

# Electrons trajectories in electromagnetically pumped free electron laser with a constant reverse guide magnetic field

Ahmad OMAR

*Physics Department, Faculty of Arts and Sciences, Arab American University  
P.O. Box (240), Jenin, West Bank-PALESTINE  
e-mail: ahmad.omar@aaup.edu*

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## Abstract

An exact solution to the quartic equation in the wiggler frequency  $\omega_w$ , which represents the dispersion relation for this system, is obtained. The effects of the frequency of the wiggler  $\omega_w$  on the electron's velocities ( $v_{||}$  and  $v_w$ ) are investigated. The regions of interest for the allowed wiggler frequency  $\omega_w$  for both group I and group II electron trajectories are discussed.

**Key Words:** Free-Electron Laser (FEL), electromagnetic wiggler, wiggler frequency, reverse guide magnetic field

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## 1. Introduction

Free-Electron Lasers (FEL) represent an incredible departure from classical lasers, where they provide widely tunable, highly intense, ultrashort laser pulse in any part of the electromagnetic spectrum [1–4].

Several studies have been reported that investigated trajectories in different electromagnetic wiggler models. An exact solution to the relativistic equation of motion for a charged particle in a constant magnetic field and a transverse electromagnetic wave propagating along the field had been found by Roberts and Buchsbaum [5]. Freund et al. studied the interaction between a relativistic electron beam and an electromagnetic wiggler in the second stage [6]. We have investigated the relation between the energy of a system consisting of a large amplitude electromagnetic wiggler FEL in the presence of a constant axial guide magnetic field and both the parallel and perpendicular components of canonical momentum [7]. Hiddleston et al. studied the equations of motion for the electromagnetically pumped FEL with an axial electrostatic field [8]. Bourdier and Gond studied the dynamics of a charged particle in both circularly and linearly polarized traveling electromagnetic wave [9, 10]. A theory on a free-electron laser with electromagnetic wiggler and ion-channel guiding have been developed by Esmaeilzadeh et al. The electron trajectories for this system are found and the stability of orbits is studied

[11]. Electromagnetic wiggler free electron laser was constructed and operated experimentally [12, 13]. The effects of the reverse guide magnetic field have been investigated for different free electron lasers configurations. Tsui, K. H. studied a system consisting of circularly polarized external wiggler with a reverse guide field by introducing the self-fields of the electron beam. He found that the competition between the self fields and the wiggler field plus the action of the guide field are responsible for a new reverse guide field singularity [14]. The behavior of electron beam in combined self generated field and a reversed axial-guide field in the tapered helical wigglers has been studied by S. K. Nam, et al. They reported that the electron beam loss is reduced by optimizing the magnetic field strength and tapering the parameter of the reversed axial guide field [15]. Conde and Bekefi reported a new regime of free electron laser operation using a helical wiggler field and a reversed axial guide magnetic field. The orientation of the axial field is such as to oppose the rotation of the electrons imparted by the helical field. The 33.3 GHz electron laser amplifier is driven by a mildly relativistic electron beam (750 kV, 300 A, 300 ns) and generates 61 MW of radiation with a 27% conversion efficiency. The results are compared with those obtained when the axial guide field is in its conventional orientation where considerable loss of power and efficiency is observed [16]. G. Zhang et al. studied the FEL amplifier performance with a reversed axial guide field theoretically and numerically. They found that group I orbits can be reproduced by assuming an anomalously large emittance, while group II results are not consistent with the single frequency model [17]. Xiao-jian Shu studied the single-particle trajectories of relativistic electrons in FEL with a reversed axial guide magnetic field. He reported that when the cyclotron wavelength approaches the wiggler period, there is a beat between two rotations, which makes a large dip occur in the radiation power [18]. Bazylev et al. studied the electron trajectories for a FEL configuration in the presence of a reversed axial guide magnetic field analytically. He considered two types of electron trajectories close to antiresonance. One belongs to a linearly polarized motion and leads to a moderate coupling with radiation field [19, 20].

A previous study on an electromagnetic wiggler FEL with a constant forward guide magnetic field had been reported by H. P. Freund et al. [6]. Numerical solution for the dispersion relation have been investigated.

In Section II we investigate the electrons trajectories for the same wiggler given in [6], but with constant reverse guide magnetic field. To study the effects of the wiggler field frequency  $\omega_w$  on the electrons velocities ( $v_{||}$  and  $v_w$ ) for our system we solved the quartic equation in the wiggler frequency  $\omega_w$  for this system exactly. Section III concentrates on studying the analytical results we have derived numerically in Section II. Dependence of the parallel and transverse components of the electrons velocities ( $v_{||}$  and  $v_w$ ) on wiggler frequency  $\omega_w$  is investigated.

## 2. Formulation of the problem

A system consisting of a uniform reverse axial magnetic field,  $-B_0 \hat{e}_z$ , and a backward propagating electromagnetic wave described by

$$\mathbf{B}_w = B_w [\hat{e}_x \cos(k_w z + \omega_w t) + \hat{e}_y \sin(k_w z + \omega_w t)] \quad (2.1)$$

$$\mathbf{E}_w = \frac{\omega_w}{k_w c} B_w [-\hat{e}_x \sin(k_w z + \omega_w t) + \hat{e}_y \cos(k_w z + \omega_w t)] \quad (2.2)$$

where  $B_w$  is the amplitude of the wiggler magnetic field,  $B_0$  is the guide magnetic field amplitude, while  $k_w$  is the wiggler field wave vector and  $\omega_w$  is the corresponding angular frequency.

The relativistic equations of motion for this system are

$$\frac{d\mathbf{P}}{dt} = \frac{d}{dt} (m \gamma \mathbf{v}) = -e \left[ \mathbf{E}_w + \frac{\mathbf{v}}{c} \times (\mathbf{B}_0 + \mathbf{B}_w) \right] \quad (2.3)$$

and

$$\frac{d\gamma}{dt} = -\frac{e}{m c^2} \mathbf{v} \cdot \mathbf{E}_w, \quad (2.4)$$

where  $\mathbf{P}$  is electron momentum,  $m$  is electrons mass,  $\mathbf{v}$  is electron velocity,  $e$  is electron charge,  $c$  is the velocity of light, and  $\gamma$  is the relativistic factor. To simplify the analysis we will introduce the following set of basis vectors which are rotating with the wiggler

$$\hat{e}_1 = \hat{e}_x \cos(k_w z + \omega_w t) + \hat{e}_y \sin(k_w z + \omega_w t) \quad (2.5a)$$

$$\hat{e}_2 = -\hat{e}_x \sin(k_w z + \omega_w t) + \hat{e}_y \cos(k_w z + \omega_w t) \quad (2.5b)$$

$$\hat{e}_3 = \hat{e}_z. \quad (2.5c)$$

So, the velocity of the electron in this wiggler frame of reference can be written as

$$v = v_1 \hat{e}_1 + v_2 \hat{e}_2 + v_3 \hat{e}_3. \quad (2.6)$$

The equations of motion for this system are given in the Appendix. Steady state orbits are found for constant energy ( $\gamma = \text{constant}$ ). This implies that  $v_2=0$  and  $\frac{dv_1}{dt} = \frac{dv_3}{dt} = 0$ . If we define  $v_1 = v_w$  and  $v_3 = v_{||}$ , equation (2.6) becomes

$$\mathbf{v}_0 = v_w \hat{e}_1 + v_{||} \hat{e}_3, \quad (2.7)$$

where the set of the steady state trajectory equations are given by

$$v_w = \frac{-\Omega_w (\omega_w + k_w v_{||})}{k_w [\Omega_0 + \gamma (\omega_w + k_w v_{||})]} \quad (2.8)$$

and

$$\frac{v_{||}^2}{c^2} + \frac{v_w^2}{c^2} = 1 - \frac{1}{\gamma^2} \quad (2.9)$$

where

$$\Omega_{0,w} = \frac{e B_{0,w}}{m c}.$$

Here,  $\Omega_0$  is the cyclotron frequency of the guide magnetic field, and  $\Omega_w$  is the cyclotron frequency of the wiggler field.

To analyze the effects of frequency  $\omega_w$  and wave vector of the wiggler  $k_w$  on the electrons trajectories, we will use the following Maxwell's Equation:

$$\left[ \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] A_w = \frac{4\pi}{c} n e v_w, \quad (2.10)$$

where  $\mathbf{A}_w$  is the wiggler vector potential given by

$$\mathbf{A}_w = -\frac{B_w}{k_w} [\hat{e}_x \cos(k_w z + \omega_w t) + \hat{e}_y \sin(k_w z + \omega_w t)] . \quad (2.11)$$

Substituting equation (2.8) for  $\mathbf{v}_w$  and equation (2.12) for the vector potential  $\mathbf{A}_w$  in equation (2.10) results in the dispersion relation

$$\omega_w^2 - c^2 k_w^2 - \omega_b^2 \frac{(\omega_w + k_w v_{||})}{[\Omega_0 + \gamma (\omega_w + k_w v_{||})]} = 0. \quad (2.12)$$

Here,  $\omega_b^2 = \frac{4\pi e^2 n_b}{m}$  is the plasma frequency and  $n_b$  is the bulk density of the electron's beam.

To show the dependency of the orbital velocities ( $v_{||}$  and  $v_w$ ) on the wiggler frequency  $\omega_w$ , we merged equations (2.8), (2.9) and (2.12) so that we have the following quartic equation in  $\omega_w$  in normalized (dimensionless) form:

$$\omega_w^4 - 2\omega_w^2 + \frac{\omega_b^4}{\Omega_w^2} \left( v_{||}^2 - 1 + \frac{1}{\gamma^2} \right) + 1 = 0. \quad (2.13)$$

In the above, we introduced the following set of normalized quantities:

$$\frac{v_{||}}{c} \rightarrow v_{||}, \quad \frac{v_w}{c} \rightarrow v_w, \quad \frac{\omega_w}{k_w c} \rightarrow \omega_w, \quad \frac{\Omega_w}{\gamma_0 k_w c} \rightarrow \Omega_w, \quad \frac{\Omega_0}{\gamma_0 k_w c} \rightarrow \Omega_0, \quad \frac{\omega_b}{\gamma_0^2 k_w c} \rightarrow \omega_b. \quad (2.14)$$

This fourth degree polynomial equation (2.13) has the following set of exact analytical solutions:

$$\omega_{w_1} = \pm \sqrt{\alpha_1} \quad (2.15)$$

$$\omega_{w_2} = \pm \sqrt{\alpha_2} \quad (2.16)$$

where

$$\alpha_1 = 1 + \sqrt{\beta} \quad (2.17)$$

$$\alpha_2 = 1 - \sqrt{\beta} \quad (2.18)$$

and

$$\beta = \frac{\omega_b^4}{\Omega_w^2} \left( 1 - v_{||}^2 - \frac{1}{\gamma^2} \right). \quad (2.19)$$

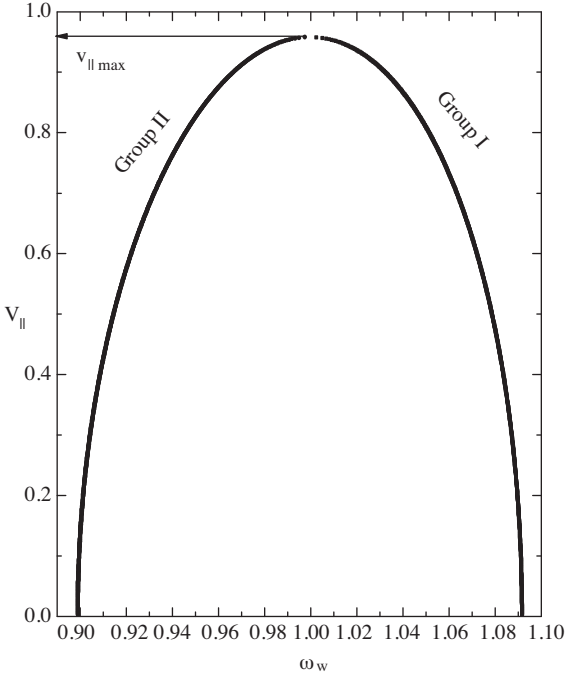
### 3. Results

We conclude from Equation (2.8) that the usage of the reverse axial guide magnetic field results in a reduction in the wiggler velocity compared to that without a guide magnetic field. This ends with a decrease in the linearized gain related to the expected values for group I and II [4]. Also, there is no resonant enhancement in the wiggler velocity if the Larmor period belongs to the axial reverse field is comparable to the wiggler period. Previous studies concluded that the inhomogeneity in the wiggler field introduces a sinusoidal driving term to the electron trajectories [4]. The inhomogeneity of the wiggler field will not affect the beam transport compared to Group I and II orbital instabilities appeared when the axial guide magnetic field is pointed in the forward direction with respect to the wiggler field.

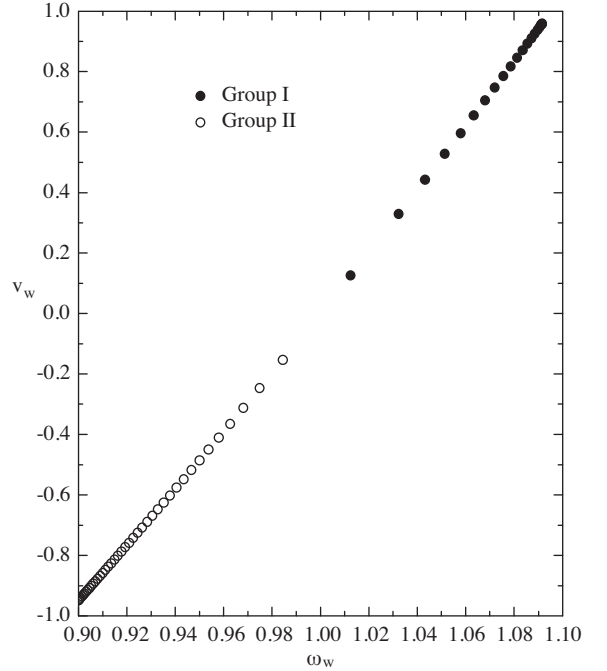
Since we are using a backward propagating electromagnetic wiggler field, the analysis is concentrated on positive wiggler frequencies. This reduces the values of  $\omega_w$  from four to just two only. Equations (2.15) and (2.16) restrict the values of  $\beta$  to  $\beta \geq 0$  which put an upper limit on the value of the normalized parallel velocity of the beam  $v_{||} \leq \sqrt{1 - \frac{1}{\gamma^2}}$ . Also, the values of  $\alpha_1$  and  $\alpha_2$  are restricted to  $\alpha_1 \geq 1$  and  $0 < \alpha_2 \leq 1$ . These values for  $\alpha_1$  and  $\alpha_2$  constraint the allowed range of  $\omega_{w1}$  and  $\omega_{w2}$  to  $\omega_{w1} \geq 1$  and  $0 < \omega_{w2} \leq 1$ , where  $\omega_{w1}$  belongs to group I trajectories, while  $\omega_{w2}$  belongs to group II trajectories.

In addition to our exact results we have obtained in section II and presented by equations (2.15) and (2.16), we show these results numerically. In Figure 1 we show the dependence of the parallel particle's velocity  $V_{||}$  on the wiggler field angular frequency  $\omega_w$ . The range of the allowed values for both group I and group II orbits are shown. There is an upper limit on the axial velocity  $V_{||} = 0.98$  when the angular wiggler frequency  $\omega_w$  approaches 1 for both group I and group II orbits. The plot shows that the axial velocity  $V_{||}$  increases with respect to the wiggler frequency  $\omega_w$  for group II orbits, while it decreases with respect to the wiggler frequency  $\omega_w$  for group I orbits. So, we can adjust the angular wiggler frequency  $\omega_w$  to reach the desired axial velocity  $V_{||}$ .

In Figure 2 is shown the relation between the electron wiggler velocity  $v_w$  and the angular frequency  $\omega_w$ , clarifying the linear dependence for both group I and group II. The specific range of the wiggler field angular frequency  $\omega_w$  is shown in the figure for both group (I) and group (II) orbits.



**Figure 1.** The axial electron's velocity  $V_{||}$  as a function of the wiggler field frequency  $\omega_w$ . The normalized parameters are  $\gamma_0 = 3.5$ ,  $\omega_b = 0.1$ ,  $\omega_w = 0.05$ .



**Figure 2.** The wiggler electron's velocity  $v_w$  as a function of the wiggler field frequency  $\omega_w$ . The normalized parameters are  $\gamma_0 = 3.5$ ,  $\omega_b = 0.1$ ,  $\omega_w = 0.05$ .

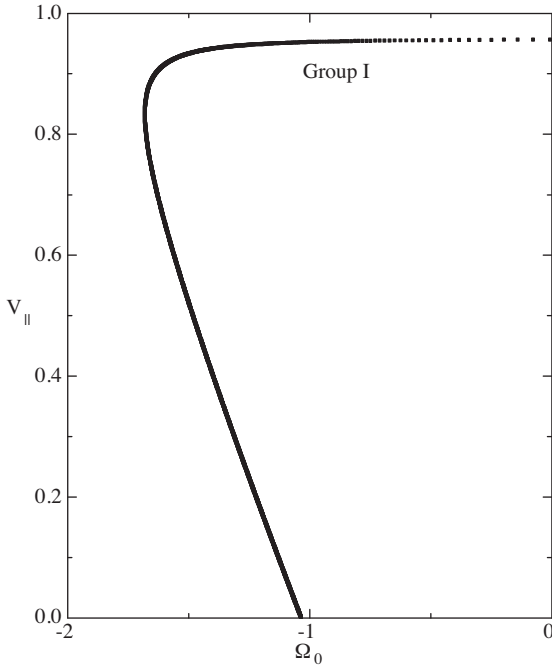
To investigate the effect of the reverse guide magnetic field  $B_0$  (included in the reverse guide magnetic field cyclotron frequency  $\Omega_0$ ) on the axial velocity of the electrons,  $V_{||}$ , we rearrange equation (2.8) and rewrite

it in the normalized form

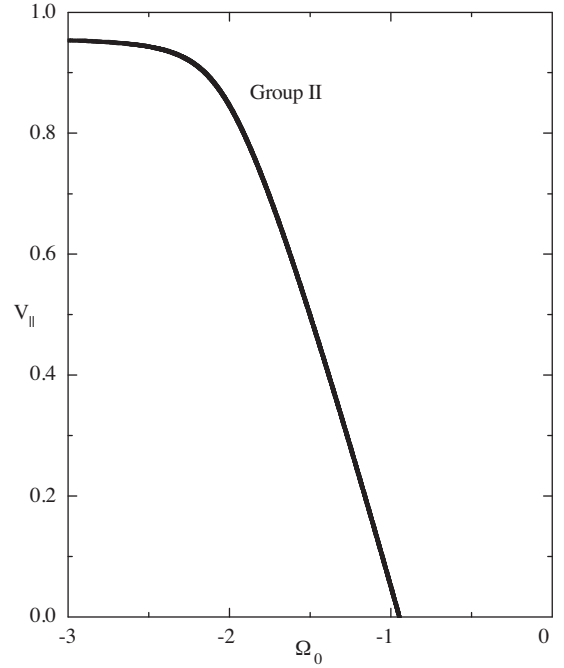
$$\Omega_{01,02} = -(\omega_{w1,w2} + v_{||}) \left( \frac{\Omega_w}{\gamma_0 v_w} + 1 \right). \quad (3.1)$$

The variation of the axial electron velocity  $v_{||}$  with respect to the reverse guide magnetic field frequency  $\Omega_0$  is shown for group I orbits, which are characterized by  $\omega_{w1} \geq 1$  in Figure 3. The lower bound imposed on the allowed values for the reverse guide magnetic field frequency  $\Omega_0$  for group I orbits is limited to  $\Omega_0 > -1.68$ .

Figure 4 shows the variation of the axial electron velocity  $V_{||}$  with respect to the reverse guide magnetic field frequency  $\Omega_0$  for group II orbits which are characterized by  $0 < \omega_{w2} \leq 1$ . The allowed values for the reverse guide magnetic field frequency  $\Omega_0$  for group II orbits are restricted to  $-3 < \Omega_0 < -0.94$ .



**Figure 3.** The axial electron's velocity  $V_{||}$  as a function of the reverse axial guide magnetic field frequency  $\Omega_0$  for group I orbits. The normalized parameters are  $\gamma_0 = 3.5$ ,  $\omega_b = 0.1$ ,  $\omega_w = 0.05$ .



**Figure 4.** The axial electron's velocity  $V_{||}$  as a function of the reverse axial guide magnetic field frequency  $\Omega_0$  for group II orbits. The normalized parameters are  $\gamma_0 = 3.5$ ,  $\omega_b = 0.1$ ,  $\omega_w = 0.05$ .

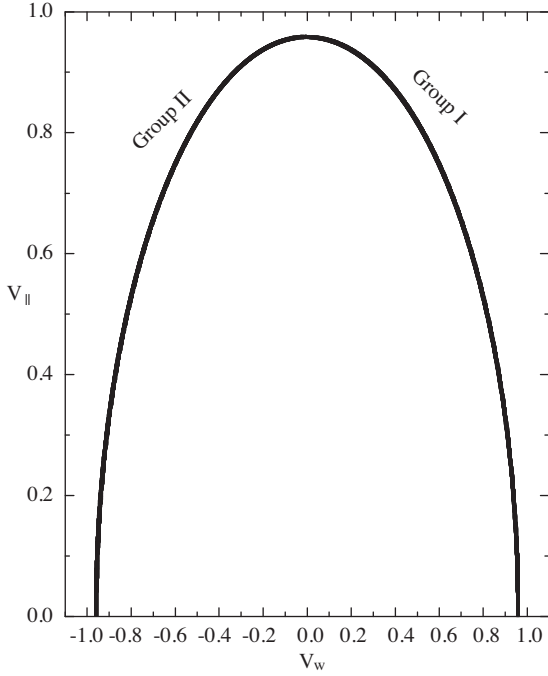
Our results in Figure 3 and Figure 4 lead to the same results found for the forward guide magnetic field system investigated in [6] if we substitute  $-\Omega_0$  instead of  $\Omega_0$  in the previous analysis.

By tuning the wiggler frequency  $\omega_w$ , we plot the variation of the electrons' parallel velocity  $V_{||}$  with respect to the electrons' wiggler velocity  $v_w$  in Figure 5.

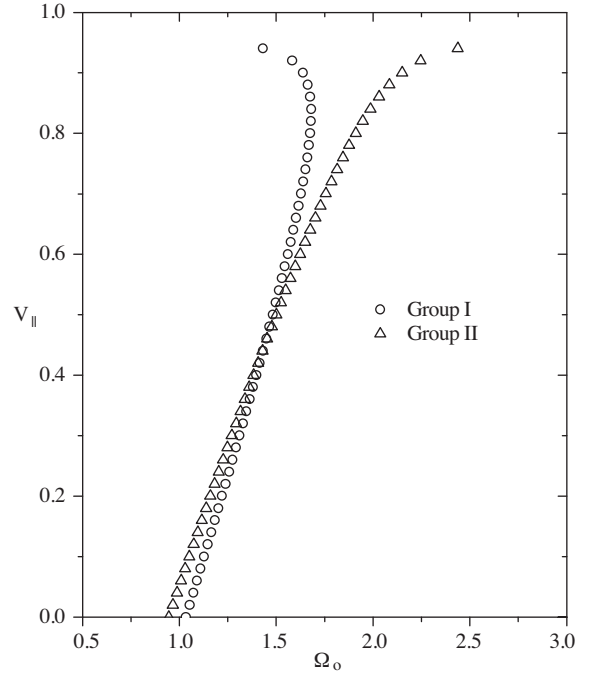
Figure 5 shows that selecting a higher axial velocity constraints the wiggler velocity to small values for both groups. The obtained results are helpful in obtaining a collimated electron's beam and limiting off axis effects.

## 4. Concluding remarks

In conclusion, we derived an exact analytical solution for the dispersion relation of our system. The effect of wiggler frequency  $\omega_w$  on the behavior of the electrons trajectories for both group I and group II orbits have been discussed. We obtained the suitable values of the wiggler field frequency  $\omega_w$  for group I and group II analytically, and show it numerically in Figure 1 and Figure 2, respectively. Our results concerning the relation between the axial electron velocity and the reverse axial magnetic field show identical behavior when transformed from reverse to forward constant guide magnetic field (see Figure 6) compared to the numerical analysis reported by Freund et al. [6].



**Figure 5.** The axial electron's velocity  $V_{||}$  as a function of the wiggler electron's velocity  $v_w$  for both group I and II orbits. The normalized parameters are  $\gamma_0 = 3.5$ ,  $\omega_b = 0.1$ ,  $\omega_w = 0.05$ .



**Figure 6.** The axial electron's velocity  $V_{||}$  as a function of the forward axial guide magnetic field frequency  $\Omega_0$  for group I & group II orbits. The normalized parameters are  $\gamma_0 = 3.5$ ,  $\omega_b = 0.1$ ,  $\omega_w = 0.05$ .

## Appendix

Rewriting equations (2.3) and (2.4) in the reference frame rotating with the beam, the equations of motion for the electrons will be

$$\dot{v}_1 = \frac{1}{\gamma} \left[ \Omega_0 + \gamma (\omega_w + k_w v_3) + \frac{\omega_w \Omega_w}{ck_w} \frac{v_1}{c} \right] v_2 \quad (\text{A1})$$

$$\dot{v}_2 = -\frac{1}{\gamma} \left[ [\Omega_0 + \gamma (\omega_w + k_w v_3)] v_1 + \Omega_w \left[ v_3 + \frac{\omega_w}{k_w} \right] - \frac{\omega_w}{ck_w} \Omega_w \frac{v_2^2}{c} \right] \quad (\text{A2})$$

$$\dot{v}_3 = \frac{1}{\gamma} \Omega_w \left[ 1 + \frac{\omega_w}{ck_w} \frac{v_3}{c} \right] v_2 \quad (\text{A3})$$

$$\dot{\gamma} = -\frac{\omega_w}{c^2 k_w} \Omega_w v_2, \quad (\text{A4})$$

where  $\Omega_{0,w} = \left| \frac{eB_{0,w}}{mc} \right|$ .

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