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Longitudinal Form Factor of Isoscalar Particle-Hole States in ^{16}O, ^{12}C and ^{40}Ca with M3Y Interaction

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Abstract

Longitudinal form factors of the low-lying, T = 0, particle-hole states of ^{16}O, ^{12}C and ^{40}Ca are studied in the framework of Random Phase Approximation RPA. The basis of single particle states is considered to include 0s, 0p, 1s-0d and 0f-1p. The Hamiltonion is diagonalized in the presence of Michigan three-rang Yakawa (M3Y) interaction and compared with our previous results depend on Modified Surface Delta Interaction MSDI interaction. Admixture of higher configuration up to 2p-1f is considered for the ground state. Effective charges are used to account for the core polarization effect. Comparisons are made to experimental data where available and the theoretical significance of the calculation and its results is discussed.

Key Words: Longitudinal Form Factors, Random Phase Approximation (RPA), Collective Model, and M3Y.

1. Introduction

The interaction of the electron with the charge distribution of the nucleus gives rise to the longitudinal or coulomb scattering. The nuclear size may be taken into account by multiplying the Mott's cross section by a fraction depending on the charge distribution in the nucleus, and this fraction is called longitudinal form factor [1]. The models of nuclear structure can be tested by comparing the calculated and measured longitudinal form factors. In this paper we study the isoscalar transitions in ^{16}O, ^{12}C and ^{40}Ca these transitions include states J^π (E_x (MeV)): 1^- (7.12) and 3^- (6.13) in ^{16}O, 2^+ (4.44) in ^{12}C and 5^- (4.48) in ^{40}Ca. In the shell model calculation, even-even nuclei are assumed to form closed shells, and excitations from these states closed shell are considered to describe the excited states. According to Random Phase Approximation RPA, the ground state as well as the excited states are treated on the same footing, and all possible configurations are constructed by removing a particle from the closed shells, and promoting it to higher shells leaving a hole state within the closed shells [2]. The ground state and the excited collective oscillations can be described as a linear combination of particle-hole states.

The basis of single particle states is consider to include 0s, 0p, 1s-0d and 0f-1p, the Hamiltonion is diagonalized in this space in the presence of Michigan three-rang Yakawa (M3Y) interaction compared with our previous results depend on Modified Surface Delta Interaction (MSDI) interaction. To add more degree of collectivity, an admixture of higher orbits up to 2p-1f is consider, and effective charges are used to account for the core polarization effect.

2. Formulation

Excited states $|\psi_{JT}\rangle$ of multipolarity J are formed as superposition of particle-hole states [3]

$$|\psi_{JT}\rangle = \sum_{ph} (X_{ph}^{JT}\, a_p^\dagger a_h - Y_{ph}^{JT}\, a_h^\dagger a_p)\, |0\rangle \tag{1}$$

Here $|0\rangle$ is the exact Hartree-Fock ground state, which is itself described as an antisymmetrized Slater determinant

$$|0\rangle = \prod_{i=0}^{A} a_i^\dagger\, |vac\rangle\,. \tag{2}$$

$a_p^\dagger$ and a_h are creation and annihilation operators for positive-energy single particle states above and below the Fermi surface. X_{ph}^{JT} and Y_{ph}^{JT} are the amplitudes for creating a particle-hole pair and for annihilating a particle-hole pair already present in the ground states.

The longitudinal form factor squared between initial and final nuclear shell-model states of spin (isospin) $J_{i,\,f}(T_{i,\,f})$ are related to the reduced matrix elements by [4]

$$\begin{aligned} F_L^2(q) \;=\; & \tfrac{4\pi}{Z^2(2J_i+1)} \left| \sum_{T=0,1} (-1)^{T_f-T_Z} \begin{pmatrix} T_f & T & T_i \\ -N_f & 0 & N_i \end{pmatrix} \right. \\ & \times \left. \left\langle \psi_{J_fT_f} \left|\left|\left| \hat{M}_{JT}^{Coul} \right|\right|\right| \psi_{J_iT_i} \right\rangle \right|^2 F_{c.m.}^2(q)\, F_{f.s.}^2(q), \end{aligned} \tag{3}$$

where $N_i(N_f)$ denote the isospin projection of the initial (final) state, respectively. $F_{c.m.}^2(q)$ and $F_{f.s.}^2(q)$ are form factor corrections [5].

The many-particle nuclear matrix elements can be written in terms of single-particle matrix elements

$$\begin{aligned} \left\langle \psi_{J_fT_f} \left|\left|\left| \hat{M}_{JT}^{Coul} \right|\right|\right| \psi_{J_iT_i} \right\rangle \;=\; & \sum_{ph} \Big[\left\langle p \left|\left|\left| \hat{M}_{JT}^{Coul} \right|\right|\right| h \right\rangle X_{ph}^{JT} \\ & + (-1)^{j_h-j_p-J-T} \left\langle h \left|\left|\left| \hat{M}_{JT}^{Coul} \right|\right|\right| p \right\rangle Y_{ph}^{JT} \Big]\,. \end{aligned} \tag{4}$$

Single-particle matrix element can be written in terms of single-particle matrix element reduced in spin only [6] as

$$\langle p \left|\left|\left| \hat{M}_{JT=0}^{Coul} \right|\right|\right| h \right\rangle = \frac{1}{\sqrt{2}} \sum_{t_Z} \langle p \left|\left| \hat{M}_{J\,t_Z}^{Coul} \right|\right| h \right\rangle \tag{5}$$

$$\langle p \left|\left|\left| \hat{M}_{JT=1}^{Coul} \right|\right|\right| h \right\rangle = \sqrt{\frac{3}{2}} \sum_{t_Z} (-1)^{\frac{1}{2}-t_Z} \langle p \left|\left| \hat{M}_{J\,t_Z}^{Coul} \right|\right| h \right\rangle \tag{6}$$

The longitudinal (Coulomb) form factor is arising from the charge density of a point nucleon. The reduced single-particle matrix element of the Coulomb operator is defined as [7]

$$\begin{aligned} \langle n_p\ell_pj_p \left|\left| \hat{M}_{J\,t_Z}^{Coul} \right|\right| n_h\ell_hj_h \right\rangle \;=\; & e(t_Z)P_J(\ell_p,\ell_h)C_J(j_p,j_h) \\ & \times \langle n_p\ell_p|\; j_J(qr)|\; n_h\ell_h\rangle \end{aligned} \tag{7}$$

where

$$e(t_Z) = \frac{1+\tau_Z(i)}{2}, \quad \tau_Z = 2t_Z, \quad P_J = \frac{1}{2}\left[1+(-1)^{\ell_p+\ell_h+J}\right]$$

and

$$\begin{aligned} C_J(j_p, j_h) &= (-1)^{j_{p+1/2}} \left[\frac{(2j_p+1)(2J+1)(2j_h+1)}{4\pi}\right]^{1/2} \\ &\times \begin{pmatrix} j_p & J & j_h \\ 1/2 & 0 & -1/2 \end{pmatrix} \end{aligned} \tag{8}$$

Using the ansatz of equation (1) for the wave function, and linearizing the equations of motion, leads (this is strictly true only if one ignore retardation corrections) to the familiar RPA eigenvalue problem [8]

$$\sum_{p_2h_2} \begin{pmatrix} A^{JT}_{p_1h_1\,,\,p_2h_2} & B^{JT}_{p_1h_1,\,p_2h_2} \\ -B^{JT}_{p_1h_1\,,\,p_2h_2} & -A^{JT}_{p_1h_1\,,\,p_2h_2} \end{pmatrix} \begin{pmatrix} X^{JT}_{p_2h_2} \\ Y^{JT}_{p_2h_2} \end{pmatrix} = \hbar\omega \begin{pmatrix} X^{JT}_{p_1h_1} \\ Y^{JT}_{p_1h_1} \end{pmatrix}, \tag{9}$$

with

$$A^{JT}_{p_1h_1\,,\,p_2h_2} = (\varepsilon_{p_1} - \varepsilon_{h_1})\delta_{h_1h_2}\delta_{p_1p_2} + V^{JT}_{p_1h_1\,,\,p_2h_2} \tag{10}$$

$$B^{JT}_{p_1h_1,\,p_2h_2} = (-1)^{1+T+j_{h_1}-j_{p_1}-J}\, V^{JT}_{p_1h_1\,,\,p_2h_2}. \tag{11}$$

The matrix elements for particle-hole states $V^{JT}_{p_1h_1\,,\,p_2h_2}$, coupled to J and T, are given by a sum of particle-particle matrix elements, $\langle p_2h_1|\, V|\, p_1h_2\rangle_{\,J'\,T'}$, coupled to different values of $J'$and T' [9]:

$$\begin{aligned} V^{J\,T}_{p_1h_1\,,\,p_2h_2} &= -\sum_{J'\,T'} (2J'+1)(2T'+1) \begin{Bmatrix} j_{p_1} & j_{h_2} & J' \\ j_{p_2} & j_{h_1} & J \end{Bmatrix} \begin{Bmatrix} 1/2 & 1/2 & T' \\ 1/2 & 1/2 & T \end{Bmatrix} \\ &\times \langle p_1h_2|\, V|\, p_2h_1\rangle_{\,J'\,T'} \end{aligned} \tag{12}$$

The particle-particle residual interaction used in this work is the Michigan three-rang Yakawa (M3Y) interaction [10, 11]. The M3Y interaction basically represents the G-Matrix for two nucleons bound near the Fermi surface, and therefore somewhat realistic, and enables us to avoid tedious computation. In its simplest form the M3Y interaction is given by two direct terms with different ranges and an exchange term represented by a delta interaction.

3. Results and Discussion

The Hamiltonion is diagonalized in the space of the single particle-hole states which include the orbits $0s_{1/2}$, $0p_{3/2}$, $0p_{1/2}$, $0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$, $0f_{7/2}$, $1p_{3/2}$, $0f_{5/2}$ and $1p_{1/2}$ in the presence of the M3Y interaction, then results are compare with our previous results depend on MSDI and with the available experimental data.

In general the value of amplitudes X^{JT}_{ph} and Y^{JT}_{ph} was used to calculate the reduced matrix elements of particle-hole states of the coulomb scattering operator $\hat{M}^{Coul}_{JT}$ in terms of the reduced single particle matrix element. The longitudinal (coulomb) form factors are calculated and interpreted with using mixing parameter γ that mixes the ground state $|n\,\ell\, j\rangle$ with the state $|n+1\ \ell\ j\rangle$ and with using effective charges to compensates for the configuration that are outside the space considered in this work.

In Figure 1, we show the longitudinal electron scattering data for Buti et al. [12] for the lowest nonspurious 1^- ($T = 0$, $E_x = 7.12$ MeV) state in ^{16}O, compared to our RPA calculation based on M3Y interaction. The comparison shows elimination of the spurious contamination for the 1^- state when the admixture of higher orbits in the ground state are taken into consideration with $\gamma = 0.9$. Core polarization effects are introduced with effective charges $e_p = 1.35e$ for protons and $e_n = 0.45e$ for neutrons. The value of the size parameter b of HO potential for single particle wave function used in the state is 1.83 fm, which is consistent with that in Reference [12]. The form factor predictions are reasonable agreement with data, but the radial scale is compressed. Our previous RPA calculations based on MSDI predicts the second state 1^- (15.58 MeV) is the nonspurious state.

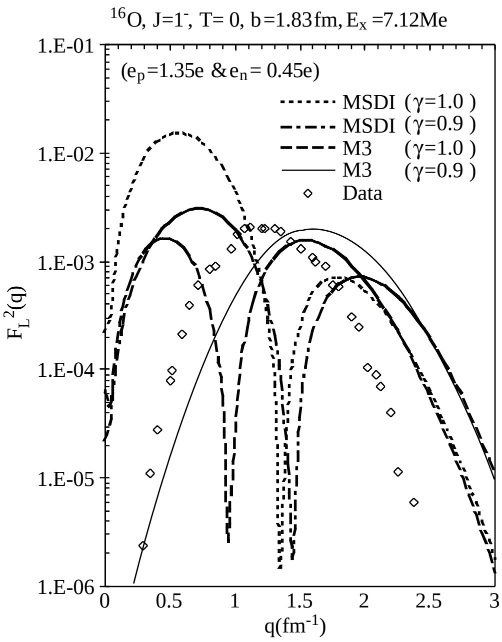

Figure 1. The Longitudinal form factor C1 data for the lowest 1^- (7.12 MeV) T=0 state in ^{16}O taken from [12] compared with RPA calculation using M3Y interaction from this work and with RPA calculation using MSDI interaction [8].

The longitudinal form factor for the 3^-($T = 0$, $E_x = 6.13$ MeV), C3 excitation in ^{16}O is shown in Figure 2. Our RPA Calculations with M3Y is based on the single-particle wave functions of the HO potential with size parameter b = 1.8 fm, and effective charges $e_p = 1.35e$ and $e_n = 0.45e$ for the protons and neutrons, respectively. Admixture of higher orbits in the ground state for the 3^- (6.13 MeV) state is less important from that in the 1^- (7.12 MeV) state. Results are consistent with our previous results of Referance [8], where their effective charges is e_p = 1.15e and e_n = 0.15e. The present and previous results agree in shape and magnitude quite well with experimental data for all momentum transfer values.

The longitudinal C2 form factor for the 2^+ ($T = 0$, $E_x = 4.44$ MeV) excitation in ^{12}C is shown in Figure 3. There is good agreement with experimental data (taken from Reference [13]) for all momentum transfer values satisfied with $b = 1.64$ fm, e_p = 1.35e, e_n = 0.45e and admixture factor γ = -0.99. The same agreement with experimental data (taken from Reference [13]) is seen in Figure 4, the longitudinal form factor of the 5^- (T=0, E_x =4.48 MeV) state in ^{40}Ca with b=1.9 fm, e$_p$=1.25e, e$_n$=0.25e and γ =1.0.

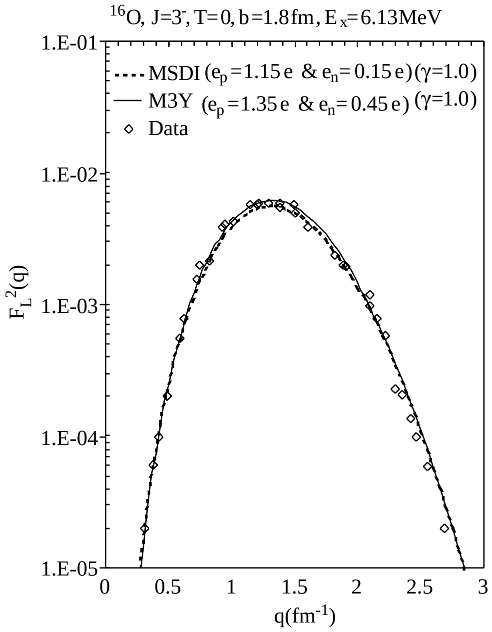

Figure 2. The Longitudinal form factor C3 data for the lowest 3^- (6.13 MeV) T=0 state in ^{16}O taken from [12] compared with RPA calculation using M3Y interaction from this work and with RPA calculation using MSDI interaction [8].

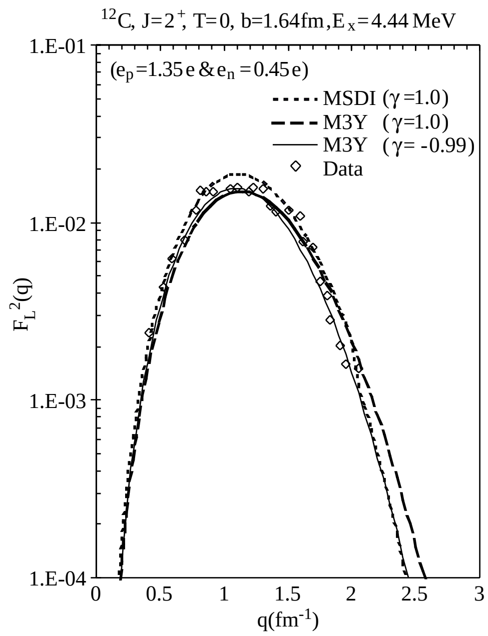

Figure 3. The Longitudinal form factor C2 data for the lowest 2^+ (4.44 MeV) T=0 state in ^{12}C taken from [13] compared with RPA calculation using M3Y interaction from this work and with RPA calculation using MSDI interaction.

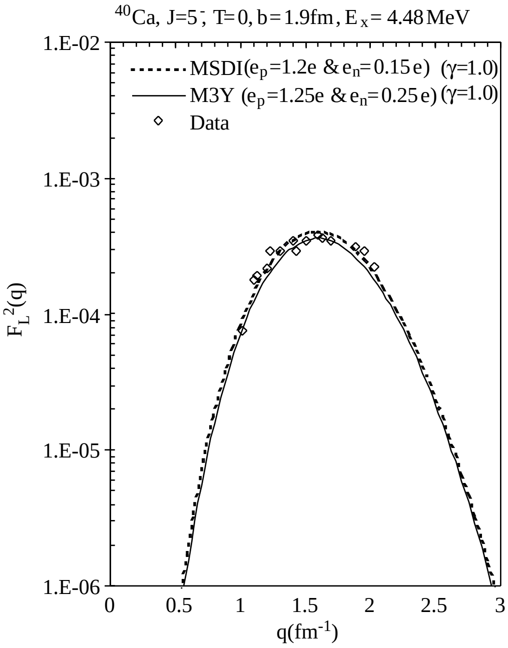

Figure 4. The Longitudinal form factor C5 data for the lowest 5^- (4.48 MeV) T=0 state in ^{40}Ca taken from [13] compared with RPA calculation using M3Y interaction from this work and with RPA calculation using MSDI interaction [5].

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