedure was made by selecting the ‘‘traditional’’ values of parameters and then allowing one parameter to vary while keeping the others constant until a best fit was obtained. This was carried out iteratively until an overall fit was achieved. Having obtained wave functions for the states in  $^{102-110}\text{Ru}$  after fitting the experimental energy levels in IBM-2, we can estimate the electromagnetic transition rates between states using the program PHINT [25]. As it is pointed out by Bijker et al. [28], nuclei with  $\chi_\pi + \chi_\nu = 0$  have properties close to those of the O(6) limit. This is not in agreement with earlier IBM [29, 30] calculations for the Sm isotopes. In this study, we take  $\chi_\pi = 0.3$  for all  $\text{Ru}$  isotopes. In particular, the spectrum of the SU(5) nuclei is dominated by value of  $\varepsilon$ , large in comparison with the other parameters, whereas O(6) nuclei are characterized by value of  $\kappa$ , large compared to  $\varepsilon$  [31]. The energy levels that are fitted by these parameters are shown in Table 2 along with experimental levels. As can be seen, the agreement between experiment and theory is quite good and the general features are reproduced well.

As it is seen in the systematics of basic observables in Ru isotopes  $E(2_1^+)$  of  $^{102-110}\text{Ru}$  nuclei drops smoothly with the neutron number. That is, it can be said that the collectivity increases smoothly. The behavior of the ratio of the energies of the first  $4^+$  and  $2^+$  states are good criterion for the shape transition; the value of  $R_{4/2}$  ratio has the limiting value 2.0 for a quadrupole vibrator, 2.5 for a non-axial gamma-soft rotor and 3.33 for an ideally symmetric rotor. The  $R_{4/2} = E(4_1^+)/E(2_1^+)$  ratio increases gradually with neutron number until  $N = 62$ , then remains nearly constant for  $N = 64$  and  $66$ . The estimated values changing from about 2.25 to about 2.36. It means that their structure seems to be varying from quadrupole vibrator to non-axial gamma soft.

We have estimated multipole mixing ratios ( $\delta(E2/M1)$ ) of some transitions for  $^{102-110}\text{Ru}$  isotopes and then compared them with some previous experimental and theoretical results in Table 3. There is no reference for transitions in  $^{110}\text{Ru}$  and they are in good agreement with the experimental values. The variations in

sign of the E2/M1 mixing ratios from nucleus to nucleus for the same class transitions and within a given nucleus for transitions from different spin states suggest that a microscopic approach is needed to explain the data theoretically. For that reason, we did not take into consideration the sign of mixing ratios. Sign convention of mixing ratios had explained in detail by A. M. Demidov et al. [7] and J. Lange et al. [8].

## 4. Conclusion

The results and comparisons shown in Table 2 and Table 3 indicate the quality of the fit presented in this paper. The shape transition predicted by this study is consistent with the spectroscopic data for these nuclei.  $^{102-110}\text{Ru}$  are typical examples of isotopes that exhibit a smooth phase transition from vibrational nuclei to soft triaxial rotor. As seen from the tables, the predictions show that  $^{102-110}\text{Ru}$  isotopes are lined up along the SU(5)-O(6) side of the IBM triangle. According to the results,  $^{102}\text{Ru}$  nuclei is positioned at the center of this side. The other isotopes are arranged in an order from the center to the location of deformed  $\gamma$ -soft nuclei.

As seen in Table 3, estimated multipole mixing ratios ( $\delta(\text{E2/M1})$ ) are mostly in agreement with the theoretical and experimental mixing ratios. We present here some mixing ratio values which have not yet been studied. These unstudied values can be identified in Table 3 as those values which have neither experimental nor theoretical values.

In view of the growing pursuit in this kind of theoretical interest, it is assumed that a new study investigating the properties of neutron rich full isotopic mass chains around  $A \cong 100$  mass region will also be carried out.

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