Turk J Phys
32 (2008) , 59 – 64.

Masses of Charged Leptons and Gauge Mixing Parameters

Cvavb Chandra RAJU
Department of Physics, Osmania University, Hyderabad-500 007, INDIA
e-mail: cvavb@rediffmail.com

Received 02.10.2007

Abstract

It is shown that the electroweak physics of the electron and the muon is based on the left-right gauge model with the mixing parameters 0.2254 and 0.2746. These mixing parameters are determined by the masses of the electron and the muon. The electroweak physics of the Tau-lepton and its left-handed neutrino is based on the gauge group $SU(2)_L XU(1)$.The mixing parameter for this gauge model is 0.5 and is determined by the mass of the charged Tau-lepton. The W and Z bosons that mediate the electroweak physics of Tau-lepton and its neutrino have masses 173.5 GeV and 245.4 GeV, respectively.

Key Words: Charged lepton masses, mixing parameters, Gauge models.

PACS: 14.60-z, 11.15-q, 13.10+q.

1. Introduction

The three charged leptons, the electron, muon and the Tau-lepton, have different masses. The world would be the same even without the presence of the other two leptons: muon and Tau-lepton. Then the question is: why is it that the nature has provided us these extra leptons with different masses. With the advent of gauge theories the masses of all the elementary particles are believed to be given by the Vacuum Expectation Values (VEV) of the Higgs boson.

With this motivation we found [1] exact expressions for the masses of the electron and the muon. The electron-muon mass ratio is supposed to be one of the unsolved problems of physics. In this reference we have derived expressions for the masses of the electron and the muon. But at that time we could not extend these expressions for the case of the charged Tau-lepton in a convincing way. This note is to show that the theoretical expression for the mass of the Tau-lepton can be written down from the theoretical expression for the mass of the electron/muon. In addition it will be shown here how these masses determine the mixing parameters of the gauge models. In section (2) the problem will be formulated and section (3) contains our solution and conclusion.

2. Formulation of Problem

With the advent of the gauge theories it is very clear that the masses of all elementary particles must depend on the VEVs and gauge constants, or functions of gauge constants. The mass of the electron cannot be a function of only the fine structure constant. It should depend on the weak interaction constant as well because the electron takes part in both electroweak interactions predominantly. Its mass may not depend on the very weak gravitational interaction. Whatever is said about the electron mass must also be true in the case of the muon because it is a heavy electron in so far as the mass is concerned .With this motivation we derived closed expressions for the masses of the electron and the muon in reference [1]. These are exact expressions. They are:

$$m_e^2 = m_1 M_{WL} \frac{\left({}^{g_V}/_{g_A}\right)^4 v_e}{\left({}^{g_V}/_{g_A}\right)^4 e\mu} \left\{ 1 - \left[1 - \left(\frac{g_V}{g_A}\right)^4 e\mu \right]^{1/2} \right\}, \tag{2.1}$$

and

$$m_\mu^2 = m_1 M_{WL} \frac{\left({}^{g_V}/_{g_A}\right)^4 v_\mu}{\left({}^{g_V}/_{g_A}\right)^4 e\mu} \left\{ 1 + \left[1 - \left(\frac{g_V}{g_A}\right)^4 e\mu \right]^{1/2} \right\}. \tag{2.2}$$

In the above expressions, $m_1$is the majorana mass of the electron and muon-neutrino. In other words, the electron-neutrino and the muon-neutrino have equal majorana mass m_1, and together these two neutrinos are equivalent to one Dirac neutrino with the same mass, m_1. M_{WL} is the mass of W boson of the standard electro-weak model. Moreover,

$$\left(\frac{g_V}{g_A}\right)^2 v_e = \left(\frac{g_V}{g_A}\right)^2 v_\mu = 1 \tag{2.3}$$

and

$$\left(\frac{g_V}{g_A}\right)^2 e\mu = (-1 + 4\sin^2\vartheta_W)^2 \tag{2.4}$$

Here θ_W is the Weinberg mixing parameter of the standard electroweak model. From equations (2.1) and (2.2) it just follows that

$$\frac{2m_e m_\mu}{m_e^2 + m_\mu^2} = \left(\frac{g_V}{g_A}\right)^2 e\mu = (-1 + 4\sin^2\vartheta_W)^2 \tag{2.5}$$

Equation (2.5) is an important relation. It connects the masses of the electron and the muon with the Weinberg mixing parameter. Weinberg mixing parameter is a free parameter in the Standard model. Many group theoretical and perturbation techniques were used to predict this parameter. In all these attempts the electron and the muon masses are supposed to have no predominant influence on the calculation of this mixing parameter. But here we find that these masses alone predict the mixing parameter. We treat $x = \sin^2\theta_w$ as an unknown parameter, in equation (2.5). Inserting the well known masses of the electron and the muon we readily find that,

$$x^2 - 0.5x + 0.061895771 = 0. \tag{2.6}$$

The two roots of the above quadratic equation are

$$\begin{aligned} x &= 0.2254 \\ x &= 0.2746 \end{aligned} \tag{2.7}$$

The sum of the two roots is 0.5 as it should be. The first root agrees pretty well with the measured value of the Weinberg mixing parameter of the standard model [2]. The second root is also very important.

In a left-right gauge model based on the gauge groups $SU(2)_L$X $SU(2)_R$X$U(1)$ there will be two mixing parameters given by

$$x_L = \frac{e^2}{g_L^2}, \tag{2.8}$$

and,

$$x_R = \frac{e^2}{g_R^2} \tag{2.9}$$

where g_L and g_R are the gauge constants corresponding to the left and right gauge groups. The left mixing parameter defined by equation (2.8) is identical to the Weinberg mixing parameter. The Electroweak physics of the electron (muon) must be based on the left-right gauge group with the above mixing parameters. There will be two weak neutral currents in such a left-right electro weak model (see reference [3, 4]). There will be two neutral vector bosons, Z boson, and D boson in addition to the photon. These bosons are mass eigenstates. The weak part of the neutral interaction is given by,

$$H_{\text{weak}}^{\text{int}} = g_z J_{ZL} Z + g_Z[\beta J_{ZL} - (\alpha+\beta) J_{ZR}]D \tag{2.10}$$

where

$$J_{ZL} = j_{3l} - x_l j_{em}, \tag{2.11}$$

$$J_{ZR} = j_{3R} - x_R j_{em}, \tag{2.12}$$

$$g_Z = \frac{e}{(1-x_L)^{1/2} x_L^{1/2}}, \tag{2.13}$$

$$\beta = \frac{\sqrt{x_L x_R}}{\sqrt{1 - x_L - x_R}}, \tag{2.14}$$

$$\alpha + \beta = \frac{(1-x_L)x_L^{1/2}}{(1-x_L-x_R)^{1/2} x_R^{1/2}}, \tag{2.15}$$

The above interaction is triangle anomaly free [3, 4]. For our consideration here, it is not necessary to require that the interaction be triangle anomaly free. But this has close resemblance to the standard model. In equation (2.10) the currents are given by,

$$j_{3L} = \frac{1}{4}\overline{v}_e \gamma_\mu (1+\gamma_5) v_e + (v_e \to v_\mu) - \frac{1}{4}\overline{e}\gamma_\mu(1+\gamma_5)^e - (e \to \mu) + \text{ quarks} \tag{2.16}$$

$$j_{em} = -\overline{e}\gamma_\mu e - \overline{\mu}\gamma_\mu \mu + \text{ quarks} \tag{2.17}$$

The left-right electroweak gauge model has two weak neutral currents. The vector coupling constant g_V and the axial vector coupling constant g_A have different values for the Z and D bosons. Therefore the ratios $(g_V/g_A)_{e\mu,Z}$ and $(g_V/g_A)_{e\mu,D}$ are not necessarily equal .But these ratios determine the masses of the electron and the muon as in Eqs. (2.1) and (2.2). Even in this left right model the ratios$(g_V/g_A)_{ve,Z} = (g_V/g_A)_{ve,D} = 1$. Same is the case with respect to the muon neutrino, because these are left handed and neutral. The expressions for the masses of the electron and the muon contain the ratios of the vector and axial vector coupling-constants. If the electroweak physics of these leptons is based on the left-right gauge model, then we require that

$$(g_V/g_A)^2_{e\mu,Z} = (g_V/g_A)^2_{e\mu,D} \tag{2.18}$$

The subscripts indicate the particles and the neutral current for which this ratio is evaluated. The relation appears to be true for all leptons, charged as well as neutral. Equation (2.18) is equivalent to,

$$(g_V/g_A)_{e\mu,Z} = (g_V/g_A)_{e\mu,D}, \tag{2.19}$$

$$(g_V/g_A)_{e\mu,Z} = -(g_V/g_A)_{e\mu,D}. \tag{2.20}$$

From equations (2.10)–(2.15), and from the definitions of the currents, we can easily evaluate the ratios indicated in the equation (2.19). This gives

$$(1-4x_L) = \frac{-(4\beta x_L - (\alpha+\beta)4x_R + \alpha)}{(\alpha+2\beta)} \tag{2.21}$$

Using equations (2.14) and (2.15) in the above, we readily notice that

$$x_L + x_R = 0.5 \tag{2.22}$$

While obtaining this important relation, equation (2.22), the masses of the electron and muon are not used. Only equation (2.19) is used. From a world average we have, $x_L = x_W = 0.23$. Therefore from equation (2.22),

$$x_R = 0.27 \tag{2.23}$$

Equation (2.7) also gives the same values for the two mixing parameters. These are obtained from the masses of the charged leptons, electron and the muon. The two mixing parameters are determined by the masses of the charged leptons. The basic gauge group is also required to be a left-right gauge group because there are two mixing parameters. In other words, the two charged leptons, electron and muon fix the mixing parameters and the basic gauge group. From equation (2.20) we also note that

$$x_R = \frac{2}{3}\frac{(1-x_L)x_L}{(1-2x_L)} \tag{2.24}$$

If we use $x_L = x_W = 0.23$ in this, we find the above parameter is 0.219. As it does not tally with the second value of the equation (2.7), we ignore equation (2.24) for the time being. Nature has assigned an important role to the two charged leptons, the electron and muon. Their masses fix the mixing parameters and thereby we can infer the basic gauge group for their electroweak model. These leptons obey a left-right gauge model whose mixing parameters are given by equation (2.7). Our analysis here also shows why it is necessary that the muon should exist. If this is the case with the electron and muon what role does the charged Tau-lepton play? Can we express the mass of the charged Tau-lepton by means of expressions similar to the equations (2.1) and (2.2)? We will address these problems and the consequences in the next section.

3. The Solution and Conclusions

To apply the previous results to the case of the charged Tau-lepton, we should remember that there are only three generations of leptons. The masses of the two charged leptons, electron and muon together satisfy equation (2.5). It can be easily seen that a similar relation holds good for the electron neutrino and muon neutrino masses. In our scenario these neutrino masses are equal and the ratio of vector to axial vector coupling constants of these neutrinos is unity, equation (2.3). If such a relation equation (2.5) is true for the electron and muon and their neutrinos why should it not be true for the Tau-lepton as well. For the electron there is a partner, the muon or vice-versa. For the Tau-lepton there is no such partner. It must be its own partner and hence

$$\frac{2m_{Tau}m_{Tau}}{m_{Tau}^2 + m_{Tau}^2} = (-1 + 4\sin^2\vartheta)^2 \tag{3.1}$$

In the above expression in place of the electron and the muon masses of equation (2.5) we used only the mass of the Tau-lepton, because there are only three generations of leptons. That means the LHS of equation (3.1) is unity. For this reason only we took the mixing parameter on the RHS of equation (3.1) as an unknown. It is something like this - The electron has a charged partner, the muon, whereas the charged Tau lepton has no charged partner. Equation (3.1) yields two roots for the mixing parameter:

$$\sin^2\vartheta = 0.5 \text{ and } 0. \tag{3.2}$$

The sum of the mixing parameters is again equal to 0.5. The meaning is very clear. The Tau lepton and its neutrino obey an electroweak model based on the gauge group $SU(2)_L XU(1)$ as there is only one mixing parameter 0.5. The other mixing parameter is zero. The mass of the charged Tau lepton is given by,

$$m_{Tau}^2 = mM_W' \frac{\left({}^{g_V}\!/_{g_A}\right)_{v_\tau}^4}{\left({}^{g_V}\!/_{g_A}\right)_{Tau}^4} \left\{1 \pm \left(1 - \left(\frac{g_V}{g_A}\right)_{Tau}^4\right)^{1/2}\right\}. \tag{3.3}$$

In view of equation (3.1) the factor within the brackets after the plus or minus sign is zero. Also the ratio of vector to axial vector coupling constants in the denominator of equation (3.3) is unity because of equation (3.1). But the resemblance of equation (3.3) to equations (2.1) and (2.2) should be noticed. For the Tau-neutrino,

$$(g_V/g_A)_{v_\tau}^2 = 1. \tag{3.4}$$

From equation (3.3), for the reasons mentioned above, the mass of the charged Tau-lepton is given by

$$m_{Tau}^2 = mM_W'. \tag{3.5}$$

In the above, m is the Dirac mass of the Tau-neutrino and M_W' is the mass of the W-boson of this gauge model. It is different from the W-boson of the Standard model. From reference [2], the masses are given by,

$$m_{Tau} = 1.777GeV \text{ and } m = 18.2MeV. \tag{3.6}$$

These are the measured masses of the charged Tau-lepton and its neutrino. The charged Tau-lepton and its neutrino obey an electro-weak gauge model based on the gauge group $SU(2)_L XU(1)$ with the mixing parameter 0.5. This mixing parameter is obtained from the mass of the charged Tau-lepton. From equations (3.5) and (3.6) the mass of the charged W-boson of this gauge model is given by

$$M_W' = 173.5GeV. \tag{3.7}$$

On the other hand, the mass of the neutral Z-boson of this gauge model is given by

$$M_Z' = M_W'/\cos\vartheta = 245.366GeV. \tag{3.8}$$

Before concluding, we wish to clarify our scheme of the neutrinos given in reference [1]. The electron neutrino is assumed to be a left handed neutrino whereas the muon neutrino is assumed to be a left handed anti neutrino. If CP invariance holds in the leptonic sector, the Majorana mass matrix will be real. If the main diagonal elements are arranged to be zero, then electron and the muon neutrinos will have equal Majorana mass, while their CP parities are opposite. When these two conditions (equal majorana mass, and opposite CP parities) are satisfied, the Majorana mass term is reduced to a Dirac mass term. Then we introduce the field $\nu(x)$so that

$$\begin{aligned} v_L &= v_{eL}, \\ v_R &= (v_{\mu L})^C \end{aligned} \tag{3.9}$$

Here $\nu(x)$is a four component Dirac field. In other words the electron neutrino and the muon neutrino are together equivalent to a single Dirac neutrino of mass m_1. The situation in the case of the Tau-neutrino is different. The Dirac mass of this neutrino is m, given by equation (3.6) to be 18.2 MeV. That means it should have a left-handed component and a right-handed component to generate Dirac mass. The electron-neutrino has a counter partner the muon-neutrino and vice-versa in the sense described above leading to equation (3.9). But in the case of the Tau-neutrino there is no such counter-partner to enable us to define a right-handed component as in equation (3.9). The Tau-lepton and its left-handed neutrino participate in the gauge model with the gauge group $SU(2)_L XU(1)$ whereas the right-handed Tau-neutrino takes part only in the Lagrangian that generates its mass. In this sense the right-handed Tau-neutrino is sterile.

In conclusion the following observations may be noted. (a) The electroweak physics of the electron and the muon is based on a left -right gauge model with the mixing parameters given equation (2.7). (b) These mixing parameters are determined by the masses of the electron and the muon. (c) The electron and muon-neutrinos are strictly left-handed such that together these are equivalent to a single Dirac neutrino of mass of about 7eV [1]. (d) The charged Tau lepton and its left- handed neutrino participate in an electro-weak gauge model based on the gauge group $SU(2)_L XU(1)$ with a mixing parameter 0.5. (e)This mixing parameter is determined by the mass of the charged tau-lepton. (f) The electro-weak physics of the Tau lepton and its neutrino is mediated by the W and Z bosons whose masses are given by equations (3.7) and (3.8) in addition to the photon. (g) The right handed component of the Tau-neutrino remains sterile.

References

[1] Cvavb Chandra Raju, *Int. Jourl. Theort. Phys.,* ***36***, (1997), 2937.

[2] Review of particle Phys-1, *Phys.Rev.(D),* **66**, (2002), 106; *Phys. Rev. (D),* **66,** (2002), 25.

[3] Cvavb Chandra Raju, *Pramana,* **24 L**, (1985), 657.

[4] Cvavb Chandra Raju, *Int. Jourl. Theort. Phys,* **25**, (1986), 1105.