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Five Dimensional Stiff Fluid Models in Lyra Geometry

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Abstract

We have constructed stiff fluid cosmological models in five dimensional space-time based on Lyra geometry. Some physical and geometrical properties of the models are discussed.

Key Words: Five dimensional cosmological model, stiff fluid, Lyra geometry.

1. Introduction

As the evolving early universe was much smaller than today, the present four-dimensional space-time of the universe could have been preceded by a higher dimensional space-time. Thus space-time of the early universe is commonly modeled as having more than four dimensions. Due to spontaneous compactification, the extra dimensions contract to the unobserved Planckian length scale, or remain constant [1–3]. The five dimensional space–time is particularly attractive because both 10D and 11D super gravity theories admit solutions which spontaneously reduced to 5D [4].

Einstein's idea of geometrizing gravitation in general theory of relativity motivated others to geometrize other physical fields. Weyl [5] proposed a modification of Riemannian manifold in order to unify gravitation and electromagnetism. But due to the non-integrability of length transfer this theory was never considered seriously. Later, Lyra [6] proposed a modification of Riemannian geometry in which he introduced a gauge function to remove the non-integrability of length of a vector under parallel transport. In this theory both the scalar and tensor fields have intrinsic geometrical significance. Halford [7] pointed out that the energy conservation law does not hold in the cosmological theory based on Lyra geometry. Various five dimensional cosmological models in Lyra manifold are constructed by Rahaman et al. [8–10] and Singh et al. [11], Mohanty et al. [12–14].

In this paper we have considered a five dimensional spherically symmetric space-time with stiff fluid distribution in Lyra geometry. Exact solutions of the field equations are obtained for two cases, viz. constant displacement vector and time dependent displacement vector. Some physical and geometrical properties of the models are also discussed.

2. Field Equations

Here we consider the five dimensional spherically symmetric space-time in the form

$$ds^2 = dt^2 - e^{\lambda}\left(dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right) - e^{\mu}\, dy^2, \tag{1}$$

where λ and μ are functions of cosmic time t only.

The field equations in normal gauge for Lyra's manifold as proposed by Sen [15] and Sen and Dunn [16] are given by

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \phi_i \phi_j - \frac{3}{4} g_{ij} \phi_k \phi^k = -\chi T_{ij}, \tag{2}$$

where ϕ_k is the displacement vector and other symbols have their usual meanings as in the Riemannian geometry. The displacement vector ϕ_k is defined as

$$\phi_k = (\beta, 0, 0, 0, 0)\,. \tag{3}$$

The energy momentum tensor is taken as

$$T_{ij} = (p + \rho)\, u_i\, u_j - p\, g_{ij}, \tag{4}$$

together with the commoving co-ordinates

$$g_{ij}\, u^i\, u^j = 1. \tag{5}$$

Here, p, ρ and u_i are isotropic pressure, energy density and five velocity vector of the cosmic fluid distribution respectively.

The field equations (2) together with (3), (4) and (5) for the space-time metric (1) yield the following equations:

$$\frac{3}{4}\left(\dot{\lambda}^2 + \dot{\lambda}\dot{\mu}\right) - \frac{3}{4}\beta^2 = \chi\, \rho \tag{6}$$

$$\ddot{\lambda} + \frac{3}{4}\dot{\lambda}^2 + \frac{1}{2}\ddot{\mu} + \frac{1}{4}\dot{\mu}^2 + \frac{1}{2}\dot{\lambda}\dot{\mu} + \frac{3}{4}\beta^2 = -\chi\, p \tag{7}$$

and

$$\frac{3}{2}\left(\ddot{\lambda} + \dot{\lambda}^2\right) + \frac{3}{4}\beta^2 = -\chi\, p, \tag{8}$$

where overhead dot denotes differentiation with respect to t.

3. Cosmological Solutions

Mohanty et al. [12] showed that the general perfect fluid does not survive in Lyra manifold in this space-time. Therefore in this section we intend to derive the exact solutions of the field equations (6)–(8) for stiff fluid distribution, i.e.

$$p = \rho. \tag{9}$$

Here, there are four unknowns λ, μ, β *and* p involved in three field equations (6)–(8).

In order to derive explicit solutions we consider the following cases.

Case I: $\beta =$ *Constant*

In this case there are the unknowns λ, μ and p involved in field equations (6)–(8).

Solving equations (6) and (8), we obtain

$$\frac{\ddot{\lambda}}{\dot{\lambda}} + \frac{3}{2}\dot{\lambda} = -\frac{\dot{\mu}}{2} = k_1, \tag{10}$$

where k_1 is an arbitrary constant.

Now equation (10) yields

$$\mu = -(2\, k_1\, t + k_2) \tag{11}$$

and

$$\lambda = \ln(a\, e^{k_1\, t} + b)^{2/3}, a > 0, \tag{12}$$

where k_2 and b are constants of integration.

Substituting equations (11) and (12) in equation (7) we get

$$p = \frac{a\,k_1\,e^{k_1 t}}{3\chi\left(a\,e^{k_1 t} + b\right)^2}\left[2b\,(k_1 + 1) + a\,e^{k_1 t}(k_1 + 2)\right] + C, \tag{13}$$

where $C = \frac{k_2^2 - 2k_2 + 3\beta^2}{4}$. In this case, the metric (1) takes the form

$$ds^2 = dt^2 - \left(a\,e^{k_1 t} + b\right)^{2/3}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) - e^{-(2k_1 t + k_2)}\,dy^2. \tag{14}$$

Case II: $\beta = \beta(t)$

In this case, there are four unknowns λ, μ, p and β involved in the three field equations (6)–(8). One more equation relating these parameters is required to obtain explicit exact solutions of the field equations. Therefore we consider an analogue of power law, i.e.

$$\mu = a\lambda, \tag{15}$$

where $a$is an arbitrary constant.

Substituting this equation in field equations (6) and (7), we obtain

$$\frac{3}{4}\dot{\lambda}^2(a+1) - \frac{3}{4}\beta^2 = \chi\rho \tag{16}$$

and

$$\left(\frac{a}{2} + 1\right)\ddot{\lambda} + \frac{(a^2 + 2a + 3)}{4}\dot{\lambda}^2 + \frac{3}{4}\beta^2 = -\chi\rho. \tag{17}$$

Further, substituting the value of β^2 from equation (8) in equations (16) and (17), we get

$$\ddot{\lambda} + \frac{(a+3)}{2}\dot{\lambda}^2 = 0 \tag{18}$$

and

$$\frac{(a-1)}{2}\left[\ddot{\lambda} + \frac{(a+3)}{2}\dot{\lambda}^2\right] = 0. \quad . \tag{19}$$

For $a \neq 1$, equations (18) and (19) are identical, which on integration yields

$$\lambda = \ln\left(a_2 t + b_2\right)^{\frac{2}{a+3}},\ a \neq -3. \tag{20}$$

From equation (15), we obtain

$$\mu = \ln\left(a_2 t + b_2\right)^{\frac{2a}{a+3}}. \tag{21}$$

Substituting the value of λin equation (16), we get

$$\chi\rho = \frac{3a_2^2(a+1)}{(a+3)^2(a_2 t + b_2)^2} - \frac{3}{4}\beta^2. \tag{22}$$

It is impossible to obtain separate values of ρ and β. Therefore in view of the physical behavior of the gauge function β, we consider

$$\beta = \frac{1}{a_3 t + b_3}. \tag{23}$$

With the help of equation (23) equation (22) yields

$$\chi\rho = \frac{3a_2^2(a+1)}{(a+3)^2(a_2 t + b_2)^2} - \frac{3}{4(a_3 t + b_3)^2}. \tag{24}$$

In this case, the line element (1) can be written as

$$ds^2 = dt^2 - (a_2t + b_2)^{\frac{2}{a+3}} \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) - (a_2t + b_2)^{\frac{2a}{a+3}} dy^2. \tag{25}$$

Case II(A): A modification

For $a = -3$, equation (18) yields

$$\lambda = a_4t + b_4. \tag{26}$$

From equation (15) we get

$$\mu = -3(a_4t + b_4). \tag{27}$$

Substituting the value of λ in equation (16), we obtain

$$\chi\rho = -\frac{3}{2}a_4^2 - \frac{3}{4}\beta^2. \tag{28}$$

Due to paucity of independent equations it is impossible to solve for explicit values of ρ and β from one equation. Therefore in view of the physical nature of $\beta(t)$ we assume

$$\beta = e^{-t}. \tag{29}$$

With the help of equation (29), equation (28) becomes

$$\chi\rho = -\frac{3}{2}a_4^2 - \frac{3}{4}e^{-2t}. \tag{30}$$

In this case the metric (1) takes the form

$$ds^2 = dt^2 - e^{a_4t+b_4} \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) - e^{-3(a_4t+b_4)} dy^2. \tag{31}$$

4. Discussion

In the preceding section we derived the exact solutions of the field equations for two cases viz. $\beta =$ Constant and $\beta = \beta(t)$. In case I, equation (14) shows that at initial epoch $t = 0$ the metric becomes flat. As time increases the three space coordinates expands while the fifth coordinate, i.e. the extra dimension contracts. At infinite time the extra dimension becomes unobservable. The scalar of expansion θ and shear scalar σ^2 in this case are obtained as

$$\theta = \frac{-bk_1}{ae^{k_1t} + b}, a > 0, k_1 > 0, b < 0 \tag{32}$$

and

$$\sigma^2 = \frac{a^2k_1^2e^{2k_1t}}{6(ae^{k_1t} + b)^2} - \frac{ak_1e^{k_1t}}{3(ae^{k_1t} + b)} + \frac{k_1^2}{2} + \frac{k_1}{3} + \frac{2}{9}. \tag{33}$$

Here the expansion scalar is finite at $t = 0$ and decreases as t increases. The expansion in the model stops at $t = \infty$. As $\lim\limits_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$, the model does not approach isotropy for large values of t.

In case II, equation (25) indicates that the line element is flat at the initial epoch $t = 0$. For $-3 < a < 0$ the three space coordinates expand with the passage of time while the extra one contracts. The scalar of expansion θ and shear scalar σ^2 in this case are

$$\theta = \frac{a_2}{a_2t + b_2} \tag{34}$$

and

$$\sigma^2 = \frac{2}{9} + \frac{(a^2 + 3)a_2^2}{2(a_2t + b_2)^2(a + 3)^2} - \frac{a_2}{3(a_2t + b_2)}. \tag{35}$$

This model does not approach isotropy for large values of t.

In sub case II(A), from equation (31) it is clear that the line element is flat at the initial epoch $t = 0$. With increase of time t the extra coordinate contracts while the other three space coordinates expand. At infinite time the extra dimension becomes very small and is unobservable. In this case the expansion scalar vanishes while the shear scalar becomes constant. This behavior is similar to that of cosmic string model obtained by Reddy [17] in four dimensional Lyra manifold.

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