

EMC Effect Using Thermodynamical Bag Model

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Abstract

The nuclear EMC effect has been evaluated in the entire range of Bjorken variable x for the value of $Q^2 = 5 \text{ GeV}^2$ using the phenomenological model known as the Thermodynamical bag model (TBM) and the results have been compared with the experimental values obtained in the deep inelastic scattering by SLAC and BCDMS. Also the values of structure function ratio are obtained as a function of Nachtmann variable ξ , which is used to represent the quark momentum in the Bjorken limit.

Key Words: EMC effect and Nuclear structure function.

1. Introduction

The distribution of quarks in a nucleus differs significantly from the distribution of quarks in the nucleon. This became clear when two decades ago the EMC measured the ratio of structure functions of Iron to Deuterium in the Deep Inelastic Scattering (DIS) of muons [1]. This discovery gave rise to a variety of theoretical models on both the nucleon and nuclear nucleon structure functions. Initially, in general the nuclear dependence is accounted in one of the following ways: conventional nuclear physics employing nucleon's degrees of freedom, quark clusters [2] and rescaling model [3]. Detailed reviews of data and models of the EMC effect are also available [4]. While many of the models have had some success, they typically reproduce only part of the observed enhancement or suppression, explain a limited range or differ from other measurements. In the present work, the EMC effect is explained by statistical distribution of quarks using TBM [5, 6].

2. Thermodynamical Bag Model (TBM)

TBM is a modified form of MIT Bag model where the quarks and gluons are treated as fermions and bosons, respectively. Considering the thermodynamical bag to be an unpolarized proton or neutron and restricting the consideration to u and d valence quarks, the four equations of state [5–7] are expressed as:

$$\epsilon(T)V + BV = W = [M^2 + 2M\nu - Q^2]^{1/2} \quad (1)$$

$$6(n_u - n_{\bar{u}}) = \mu_u T^2 + \mu_u^3 / \pi^2 \quad (2)$$

$$6(n_d - n_{\bar{d}}) = \mu_d T^2 + \mu_d^3 / \pi^2 \quad (3)$$

$$P = \frac{\epsilon(T)}{3} - B = 0 \quad (4)$$

where $\in(T)$ is the energy density of the excited system at temperature T , V the volume of the bag, B the bag constant, W the invariant mass of the final excited state of the nucleon at some finite momentum transfer, M the mass of the nucleon and ν the energy transfer. Here μ_u and μ_d are the chemical potentials of u and d quarks, and P the pressure necessary for confining the bag. Also $(n_u - n_{\bar{u}})$ and $(n_d - n_{\bar{d}})$ represent u and d valence quarks respectively. This model suggests an intimate relation between x , T and Q^2 [7, 8]. The smaller the value of x , the greater is the temperature, thus explaining the copious production of sea quarks at smaller values of x .

Solving the above equations of state of the bag self-consistently, with volume and temperature for fixed $Q^2 = 5 \text{ GeV}^2$, corresponding chemical potentials μ_u and μ_d are obtained. The energy density of the system as given by equation (1) at a temperature T is expressed as the sum of energy densities of quarks, antiquarks and gluons as:

$$\in(T) = 6(\in_u + \in_{\bar{u}}) + 6(\in_d + \in_{\bar{d}}) + 16\in_g \quad (5)$$

where

$$\in_u + \in_{\bar{u}} = \left(\frac{1}{8\pi^2}\right)\mu_u^4 + \left(\frac{1}{4}\right)\mu_u^2 T^2 + \left(\frac{7\pi^2}{120}\right)T^4 \quad (6)$$

$$\in_d + \in_{\bar{d}} = \left(\frac{1}{8\pi^2}\right)\mu_d^4 + \left(\frac{1}{4}\right)\mu_d^2 T^2 + \left(\frac{7\pi^2}{120}\right)T^4 \quad (7)$$

$$\in_g = \frac{\pi^2 T^4}{30} \quad (8)$$

Starting from the Fermi and Bose distribution functions and transforming them to the Infinite Momentum Frame, the u and d quark distribution functions [5] are obtained as:

$$u(x) = \left(\frac{6V}{4\pi^2}\right)M^2 T x \ln\left[1 + \exp\left\{\left(\frac{1}{T}\right)\left(\mu_u - \frac{Mx}{2}\right)\right\}\right] \quad (9)$$

$$d(x) = \left(\frac{6V}{4\pi^2}\right)M^2 T x \ln\left[1 + \exp\left\{\left(\frac{1}{T}\right)\left(\mu_d - \frac{Mx}{2}\right)\right\}\right] \quad (10)$$

The anti-quark distribution can be obtained by putting $\mu \rightarrow -\mu$ and hence we get the following two relations:

$$\bar{u}(x) = \left(\frac{6V}{4\pi^2}\right)M^2 T x \ln\left[1 + \exp\left\{\left(\frac{1}{T}\right)\left(-\mu_u - \frac{Mx}{2}\right)\right\}\right], \quad (11)$$

$$\bar{d}(x) = \left(\frac{6V}{4\pi^2}\right)M^2 T x \ln\left[1 + \exp\left\{\left(\frac{1}{T}\right)\left(-\mu_d - \frac{Mx}{2}\right)\right\}\right]. \quad (12)$$

In the parton model, the distribution function of the quark momenta is denoted by $q_i(x)$ and it is the expectation value of the number of quarks of type i in the hadron whose momentum fraction lies within the interval $[x, x + dx]$. The momentum distribution of antiquarks is denoted by $\bar{q}_i(x)$. Instead of parameterized structure function F_2 , we are using the sum of the momentum distributions weighted by x and z_i^2 and hence we get the relation

$$F_2(x) = x \sum_i z_i^2 [q_i(x) + \bar{q}_i(x)] \quad (13)$$

The structure function of the deuteron F_2^D is assumed as to be the sum of the proton and neutron and is given by

$$F_2^D \approx F_2^p + F_2^n \quad (14)$$

The TBM has an inbuilt mechanism by which the sea quarks and gluons are produced at small values of x .

2.1. Theoretical evaluation of EMC

Within the conventional picture, the nucleus as a collection of bound nucleons, it is quite natural to take into account the effects of Fermi motion and nuclear binding. The importance of the nuclear binding to describe the whole EMC effect was first pointed out by Akulinichev et al [9]. In the convolution model, the calculation of the nuclear structure function $F_2^A(x)$ which describes the influence of the nucleon binding energy and Fermi motion, is provided by

$$F_2^A(x) = \int f^A(z)F_2^N(x/z)dz \quad (15)$$

where $f^A(z)$ describes the momentum and energy distribution of nucleons and F_2^N is for single nucleon structure function. For the description of $F_2^N(x/z)$ in terms of quark degrees of freedom, the distance scale x is modified to a new scale variable x/z , which increases with mass number A . In order to calculate the effect of nuclear binding on the structure function, the momentum spectrum of the target nucleons has to be evaluated. In the simple Fermi gas model, the momentum distribution ($3/4\pi k_f$) is constant up to the maximum Fermi momentum k_f and is zero above k_f . With this distribution one gets

$$f^A(z) = (3/4)(M/k_f)^3[(k_f/M)^2 - (z - \eta)^2] \quad \text{for } -k_f < z < +k_f \quad (16)$$

and $f^A(z) = 0$ otherwise. As $f^A(z)$ is maximum at $z = x/\eta$, the new scaling variable ($x = x/z$) is used and this leads to the observed depletion at medium x in the EMC effect. Using (14), (15) and (16) the EMC ratio of iron to deuterium is obtained for $Q^2 = 5 \text{ GeV}^2$ as

$$R_{EMC} = \frac{\int f^A(z)F_2^N(x/z)dz}{F_2^D(x)}. \quad (17)$$

3. Results and Discussion

The theoretical evaluation of EMC effect using TBM is plotted in Figure 1 along with SLAC and BCDMS data. The experimental data of available [10] averaged Q^2 values, are compared with values of TBM at $Q^2 = 5 \text{ GeV}^2$. From the analysis an agreement with experimental data is observed. A slight increase at low value of 'x' may be attributed to the sea quark and gluon contribution. Also Table 1 gives the ratio of the structure function for iron to deuterium as measured by the BCDMS collaboration [10] using a 200 GeV muon beam along with TBM values for comparison. The areas enclosed by the curves of $u(x)$ and $d(x)$ remain a constant and are equal to the number of quarks of that flavor and are shown in Figure 2. This observation leads to the formulation of TBM to obtain the quark distribution functions of the correct asymptotic behavior. The value of $\int_{-k_f}^{+k_f} f^A(z)dz = 1$ is also verified and it proves the conservation of momentum. Unlike other models, TBM has an inbuilt function for sea quarks even at small x , especially at $x < 0.1$, and shows the dominance of sea quarks and gluons [5]. At medium range ($x > 0.3$) the EMC effect is observed and at higher values ($x > 0.7$) the steady increase may be attributed to Fermi motion. The difference between x and ξ is often ignored in high energy scattering or at low x , but cannot be ignored at large x or low Q^2 [11] to obtain the conservation of valence quark numbers. The EMC ratio of Iron to Deuterium calculated using Nachtmann variable $\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2})$ for $Q^2 = 4 \text{ GeV}^2$ is compared with the extracted data [12] from JLab ($1.2 < W^2 < 3 \text{ GeV}^2$, $Q^2 \approx 4 \text{ GeV}^2$) and is given in Table 2. Figure 3 shows ratio of structure functions using TBM as a function of Nachtmann variable. From the observation, the data are in close agreement with previous measurements of the EMC effect. We conclude that the TBM is a very successful model in explaining the quark momentum distributions which is used for extracting the nuclear effects.

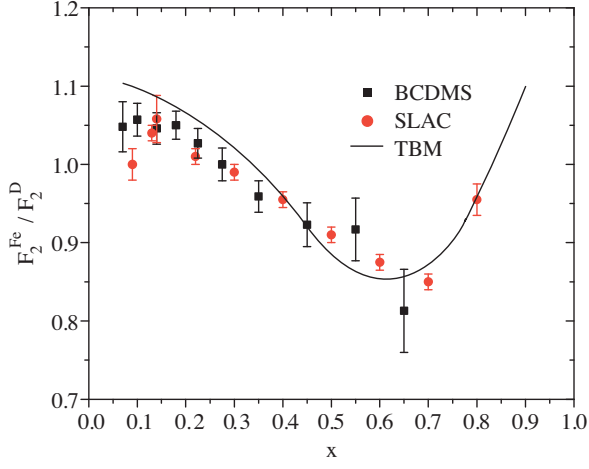


Figure 1. Structure function ratio of Iron to Deuterium as a function of x from BCDMS, SLAC and TBM. The scale uncertainties are also shown in the figure.

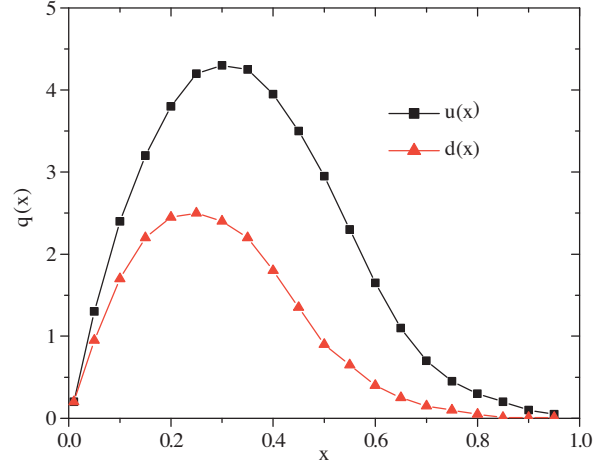


Figure 2. The quark distribution functions $u(x)$ and $d(x)$ as a function of Bjorken variable x .

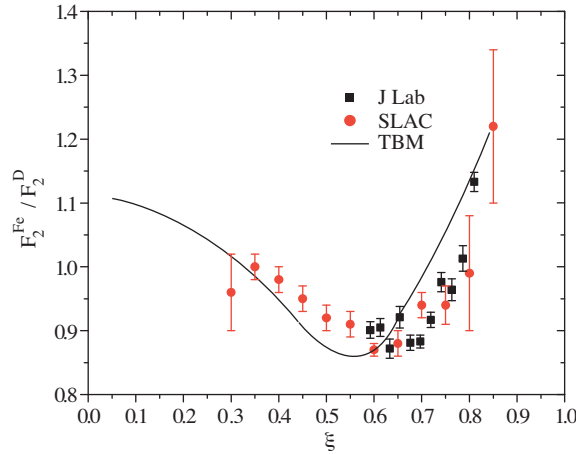


Figure 3. EMC ratio of Iron to Deuterium as a function of Nachtmann variable ξ using TBM compared with JLab and SLAC E87 experimental data. The scale uncertainties are also shown in the figure.

Table 1. The ratio of structure function of Iron to Deuterium as measured by the BCDMS collaboration using a 200GeV muon beam and computed values of TBM.

x	Q^2 GeV ²	BCDMS ref[10]	TBM
0.100	22	1.057 ± 0.021	1.095
0.140	25	1.046 ± 0.020	1.085
0.180	29	1.050 ± 0.018	1.073
0.225	46	1.027 ± 0.019	1.057
0.275	49	1.000 ± 0.021	1.035
0.350	59	0.959 ± 0.020	0.995
0.450	72	0.923 ± 0.028	0.934
0.550	72	0.917 ± 0.040	0.881
0.650	72	0.813 ± 0.053	0.872

Table 2. EMC ratio extracted from the data [12] compared with calculated values of TBM using Nachtmann variable ξ .

ξ	W^2 GeV ²	$(F_2^{Fe}/F_2^D)_{IS}$	$(F_2^{Fe}/F_2^D)_{TBM}$
0.592	2.86	0.901 ± 0.013	0.878
0.613	2.70	0.905 ± 0.014	0.890
0.633	2.55	0.872 ± 0.015	0.913
0.654	2.39	0.921 ± 0.017	0.939
0.676	2.22	0.881 ± 0.012	0.958
0.697	2.07	0.883 ± 0.010	0.987
0.719	1.91	0.917 ± 0.012	1.016
0.741	1.75	0.976 ± 0.015	1.026
0.763	1.59	0.964 ± 0.017	1.057
0.786	1.43	1.013 ± 0.020	1.088
0.810	1.26	1.133 ± 0.015	1.152

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