# Study of second-class currents effects on polarization characteristics in quasi-elastic neutrino (anti-neutrino) scattering by nuclei 

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#### Abstract

The differential cross section of quasi-elastic neutrino scattering by nuclei is computed. The numerical analysis of energy dependence of the spin asymmetry, the angular electron-neutrino (e $\nu$ ) correlation and the charge asymmetry coefficients shows that the relative contribution of the second-class current (SCC) tensor form factor can reach some tens of percents in the case of particular values of final nuclei alignment and polarization.


Key Words: Second-class currents, nuclei polarization, neutrino scattering, spin asymmetry and e $e$ correlation.

## 1. Introduction

The most general expressions for the matrix elements of vector currents $V_{\mu}$ and axial - vector currents $A_{\mu}$ between nuclei states $p_{1}$ and $p_{2}$ are given by the formulae [1]

$$
\begin{align*}
\left\langle p_{1}\right| V_{\mu}\left|p_{2}\right\rangle & =\bar{u}\left(p_{1}\right)\left[F_{1} \gamma_{\mu}+F_{2} \sigma_{\mu \nu} q_{\nu}+i F_{S} q_{\mu}\right] u\left(p_{2}\right)  \tag{1}\\
\left\langle p_{1}\right| A_{\mu}\left|p_{2}\right\rangle & =\bar{u}\left(p_{1}\right)\left[F_{A} \gamma_{\mu}+F_{T} \sigma_{\mu \nu} q_{\nu}+i F_{P} q_{\mu}\right] \gamma_{5} u\left(p_{2}\right),
\end{align*}
$$

where $q_{\mu}=\left(p_{1}-p_{2}\right)_{\mu}$ is the 4-momentum transfer to the nucleus, $u\left(p_{1}\right), u\left(p_{2}\right)$ are Dirac spinor amplitudes, $F_{1}, F_{A}$ are vector and axial-vector form factors, $F_{2}, F_{S}, F_{T}$ and $F_{p}$ are respectively weak magnetic, effective scalar, induced and induced pseudo-scalar form factors. All these form factors depend on the square transfer momentum $q^{2}$ :

$$
\begin{aligned}
& F_{x}\left(q_{\mu}^{2}\right)=F_{x}(0) /\left(1+q_{\mu}^{2} /(855 M e V)^{2}\right)^{2}, \\
& F_{p}\left(q_{\mu}^{2}\right)=2 M F_{A}(0) /\left(q_{\mu}^{2}+m_{\pi}^{2}\right)
\end{aligned}
$$

where $x=1,2, A, T, S, m_{\pi}$ and $m_{\pi}$ is the pion mass, $F_{1}(0)=1.0, F_{A}(0)=-1.23, F_{1}(0)+2 M F_{2}(0)=\mu(0)=$ 4.706, $F_{T}(0)=5 \cdot 10^{-3} \mathrm{MeV}^{-1}$. We assume the conserved-vector-current hypothesis (CVC) in which case there are no induced scalar currents, and the form factor $F_{S}$ is zero.

According to S. Weinberg classification [2], based on the G parity transformation (interchanging particles with their anti-particles and rotating the system in isospin space around the $\mathrm{T}_{2}$ axis), vector and axial-vector currents can be decomposed into first-class currents (FCC) and second-class currents (SCC). $F_{1}, F_{2}, F_{A}$ and $F_{p}$ are the form factors of the first-class currents and $F_{S}$ and $F_{T}$ are the form factors of the second class current.

Issue of second-class currents and their existence are being widely discussed. Some contradictions between different experimental data do not allow a definite solution of this issue. For example, the experimental study of angular distribution of positrons in $\beta$ decay process of polarized ${ }^{19} N e$ nucleus [3] and the study of angular distribution of electron and positron in $\beta$ decay from polarized ${ }^{12} B$ and ${ }^{12} N[4,5]$, show the existence of SCC whose tensor form factor is of the same order of magnitude as the form factor of weak magnetic interaction. Yet, Morita's data [6] show that the value of tensor form factor is next to zero within the limit of experimental errors, in agreement with the hypothesis of SCC shutting.

An important development has arose in recent years, in which the experimental study of neutrino interactions with nuclei offers opportunity to investigate subtle detail in the structure of the standard model as well as that of the nucleon. Study of neutrino interaction can offer a solid understanding of cross sections in neutrino-induced reactions, particularly with nuclei such as ${ }^{12} \mathrm{C}$, a component of liquid scintillators, and ${ }^{16} \mathrm{O}$, the basic component of water Cerenkov detectors [7].

The present study deals with effects due to the possible existence of SCC in quasi-elastic neutrino (and anti-neutrino) scattering by nuclei and the influence of polarization on the final nuclei on these effects.

## 2. Differential cross section of quasi-elastic neutrino (anti-neutrino) scattering by nuclei

Second class-currents are difficult to detect without any assumption [8], so we try to extract SCC effects by calculating the differential cross section of neutrino scattering by nuclei.

In first order of perturbation theory, the process of neutrino (anti-neutrino) scattering by nuclei described here can be written as

$$
\begin{equation*}
\nu(\tilde{\nu})+(A, Z) \rightarrow(A, Z \pm 1)+\ell^{-}\left(\ell^{+}\right) \tag{2}
\end{equation*}
$$

with matrix elements expressed as

$$
\begin{equation*}
M_{f i}=-\frac{G_{F}}{\sqrt{2}} \ell_{\mu} J_{\mu} \tag{3}
\end{equation*}
$$

where $G_{F} \cong 10^{-5} M^{-2}$ is Fermi coupling constant for weak interaction; $M$ is the mass of the nucleus; $\ell_{\mu}=\bar{u}_{2} \gamma_{\mu}\left(1+\gamma_{5}\right) \bar{u}_{1}$ and $J_{\mu}=\langle f| \int d x \exp (-i q x) \hat{J}_{\mu}(x)|i\rangle$ are, respectively, leptonic and hadronic currents;
$u_{j}(j=1,2)$ are Dirac spinor amplitudes; $\hat{J}_{\mu}(x)$ is local current operator of nucleus; $q_{\mu}=\left(q, i q_{0}\right) \equiv\left(P_{\ell}-P_{\nu}\right)_{\mu}$ is the 4 -momentum transfer to the nucleus; $P_{\ell}$ is the 4 -momentum of electron for $\ell=e^{-}$(and positron for $\ell=e^{+}$) ; $P_{\nu}$ is the 4-momentum of neutrino (anti-neutrino). The initial (final) state of nucleus is determined by the parity $\pi_{i}\left(\pi_{f}\right)$, the spin $J_{i}\left(J_{f}\right)$, the isotopic spin $T_{i}\left(T_{f}\right)$ and also by their projections $M_{i}, M_{T_{i}}$ $\left(M_{f}, M_{T_{f}}\right)$.

Using multipole decomposition $[1,9,10]$ in a cyclical basis, and with respect to the spin orientation of the final nuclei, we can put the components of the hadronic current into the following forms:

$$
\begin{gather*}
J_{0}=\sum_{J \geq 0, M_{f}^{\prime}}(-i)^{J}(4 \pi(2 J+1))^{1 / 2}\left\langle J_{f} M_{f}^{\prime}\right| \hat{M}_{J 0}\left|J_{i} M_{i}\right\rangle D_{M_{f} M_{f}^{\prime}}^{J_{f} *}(\theta *, \varphi *), \\
J_{3}=\sum_{J \geq 0, M_{f}^{\prime}}(-i)^{J}(4 \pi(2 J+1))^{1 / 2}\left\langle J_{f} M_{f}^{\prime}\right| \hat{L}_{J 0}\left|J_{i} M_{i}\right\rangle D_{M_{f} M_{f}^{\prime}}^{J_{f} *}(\theta *, \varphi *),  \tag{4}\\
J_{\lambda}=-\sum_{J \geq 1, M_{f}^{\prime}}(-i)^{J}(2 \pi(2 J+1))^{1 / 2}\left\langle J_{f} M_{f}^{\prime}\right| \lambda \hat{J}_{J ;-\lambda}^{M}+\hat{J}_{J ;-\lambda}^{E}\left|J_{i} M_{i}\right\rangle D_{M_{f} M_{f}^{\prime}}^{J_{f} *}(\theta *, \varphi *), \lambda= \pm 1 .
\end{gather*}
$$

Here, $D_{M_{f} M_{f}^{\prime}}^{J_{f} *}(\theta *, \varphi *)$ is the Wigner D-matrix [11] and $\left|J_{i}-J_{f}\right| \leq J \leq J_{i}+J_{f}, \hat{M}_{J 0}, \hat{L}_{J 0}, \hat{J}_{J ;-\lambda}^{M}$ and $\hat{J}_{J ;-\lambda}^{E}$ are the coulomb, longitudinal, magnetic and electric multipole operators $[1,9] ; \theta *$ and $\varphi *$ are the angles used in determining the spin orientation of final nuclei [9].

The differential cross section of the process described in equation (2) is given by the following, by taking into account final nuclei spin orientation:

$$
\begin{align*}
& \frac{d \sigma}{d \Omega_{\ell}}=\frac{2 J_{f}+1}{2 J_{i}+1} \frac{E_{\ell} P_{\ell} G_{F}^{2}}{2 \pi}\left\{\sum_{\begin{array}{l}
L \geq 0 \\
\text { even }
\end{array}} f_{L}^{(f)} P_{L}(\cos \theta *)\left(v_{1} W_{1}^{L}+v_{2} W_{2}^{L}+v_{3} W_{3}^{L}+v_{4} W_{4}^{L}+v_{5} W_{5}^{L}\right)+\right. \\
& \sum_{L \geq 1} f_{L}^{(f)}\left[P_{L}(\cos \theta *)\left(v_{1} \bar{W}_{1}^{L}+v_{2} \bar{W}_{2}^{L}+v_{3} \bar{W}_{3}^{L}+v_{4} \bar{W}_{4}^{L}+v_{5} \bar{W}_{5}^{L}\right)+P_{L}^{1}(\cos \theta *) \cos \varphi *\left(v_{6} \bar{W}_{6}^{L}+\right.\right. \\
& o d d
\end{aligned} \begin{aligned}
& \left.\left.v_{7} \bar{W}_{7}^{L}+v_{8} \bar{W}_{8}^{L}+v_{9} \bar{W}_{9}^{L}\right)\right]+\sum_{\begin{array}{l}
L \geq 2 \\
\text { even }
\end{array}} f_{L}^{(f)}\left[P_{L}^{1}(\cos \theta *) \cos \varphi *\left(v_{6} W_{6}^{L}+v_{7} W_{7}^{L}+v_{8} W_{8}^{L}+v_{9} W_{9}^{L}\right)+\right. \\
& \left.P_{L}^{2}(\cos \theta *) \cos 2 \varphi * v_{10} W_{10}^{L}\right]+ \\
& \sum_{L \geq 3} f_{L}^{(f)} P_{L}^{2}(\cos \theta *) \cos 2 \varphi * v_{10} \bar{W}_{10}^{L}  \tag{5}\\
& \quad \text { odd }
\end{align*}
$$

Here, $0 \leq L \leq 2 J_{f}$ and $P_{L}^{m}(\cos \theta *)$ are Lagrange functions;

$$
f_{L}^{(f)}=\sum_{M_{f}}(-1)^{J_{f}-M_{f}}[L]\left(\begin{array}{ccc}
J_{f} & J_{f} & L \\
M_{f} & -M_{f} & 0
\end{array}\right) P\left(M_{f}\right)
$$

is a Fano tensor [9], where $[x]=\sqrt{2 x+1}$ and $P\left(M_{f}\right)$ is the population of magnetic substates; $v_{i}(i=$ $1,2, \cdots, 10)$ are leptonic functions and $W_{k}^{L}, \bar{W}_{k}^{L}(k=1,2, \cdots, 10)$ are hadronic functions.

Leptonic functions are defined by:

$$
\begin{array}{ll}
v_{1}=\frac{1}{2}\left(\ell_{1} \ell_{1}^{*}+\ell_{2} \ell_{2}^{*}\right) & v_{2}=-\frac{i}{2}\left(\ell_{1} \ell_{2}^{*}-\ell_{2} \ell_{1}^{*}\right) \\
v_{3}=\ell_{3} \ell_{3}^{*} & v_{4}=-2 \operatorname{Re}\left(\ell_{3} \ell_{0}^{*}\right) \\
v_{5}=\ell_{0} \ell_{0}^{*} & v_{6}=2 \operatorname{Re}\left(\ell_{1} \ell_{3}^{*}\right)  \tag{6}\\
v_{7}=-2 \operatorname{Im}\left(\ell_{2} \ell_{3}^{*}\right) & v_{8}=-2 \operatorname{Re}\left(\ell_{1} \ell_{0}^{*}\right) \\
v_{9}=2 \operatorname{Im}\left(\ell_{2} \ell_{0}^{*}\right) & v_{10}=-\frac{1}{2}\left(\ell_{1} \ell_{1}^{*}-\ell_{2} \ell_{2}^{*}\right)
\end{array}
$$

where $\ell_{i}(i=0,1,2,3)$ are the components of leptonic currents.
Hadronic functions are given by:

$$
\begin{align*}
& W_{1}^{L}=-\sum_{J^{\prime} J} A_{-1 ; 1}^{(L)}\left\{P_{J^{\prime}+J}^{+}\left(F_{E J} F_{E J^{\prime}}+F_{M J} F_{M J^{\prime}}+F_{E J}^{5} F_{E J^{\prime}}^{5}+F_{M J}^{5} F_{M J^{\prime}}^{5}\right)-\right. \\
& \left.-P_{J^{\prime}+J}^{-}\left(F_{M J} F_{E J^{\prime}}+F_{E J} F_{M J^{\prime}}+F_{E J}^{5} F_{M J^{\prime}}^{5}+F_{M J}^{5} F_{E J^{\prime}}^{5}\right)\right\}, \\
& W_{2}^{L}=-\sum_{J^{\prime} J} A_{-1 ; 1}^{(L)}\left\{P_{J^{\prime}+J}^{+}\left(F_{M J}^{5} F_{E J^{\prime}}+F_{E J} F_{M J^{\prime}}^{5}+F_{M J} F_{E J^{\prime}}^{5}+F_{E J}^{5} F_{M J^{\prime}}\right)-\right. \\
& \left.-P_{J^{\prime}+J}^{-}\left(F_{E J}^{5} F_{E J^{\prime}}+F_{M J}^{5} F_{M J^{\prime}}+F_{M J} F_{M J^{\prime}}^{5}+F_{E J} F_{E J^{\prime}}^{5}\right)\right\}, \\
& W_{3}^{L}=\sum_{J^{\prime} J} A_{0,0}^{(L)} P_{J^{\prime}+J}^{+}\left(F_{L J} F_{L J^{\prime}}+F_{L J}^{5} F_{L J^{\prime}}^{5}\right), \\
& W_{4}^{L}=\sum_{J^{\prime} J} A_{0,0}^{(L)} P_{J^{\prime}+J}^{+}\left(F_{L J} F_{C J^{\prime}}+F_{L J}^{5} F_{C J^{\prime}}^{5}\right), \\
& W_{5}^{L}=\sum_{J^{\prime} J} A_{0,0}^{(L)} P_{J^{\prime}+J}^{+}\left(F_{C J} F_{C J^{\prime}}+F_{C J}^{5} F_{C J^{\prime}}^{5}\right), \\
& W_{6}^{L}=\sum_{J^{\prime} J} A_{1 ; 0}^{(L)} P_{J^{\prime}+J}^{+}\left(F_{E J} F_{L J^{\prime}}+F_{E J}^{5} F_{L J^{\prime}}^{5}+F_{M J} F_{L J^{\prime}}^{5}+F_{M J}^{5} F_{L J^{\prime}}\right), \\
& W_{7}^{L}=\sum_{J^{\prime} J} A_{1 ; 0}^{(L)} P_{J^{\prime}+J}^{-}\left(F_{E J} F_{L J^{\prime}}^{5}+F_{E J}^{5} F_{L J^{\prime}}+F_{M J}^{5} F_{L J^{\prime}}^{5}+F_{M J} F_{L J^{\prime}}\right), \\
& W_{8}^{L}=\sum_{J^{\prime} J} A_{1 ; 0}^{(L)} P_{J^{\prime}+J}^{+}\left(F_{E J} F_{C J^{\prime}}+F_{E J}^{5} F_{C J^{\prime}}^{5}+F_{M J} F_{C J^{\prime}}^{5}+F_{M J}^{5} F_{C J^{\prime}}\right), \tag{7}
\end{align*}
$$

$$
\begin{gathered}
W_{9}^{L}=\sum_{J^{\prime} J} A_{1 ; 0}^{(L)} P_{J^{\prime}+J}^{-}\left(F_{E J} F_{C J^{\prime}}^{5}+F_{E J}^{5} F_{C J^{\prime}}+F_{M J}^{5} F_{C J^{\prime}}^{5}+F_{M J} F_{C J^{\prime}}\right), \\
W_{10}^{L}=-\sum_{J^{\prime} J} A_{1 ; 1}^{(L)}\left\{P_{J^{\prime}+J}^{+}\left(F_{E J} F_{E J^{\prime}}-F_{M J} F_{M J^{\prime}}+F_{E J}^{5} F_{E J^{\prime}}^{5}-F_{M J}^{5} F_{M J^{\prime}}^{5}\right)+\right. \\
\left.+P_{J^{\prime}+J}^{-}\left(F_{M J} F_{E J^{\prime}}-F_{E J}^{5} F_{M J^{\prime}}+F_{M J}^{5} F_{E J^{\prime}}^{5}-F_{E J} F_{M J^{\prime}}\right)\right\}, \\
\bar{W}_{3}^{L}=\sum_{J^{\prime} J} A_{0,0}^{(L)} P_{J^{\prime}+J}^{-}\left(F_{L J} F_{L J^{\prime}}^{5}+F_{L J J}^{5} F_{L J^{\prime}}\right), \\
\bar{W}_{4}^{L}=\sum_{J^{\prime} J} A_{0 ; 0}^{(L)} P_{J^{\prime}+J}^{-}\left(F_{L J} F_{C J^{\prime}}^{5}+F_{L J}^{5} F_{C J^{\prime}}\right), \\
\bar{W}_{5}^{L}=\sum_{J^{\prime} J} A_{0 ; 0}^{(L)} P_{J^{\prime}+J}^{-}\left(F_{C J} F_{C J^{\prime}}^{5}+F_{C J}^{5} F_{C J^{\prime}}\right), \\
\bar{W}_{10}^{L}=-\sum_{J^{\prime} J} A_{1 ; 1}^{(L)}\left\{P_{J^{\prime}+J}^{+}\left(F_{M J}^{5} F_{E J^{\prime}}+F_{M J} F_{E J^{\prime}}^{5}-F_{E J}^{5} F_{M J^{\prime}}-F_{E J} F_{M J^{\prime}}^{5}\right)+\right. \\
\left.+P_{J^{\prime}+J}^{-}\left(F_{E J}^{5} F_{E J^{\prime}}+F_{E J} F_{E J^{\prime}}^{5}-F_{M J J} F_{M J^{\prime}}^{5}-F_{M J}^{5} F_{M J^{\prime}}\right)\right\}, \\
\bar{W}_{1}^{L}=-W_{2}^{L}, \bar{W}_{2}^{L}=-W_{1}^{L}, \bar{W}_{6}^{L}=W_{7}^{L}, \bar{W}_{7}^{L}=W_{6}^{L}, \\
\bar{W}_{8}^{L}=W_{9}^{L}, \bar{W}_{9}^{L}=W_{8}^{L}
\end{gathered}
$$

Here, $F_{C J}, F_{L J}, F_{M J}$ and $F_{E J}\left(F_{C J}^{5}, F_{L J}^{5}, F_{M J}^{5}\right.$ and $\left.F_{E J}^{5}\right)$ are matrix elements of the vector (axial-vector), coulomb, longitudinal, magnetic and electric multipole operators.

The coefficients $A_{m^{\prime} m}^{(L)}$ and $P_{J^{\prime}+J}^{ \pm}$are defined by:

$$
\begin{aligned}
& A_{m m^{\prime}}^{(L)}=(-1)^{J_{i}+J_{f}}[J]\left[J^{\prime}\right][L]\left(\frac{(L-|M|)!}{(L+|M|)!}\right)^{1 / 2}\left(\begin{array}{ccc}
J & J^{\prime} & L \\
m & m^{\prime} & M
\end{array}\right)\left\{\begin{array}{ccc}
J & J^{\prime} & L \\
J_{f} & J_{f} & J_{i}
\end{array}\right\}, \\
& P_{J^{\prime}+J}^{+}=\frac{1}{2}(-1)^{\frac{1}{2}\left(J^{\prime}-J\right)}\left(1+(-1)^{\frac{1}{2}\left(J^{\prime}+J\right)}\right), P_{J^{\prime}+J}^{-}=\frac{1}{2}(-1)^{\frac{1}{2}\left(J^{\prime}-J+1\right)}\left(1-(-1)^{\frac{1}{2}\left(J^{\prime}+J\right)}\right) .
\end{aligned}
$$

The cross section of the process of neutrino (anti-neutrino) scattering by nuclei (2), taking into account initial nuclei spin orientation, is obtained using formulas (5), (6) and (7) in which $W_{k}^{L}$ and $f_{L}^{(f)}$ are replaced by $W_{k(i)}^{L}$ and $f_{L}^{(i)}$ defined as

$$
\begin{gathered}
W_{k(i)}^{L}=\frac{2 J_{i}+1}{2 J_{f}+1}(-1)^{J^{\prime}+J+L} W_{k}^{L} \\
f_{L}^{(i)}=\sum_{M_{f}}(-1)^{J_{i}-M_{i}}[L]\left(\begin{array}{ccc}
J_{i} & J_{i} & L \\
M_{i} & -M_{i} & 0
\end{array}\right) P\left(M_{i}\right) .
\end{gathered}
$$

After summing over the spin states of electron (positron) and, in the case of zero mass neutrino, the leptonic functions are expressed as

$$
v_{1}=1-\beta_{\ell} C_{3}, \quad v_{2}=\eta\left(C_{1}-\beta_{\ell} C_{2}\right), \quad v_{3}=1+2 \beta_{\ell} C_{3}-\cos \theta, \quad v_{4}=-2\left(C_{1}+\beta_{\ell} C_{2}\right)
$$

$$
\begin{gather*}
v_{5}=1+\beta_{\ell} \cos \theta \quad v_{6}=2\left(E_{\nu}^{2}-\beta_{\ell}^{2} E_{\ell}^{2}\right) \frac{\sin \theta}{q^{2}}, \quad v_{7}=2 \eta \beta_{\ell}\left(E_{\ell}-E_{\nu}\right) \frac{\sin \theta}{q}  \tag{8}\\
v_{8}=-2 \beta_{\ell}\left(E_{\ell}+E_{\nu}\right) \frac{\sin \theta}{q}, \quad v_{9}=2 \eta \beta_{\ell} \sin \theta, \quad v_{10}=\frac{\beta_{\ell} E_{\ell} E_{\nu} \sin ^{2} \theta}{q^{2}}
\end{gather*}
$$

Here, $E_{\ell}$ and $E_{\nu}$ are electron (positron) and neutrino (anti-neutrino) energies; $\theta$ is angle between electron (positron) and neutrino (anti-neutrino) momenta; $\beta_{\ell}$ is electron (positron) velocity; $q=|\vec{q}|$ is momentum transferred to nuclei ; $\eta$ is +1 for neutrino scattering, -1 for anti-neutrino scattering; and $C_{1}, C_{2}$ and $C_{3}$ are coefficients given by the relations

$$
C_{1}=\left(\beta_{\ell} E_{\ell} \cos \theta-E_{\nu}\right) / q, \quad C_{2}=\left(\beta_{\ell} E_{\ell}-E_{\nu} \cos \theta\right) / q, \quad C_{3}=C_{1} C_{2}
$$

## 3. The differential scattering quasi-elastic cross section of ${ }^{12} \mathrm{C}$

Consider the process

$$
\begin{equation*}
\nu(\bar{\nu})+{ }^{12} C \rightarrow{ }^{12} N\left({ }^{12} B\right)+e^{-}\left(e^{+}\right) \tag{9}
\end{equation*}
$$

for which $J_{i}=0 \rightarrow J_{f}=1$. The quanta numbers $L, J$ and $J^{\prime}$ are defined then by

$$
0 \leq L \leq 2 J_{f} \Rightarrow 0 \leq L \leq 2, \quad\left|J_{i}-J_{f}\right| \leq J \leq J_{i}+J_{f} \Rightarrow J=1, \quad\left|J_{i}-J_{f}\right| \leq J^{\prime} \leq J_{i}+J_{f} \Rightarrow J^{\prime}=1
$$

The differential cross section of the process described in equation (9), obtained from (5), is given by

$$
\begin{align*}
\frac{d \sigma}{d \Omega_{\ell}}= & \frac{E_{\ell}^{2} G_{F}^{2}}{2 \pi}\left\{R_{0}^{0}+A\left(P_{2}(\cos \theta *) R_{2}^{0}+P_{2}^{1}(\cos \theta *) \cos \varphi * R_{2}^{1}\right.\right.  \tag{10}\\
& \left.\left.+P_{2}^{2}(\cos \theta *) \cos 2 \varphi * R_{2}^{2}\right)+P\left(P_{1}(\cos \theta *) R_{1}^{0}+P_{1}^{1}(\cos \theta *) \cos \varphi * R_{1}^{1}\right)\right\}
\end{align*}
$$

$A$ and $P$ are alignment and polarization coefficients of the final nuclei [12]. Functions $R_{m}^{n}$ are given by the relations

$$
\begin{gather*}
R_{0}^{0}=v_{1} H_{1}+v_{2} H_{2}+v_{3} H_{3}+v_{4} H_{4}+v_{5} H_{5}, \\
R_{2}^{0}=\frac{1}{2}\left[v_{1} H_{1}+v_{2} H_{2}-2\left(v_{3} H_{3}+v_{4} H_{4}+v_{5} H_{5}\right)\right] \\
R_{2}^{1}=\frac{1}{2 \sqrt{6}}\left(v_{6} H_{6}+v_{8} H_{8}\right), \quad R_{2}^{2}=\frac{1}{4} v_{10} H_{10},  \tag{11}\\
R_{1}^{0}=-\frac{3}{2}\left(v_{1} H_{2}+v_{2} H_{1}\right), \quad R_{1}^{1}=-\frac{3}{2 \sqrt{6}}\left(v_{7} H_{6}+v_{9} H_{8}\right),
\end{gather*}
$$

where

$$
\begin{gathered}
H_{1}=\left(F_{M 1}\right)^{2}+\left(F_{E 1}^{5}\right)^{2}, \quad H_{2}=2 F_{M 1} F_{E 1}^{5}, \quad H_{3}=\left(F_{L 1}^{5}\right)^{2}, \\
H_{4}=F_{L 1}^{5} F_{C 1}^{5}, \quad H_{5}=\left(F_{C 1}^{5}\right)^{2}, \quad H_{6}=F_{L 1}^{5}\left(F_{E 1}^{5}+F_{M 1}\right), \\
H_{8}=F_{C 1}^{5}\left(F_{E 1}^{5}+F_{M 1}\right), \quad H_{10}=\left(F_{E 1}^{5}\right)^{2}-\left(F_{M 1}\right)^{2} .
\end{gathered}
$$

The matrix elements of the vector magnetic, axial-vector electric, coulomb and longitudinal multipole operators computed in the shell model are

$$
\begin{gather*}
F_{M 1}=\frac{\psi}{3 \sqrt{\pi}} \frac{q}{2 M}\left(F_{1}-\left(F_{1}+2 M F_{2}\right)(2-y)\right) e^{-y}, \\
F_{E 1}^{5}=-\frac{\psi}{3 \sqrt{\pi}} F_{A}(2-y) e^{-y}, \\
F_{C 1}^{5}=-\frac{\sqrt{2} \psi}{3 \sqrt{\pi}} \frac{q}{2 M}\left[\frac{3}{2} F_{A}+\left(W_{0} F_{P}+2 \eta M F_{T}\right)(1-y)\right] e^{-y},  \tag{12}\\
F_{L 1}^{5}=-\frac{\sqrt{2} \psi}{3 \sqrt{\pi}}\left(F_{A}-\frac{q^{2}}{2 M} F_{P}\right)(1-y) e^{-y},
\end{gather*}
$$

where $y=(b q / 2)^{2}, b=1.77 \mathrm{fm}$ is the oscillator parameter, and $\psi=-0.003$ [1].

## 4. Spin asymmetry coefficient

Consider the spin asymmetry coefficients defined as

$$
\begin{equation*}
A_{\nu}\left(E_{\nu}, \theta\right)=\frac{d \sigma\left(\vec{S}_{N} \uparrow \uparrow \vec{P}_{\nu}\right)-d \sigma\left(\vec{S}_{N} \uparrow \downarrow \vec{P}_{\nu}\right)}{d \sigma\left(\vec{S}_{N} \uparrow \uparrow \vec{P}_{\nu}\right)+d \sigma\left(\vec{S}_{N} \uparrow \downarrow \vec{P}_{\nu}\right)} \tag{13}
\end{equation*}
$$

where $\vec{S}_{N}$ is the final nuclei spin and $\vec{P}_{\nu}$ is the neutrino momentum. Analysis of angular dependence of spin asymmetry shows, for given values of $P$ and $A$, that the maximum $F_{T}$ contribution calculated due to $\Delta A_{\nu}$ is moving opposite of increasing neutrino energy (see Figures $1(\mathrm{a}, \mathrm{b})$ ). This maximum also depends on the angle $\theta$ for given values of $P$ and $A$ (see Table).


Figure 1. Contribution of SCC to the asymmetry coefficient for different value of the neutrino energy as a function of angle.

Table. Contribution of SCC to spin asymmetry coefficients, with $\Delta A_{\nu}=A_{\nu}\left(F_{T}=5 \cdot 10^{-3} \mathrm{MeV}\right)-A_{\nu}\left(F_{T}=0\right)$.

|  | $P=0.7, A=0.3$ |  |  | $P=1, A=1$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $E_{\nu}(\mathrm{MeV})$ | 200 | 200 | 400 | 400 | 500 | 500 | 600 | 600 |
| $\theta$ (degree $)$ | 47 | 163 | 54 | 80 | 41.5 | 66 | 34 | 62 |
| $\Delta A_{\nu}$ | 0.07 | 0.53 | 0.27 | 0.54 | 0.26 | 0.73 | 0.21 | 0.72 |

In Figure 2 we show the energy dependence of the contribution $D$ to the SCC, defined as

$$
\begin{equation*}
D=\frac{A_{v}\left(F_{T}=0\right)-A_{v}\left(F_{T}=5 \cdot 10^{-3} \mathrm{MeV}^{-1}\right)}{A_{v}\left(F_{T}=0\right)} \tag{14}
\end{equation*}
$$

relative the asymmetry coefficient for $\theta=60^{\circ}$. It seems that the SCC relative contribution is less than $16 \%$ for neutrino energy below 300 MeV . This relative contribution can take value in the range of $75 \%$ to $92 \%$ when neutrino energy is more than 400 MeV .


Figure 2. Relative contribution of SCC to the asymmetry coefficient for $\theta=60^{\circ}$.

## 5. Electron-neutrino correlation and charge asymmetry coefficients

The $e \nu$ correlation coefficient is defined by the formula

$$
\begin{equation*}
A_{e \nu}=\frac{d \sigma(\theta \approx 0)-d \sigma(\theta \approx \pi)}{d \sigma(\theta \approx 0)+d \sigma(\theta \approx \pi)} \tag{15}
\end{equation*}
$$

When the final nucleus is oriented in the direction of neutrino and in the ultra relativistic case $\left(\beta_{\ell}=1\right)$, the coefficient $A_{e \nu}$ takes the form

$$
\begin{equation*}
A_{e \nu}=\frac{D_{1}-D_{2}}{D_{1}+D_{2}} \tag{16}
\end{equation*}
$$

with

$$
D_{1}=2(1-A)\left(1-y_{1}\right)^{2}\left(\left(E_{\nu}-E_{\ell}\right) F_{T}+F_{A}\right)^{2} \exp \left(-2 y_{1}\right)
$$

$$
\begin{gathered}
D_{2}=\left(1+\frac{1}{2} A+\frac{3}{2} \eta P\right)\left(2-y_{2}\right)^{2}\left(\left(E_{\nu}+E_{\ell}\right) F_{2}-\eta F_{A}\right)^{2} \exp \left(-2 y_{2}\right), \\
y_{1,2}=\left(b\left(E_{\nu} \mp E_{\ell}\right) / 2\right)^{2}
\end{gathered}
$$

In the maximum polarization case $(A=P=1), D_{1}=0, A_{e \nu}=-1$ and $e \nu$ correlation do not depend on SCC form factor $F_{T}$. However, in the case of partial polarization of the final nucleus, the relative contribution of SCC depends on the value of the alignment coefficient $A$. For example, when $A=0.1$ and $P=0.5$ (Figure 3), this value reaches between $1 \%$ to $17 \%$ in the $80-120 \mathrm{MeV}$ neutrino energy range. When energies $E_{\nu}$ are above 200 MeV the coefficient $A_{e \nu} \cong+1$ and it is no more sensitive to SCC form factor $F_{T}$ variation.

The charge asymmetry coefficient is defined by the formula

$$
\begin{equation*}
B=\frac{d \sigma_{\nu}-d \sigma_{\tilde{\nu}}}{d \sigma_{\nu}+d \sigma_{\tilde{\nu}}} \tag{17}
\end{equation*}
$$

where $d \sigma_{\nu}\left(d \sigma_{\tilde{\nu}}\right)$ is the differential cross section for neutrino (anti-neutrino) scattering.
$B$ is determined with respect to the parameter $\eta$, which is equal to +1 for neutrino scattering and -1 for anti-neutrino scattering.

The curves (see Figure 4) for which the neutrino energy is in the range of 350 to 423 MeV show that the charge asymmetry coefficient is negative and takes values between $-2.6 \%$ to $-2.5 \%$ when $F_{T}=0$. But it becomes positive with value between $30 \%$ and $60 \%$ when $F_{T}=5 \times 10^{-3} \mathrm{MeV}^{-1}$. So, in this neutrino energy area, the coefficient $B$ presents only pure SCC effects.


Figure 3. Relative contribution of SCC to the $e \nu$ correlation coefficient.


Figure 4. Charge asymmetry coefficient for $P=0.7$, $A=0.3$ and $\theta=60^{\circ}$ as a function of energy.

## 6. Conclusion

Theoretical analysis of different characteristics possessed by processes of quasi-elastic neutrino scattering by nuclei has shown that the relative contribution of SCC to spin asymmetry, ev correlation and charge
asymmetry coefficients for $F_{T}=5 \times 10^{-3} \mathrm{MeV}^{-1}$, can reach some tens of percents for particular values of alignment $A$ and polarization $P$ of final (initial) nucleus and that of neutrino (antineutrino) energy.

Therefore, the experimental study of quasi-elastic neutrino (antineutrino) scattering processes can allow more accurate expression of the SCC tensor form factor when the nucleus polarization is taken into account.

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