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Bianchi-V cosmological models with viscous fluid and constant deceleration parameter in general relativity

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Abstract

In this paper we discuss the variation law for Hubble's parameter, average scale factor in spatially homogenous anisotropic Bianchi Type V space-time that yields a constant value deceleration parameter. Using the law of variation for Hubble's parameter, exact solutions of Einstein's field equations are obtained for Bianchi-V space time filled with viscous fluid in two different cases where the universe exhibits power law and exponential expansion. We investigate a number of solutions with constant and time varying cosmological constant together with variable and constant bulk viscosity. We find that the constant value of deceleration parameter is reasonable for the present day universe and gives an appropriate discussion of evolution of universe with the recent observations of type Ia supernovae. The detailed study of physical and kinematical properties of the model is also discussed.

Key Words: Hubble's parameter (HP), deceleration parameter (DP), anisotropy parameter (AP), cosmology

1. Introduction

At the present state of evolution, the universe is spherically symmetric and the matter distribution in it is, on the whole, isotropic and homogeneous. In its early stages the universe could not have had such a smoothed out picture because near the big bang singularity it would be in a highly dense and energetic state, hence isotropic. Hence the anisotropy of the cosmic expansion, which came to be damped out over the course of cosmic evolution, is an important quantity of study.

The cosmological models which are spatially homogenous and anisotropic play significant roles in the description of the universe at its early stages of evolution. Bianchi I-IX spaces are very useful to constructing special homogeneous cosmological models. (The importance of Bianchi type V model is due to the fact that the space of constant negative curvature is contained in it as a special case.) These models can be used to analyze aspects of the physical universe which pertain or which may be affected by anisotropy in the rate of expansion, for example, the cosmic microwave background radiation, nucleosynthesis in the early universe and the question of isotropization of the universe itself (MacCallum, [1]). Spatially homogeneous cosmologies also play an important role in the attempt to understand the structure and the properties of the space of all cosmological solutions of Einstein's field equations. A spatially homogeneous cosmology is said to be tilted (Ellis and King [2]) if the fluid velocity vector is not orthogonal to the group of orbits, otherwise the model is said to be non tilted (King and Ellis [3]).

A tilted model is spatially homogeneous relative to observers whose world line are orthogonal relative to group orbits become time like. This means that the models are no longer spatially homogeneous (Collins and Ellis [4]).

Most cosmological models assume that the mater in the universe can be described by dust (a pressureless distribution) or at the early stages of universe viscous effects do play a role (Israel and Vardalas [5], Kilmek [6], Weinberg [7]). For example, the existence of bulk viscosity is equivalent to slow process of restoring equilibrium states (Landau and Lifchitz [8]). The observed physical phenomena such as the large entropy per baryon and remarkable degree of isotropy of the cosmic microwave background radiation suggest analysis of dissipative effects in cosmology. Bulk viscous models have prime roles in getting inflationary phases of the universe [9–15]. Bulk viscosity driven inflation is primarily due to the negative bulk viscous pressure giving rise to a total negative effective pressure which may overcome the pressure due to the usual gravity of matter distribution in the universe and provide an impetus to drive it apart. Bulk viscosity is associated with the GUT phase transition and string creation. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (Gron, [16]) for a review on cosmological models with bulk viscosity.

The model studied by Murphy [17] possessed an interesting feature in that the big bang type singularity of infinite space time curvature does not occur to be finite past. However, the relationship assumed by Murphy between the viscosity coefficient and the matter density is not acceptable at large density. The effect of bulk viscosity on cosmological evolution has been investigated by a number of authors in the framework of general relativity (Pavon [18], Padmanabhan and Chitre [19], Johri and Sudarshan [20], Maartens [21] , Zimdahl [22], Santos et al. [23], Pradhan et al. [24], Kalyani and Singh [25], Singh et al. [26], Pradhan et al. [27–29]) This motivates to study cosmological bulk viscous fluid model. Banerjee and Sanyal [30] have considered Bianchi Type V cosmologies with viscosity and heat flow. It has also been shown that it is possible for dissipative Bianchi type V universe model not to be in thermal equilibrium in their early stages. Coley [31] have investigated Bianchi Type V spatially homogenous with perfect fluid cosmological model which contains both viscosity and heat flow. Bali and Meena [32] have investigated two conformally flat tilted Bianchi type V cosmological models filled with perfect fluid and heat conductivity. Conformally flat tilted Bianchi type V cosmological models in the presence of a bulk viscous fluid are investigated by Pradhan and Rai [33]. Recently Shriram et al. [34] have investigated the variation law for Hubble's parameter with average scale factor in a spatially homogenous anisotropy Bianchi type-V space-time model.

The cosmological constant Λ problem is regarded as one of the important unsolved problem in cosmology. In recent years, models with cosmological constant Λ have drawn considerable attention among researchers for various aspects such as the age problem, classical tests, observational constraints on Λ, structure formation and gravitational lenses have been discussed in the literature. Some of the recent discussions on the cosmological constant by Ratra and Peebles [35], Dolgov [36–38]. Sahni and Starbinsky [39] have pointed out that in the absence of any interaction with matter or radiation, the cosmological constant remains a "constant". However in the presence of interactions with matter or radiation, a solution of Einstein equations and the assumed equation of covariant conservation of stress-energy with a time varying Λ can be found. These recent observations strongly favour a significant and a positive value of Λ with magnitude $\Lambda\left(G\hbar/c^3\right) \approx 10^{-123}$. Riess et al. [40–43] have recently presented an analysis of 156 SNe including a few at $z > 1.3$ from the Hubble Space Telescope (HST) "GOOD ACS" Treasury survey. They conclude to the evidence for present acceleration $q_0 < (q_0 \approx -0.7)$. Observation by Knop et al. [44] of type Ia Supernovae (SNe) allow us to probe the expansion history of the universe leading to the conclusion that the expansion of the universe is accelerating.

In this paper, we consider a spatially homogenous anisotropy Bianchi type-V space-time in which the source of matter distribution is viscous fluid with a cosmological constant. The purpose of the present paper is to investigate the behaviour of a viscous fluid with a cosmological constant in the framework of a Bianchi type V space time. It is not an easy task to construct an exact solution to the Einstein's field equation due to nonlinearity of the differential equations which arise from general relativity. An attempt has been made to formulate a law of variation for Hubble's parameter in anisotropic Bianchi type-V space-time that yields a constant value of deceleration parameter. The Law together with the Einstein's field equation leads to a number of new solutions of Bianchi type-V space-time. The law explicitly determine the scale factor, explicit form of pressure, energy density and some other cosmological parameters are obtained for two different physical models. We also discuss the physical and kinematical behaviour of the different parameters such as expansion scalar, anisotropic pressure and shear scalar in these two singular and non singular cosmological models with constant DP.

2. Model and field equations

The spatially homogeneous and anisotropic Bianchi V space time is described by the line element

$$ds^2 = -dt^2 + a_1^2 dx^2 + a_2^2 e^{-2mx} dy^2 + a_3^2 e^{-2mx} dz^2, \tag{1}$$

where a_1, a_2 and a_3 are the metric functions of cosmic time t and m is constant.

The spatial volume of this model is given by the relation

$$R^3 = a_1 a_2 a_3. \tag{2}$$

We define $R = (a_1 a_2 a_3)^{\frac{1}{3}}$ as the average scale factor so that the Hubble's parameter in anisotropic models may be defined as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right), \tag{3}$$

where a dot over symbols denote derivative with respect to the cosmic time t.

Also, we have

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \tag{4}$$

where $H_1 = \frac{\dot{a}_1}{a_1}$, $H_2 = \frac{\dot{a}_2}{a_2}$, and $H_3 = \frac{\dot{a}_3}{a_3}$ are directional Hubble's factors in the directions of x, y and z, respectively.

The field equations in case of perfect fluid are

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j, \tag{5}$$

with

$$T_i^j = (\rho + \overline{p})\ u_i u^j + \overline{p} g_i^j - \Lambda, \tag{6}$$

where $\overline{p} = p - \xi\theta,\ g_{ij}u^i u^j = -1;\ u^i$ is the four velocity vector; R_{ij} is the Ricci tensor; R is the Ricci scalar; ξ is the bulk viscosity, θ is the expansion scalar; and ρ and p, respectively, are the energy density and isotropic pressure of the fluid.

In a commoving coordinate system, Einstein's field equations (5) for the anisotropic Bianchi type-V space time (1), in case of (6), read as

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{m^2}{a_1^2} = -\overline{p} + \Lambda, \tag{7}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{m^2}{a_1^2} = -\overline{p} + \Lambda, \tag{8}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} - \frac{m^2}{a_1^2} = -\overline{p} + \Lambda, \tag{9}$$

$$\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{3m^2}{a_1^2} = \rho + \Lambda, \tag{10}$$

$$2\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3} = 0. \tag{11}$$

The equation of state is taken to be of the usual form, viz.,

$$p = (\gamma - 1)\,\rho,\ (0 \le \gamma \le 2)\,. \tag{12}$$

3. Variation law for Hubble's parameter

In order to solve Einstein's field equations, we normally assume a form for the matter content or suppose that the space time admits killing vector symmetries. The Einstein field equations (7)–(11) are a coupled system of highly non linear differential equations and there are no standard methods for solving them. Kramer et al. [45] have pointed out that most authors solve the Einstein's field equations with a stress energy tensor of perfect fluid type by assuming an equation of state linking the pressure p and energy density ρ in order to build analytical methods near the singularity. Davidson [46] and later many others (Coley and Tupper [47]) have considered models with variable equation of state, which essentially deals with the Friedman Robertson Walker (FRW) metric. Law of variation for Hubble's parameter was first proposed by Berman [48] in FRW models and that yields a constant value of deceleration parameter. Recently Singh and Kumar [49–51] have proposed a similar law of variation for Hubble's parameter in locally rotationally symmetric (LRS) Bianchi type I, II space times, that yields a constant value of deceleration parameter. Reddy et al. [52, 53] have presented LRS Bianchi type I models with constant DP in scalar tensor and scale covariant theories of gravitation.

In order to obtain physically realistic solutions, one has to make assumptions generally at the cost of physics in the problem or for mathematical convenience. Solutions of field equations can be generated by applying the law of variation of Hubble's parameter proposed by Berman [54] which yields a constant value of DP. The law of variation for Hubble's parameter gives a new approach for solving field equations that is quite general and suitable for the description of present day universe. In this paper, we propose that the law to be examined for the variation of Hubble's parameter which yields a constant value of DP in anisotropic Bianchi type-V space time is

$$H = D\left(a_1 a_2 a_3\right)^{-\frac{n}{3}}, \tag{13}$$

where $D > 0$ and $n \geq 0$ are constants.

The deceleration parameter q is defined by

$$q = -\frac{3}{\theta^2}\left[\theta_{,\alpha} u^{\alpha} + \frac{1}{3}\theta^2\right].$$

Therefore,

$$q = -\frac{R\ddot{R}}{R^2}. \tag{14}$$

From equations (3) and (13), we get

$$\frac{1}{3}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) = D\left(a_1 a_2 a_3\right)^{-\frac{n}{3}}. \tag{15}$$

This on integration leads to

$$a_1 a_2 a_3 = \left(nDt + c_1\right)^{\frac{3}{n}} \text{ for } n \neq 0 \tag{16}$$

$$a_1 a_2 a_3 = c_2 e^{3Dt} \text{ for } n = 0 \tag{17}$$

Here, c_1 and c_2 are positive constants of integration.

Now substituting (16) into (14), we get

$$q = n - 1. \tag{18}$$

This shows that the deceleration parameter is constant for this model. It may be pointed out that the above law refers to anisotropic Bianchi type V space time in any context, i.e. in any theory that is based on anisotropic Bianchi type V space time.

4. Solution of field equations

From equations (7), (8) and (9), we obtain

$$\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} = \frac{k_1}{R^3} \tag{19}$$

$$\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3} = \frac{k_2}{R^3}. \tag{20}$$

From (19) and (20), the metric functions can be explicitly written as

$$a_1 = m_1 R \exp\left(x_1 \int \frac{dt}{R^3}\right) \tag{21}$$

$$a_2 = m_2 R \exp\left(x_2 \int \frac{dt}{R^3}\right) \tag{22}$$

$$a_3 = m_3 R \exp\left(x_3 \int \frac{dt}{R^3}\right), \tag{23}$$

where $x_1,\ x_2,\ x_3,\ m_1,\ m_2,\ m_3$ are arbitrary constants of integration satisfying the equality

$$m_1 m_2 m_3 = 1, \quad x_1 + x_2 + x_3 = 0. \tag{24}$$

4.1. Cosmology for $n \neq 0$

Using relation (16) in (21)–(23), we get the following expressions for scale factors:

$$a_1 = m_1 (nDt + c_1)^{\frac{1}{n}} \exp\left(\frac{x_1}{D\,(n-3)} (nDt + c_1)^{\frac{n-3}{n}}\right), \tag{25}$$

$$a_2 = m_2 (nDt + c_1)^{\frac{1}{n}} \exp\left(\frac{x_2}{D\,(n-3)} (nDt + c_1)^{\frac{n-3}{n}}\right), \tag{26}$$

$$a_3 = m_3 (nDt + c_1)^{\frac{1}{n}} \exp\left(\frac{x_3}{D\,(n-3)} (nDt + c_1)^{\frac{n-3}{n}}\right). \tag{27}$$

The pressure and energy density are given by

$$\begin{aligned} p - 3D\xi (nDt + c_1)^{-1} - \Lambda &= D^2 (2n-3)(nDt + c_1)^{-2} - \left(x_2^2 + x_3^2 + x_2 x_3\right)(nDt + c_1)^{-\frac{6}{n}} \\ &\quad + m^2 m_1^{-2} (nDt + c_1)^{-\frac{2}{n}} \exp\left(-\frac{2x_1}{D\,(n-3)} (nDt + c_1)^{\frac{n-3}{n}}\right), \end{aligned} \tag{28}$$

$$\begin{aligned} \rho + \Lambda &= 3D^2 (nDt + c_1)^{-2} + (x_1 x_2 + x_2 x_3 + x_1 x_3)(nDt + c_1)^{-\frac{6}{n}} \\ &\quad - 3m^2 m_1^{-2} (nDt + c_1)^{-\frac{2}{n}} \exp\left(-\frac{2x_1}{D\,(n-3)} (nDt + c_1)^{\frac{n-3}{n}}\right). \end{aligned} \tag{29}$$

In view of (24), one may observe that the solutions (25)–(29) represent exact solutions of the Einstein field equations (7)–(11). Now we find expressions for some other cosmological parameters of the model. The anisotropy parameter A is defined as

$$A = \frac{1}{3} \sum_{i=1}^{3} \left[\frac{H_i - H}{H}\right]^2. \tag{30}$$

The directional Hubble factors $H_i (i = 1, 2, 3)$ as defined in (4) are given by

$$H_i = D (nDt + c_1)^{-1} + x_i (nDt + c_1)^{-\frac{3}{n}} \tag{31}$$

The expansion scalar is given by the equality

$$\theta = 3H = 3D (nDt + c_1)^{-1}. \tag{32}$$

Using (31) and (32), in (30) we get

$$A = \frac{1}{3D^2} \left(x_1^2 + x_2^2 + x_3^2\right) c_2^{-2} (nDt + c_1)^{\frac{2n-6}{n}}. \tag{33}$$

The volume and shear scalar of the model are given by

$$R^3 = (nDt + c_1)^{\frac{3}{n}} \tag{34}$$

$$2\sigma^2 = \frac{1}{3}\left[(x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_1)^2\right](nDt + c_1)^{-\frac{6}{n}}. \tag{35}$$

4.1.1. Model with constant Λ-term and $\xi(t)$

In this case equations (28) and (29) together with (12) yield the expressions for energy density, isotropic pressure and bulk viscosity are, respectively, given by

$$\begin{aligned}\rho &= 3D^2 (nDt + c_1)^{-2} + (x_1x_2 + x_2x_3 + x_1x_3)(nDt + c_1)^{-\frac{6}{n}} \\ &\quad -3m^2m_1^{-2}(nDt + c_1)^{-\frac{2}{n}} \exp\left(-\frac{2x_1}{D(n-3)}(nDt + c_1)^{\frac{n-3}{n}}\right) - \Lambda,\end{aligned} \tag{36}$$

$$\begin{aligned}p &= (\gamma - 1)\Big[3D^2 (nDt + c_1)^{-2} + (x_1x_2 + x_2x_3 + x_1x_3)(nDt + c_1)^{-\frac{6}{n}} \\ &\quad -3m^2m_1^{-2}(nDt + c_1)^{-\frac{2}{n}} \exp\left(-\frac{2x_1}{D(n-3)}(nDt + c_1)^{\frac{n-3}{n}}\right) - \Lambda\Big],\end{aligned} \tag{37}$$

$$\begin{aligned}\xi &= \left(\gamma - \frac{2n}{3}\right) D (nDt + c_1)^{-1} + \left[\gamma (x_1x_2 + x_2x_3 + x_1x_3) - x_1x_2 - x_1x_3 + x_2^2 + x_3^2\right] \cdot \frac{(nDt + c_1)^{\frac{-6+n}{n}}}{3D} \\ &\quad + \left(\frac{4}{3} - \gamma\right) m^2m_1^{-2}(nDt + c_1)^{\frac{-2+n}{n}} \exp\left(-\frac{2x_1}{D(n-3)}(nDt + c_1)^{\frac{n-3}{n}}\right) - \frac{\gamma\Lambda}{3D}(nDt + c_1).\end{aligned} \tag{38}$$

It is observed that all the quantities diverge at $t = 0$. At late times, the energy density converges to $-\Lambda$. So positivity of ρ is ensured only for $\Lambda < 0$. The bulk viscosity diverges as $t \to \infty$, even for $\Lambda < 0$, this shows unphysical nature of the model. Thus, we find that the solutions do not provide a physically realistic model in the presence of a non-zero and constant Λ.

4.1.2. Model with variable Λ and constant ξ

Assuming that the coefficient of bulk viscosity is constant i.e. $\xi(t) = \xi_0 = cons\tan t$ then

(28) and (29) together with (12) yield the following expressions for energy density, pressure and cosmological constant

$$\begin{aligned}\rho &= \frac{1}{\gamma}\Big[2nD^2 (nDt + c_1)^{-2} + \left(x_1x_2 + x_1x_3 - x_2^2 - x_3^2\right)(nDt + c_1)^{-\frac{6}{n}} \\ &\quad -2m^2m_1^{-2}(nDt + c_1)^{-\frac{2}{n}} \exp\left(-\frac{2x_1}{D(n-3)}(nDt + c_1)^{\frac{n-3}{n}}\right) + 3D\xi_0 (nDt + c_1)^{-1}\Big],\end{aligned} \tag{39}$$

$$\begin{aligned}p &= \frac{(\gamma - 1)}{\gamma}\Big[2nD^2 (nDt + c_1)^{-2} + \left(x_1x_2 + x_1x_3 - x_2^2 - x_3^2\right)(nDt + c_1)^{-\frac{6}{n}} \\ &\quad -2m^2m_1^{-2}(nDt + c_1)^{-\frac{2}{n}} \exp\left(-\frac{2x_1}{D(n-3)}(nDt + c_1)^{\frac{n-3}{n}}\right) + 3D\xi_0 (nDt + c_1)^{-1}\Big]\end{aligned} \tag{40}$$

$$\Lambda = \left(3-\frac{2n}{\gamma}\right) D^2 (nDt+c_1)^{-2} + \left[(x_1x_2+x_2x_3+x_1x_3) - \frac{(x_1x_2+x_1x_3-x_2^2-x_3^2)}{\gamma}\right](nDt+c_1)^{-\frac{6}{n}}$$
$$-\left(3-\frac{2}{\gamma}\right) m^2 m_1^{-2} (nDt+c_1)^{-\frac{2}{n}} \exp\left(-\frac{2x_1}{D(n-3)}(nDt+c_1)^{\frac{n-3}{n}}\right) - 3\frac{D\xi_0}{\gamma}(nDt+c_1)^{-1}\Bigg], \quad (41)$$

where $\gamma \neq 0$.

As the evolution progress, the energy density, pressure and cosmological constant decreases, the solutions are singular at $t = 0$. At late times these quantities are negligible.

4.1.3. Model with variable Λ and $\xi\alpha\rho$

Let us assume that $\xi(t) = \xi_0\rho$. Then from equations (28) and (29) together with (12) we obtain the expression for energy density, pressure, bulk viscosity and cosmological constant as follows:

$$\rho = \frac{1}{\left[\gamma - 3D\xi_0 (nDt+c_1)^{-1}\right]}\left[2nD^2 (nDt+c_1)^{-2} + \left(x_1x_2+x_2x_3-x_2^2-x_3^2\right)(nDt+c_1)^{-\frac{6}{n}} - \right.$$
$$\left. 2m^2 m_1^{-2}(nDt+c_1)^{-\frac{2}{n}} \exp\left(-\frac{2x_1}{D(n-3)}(nDt+c_1)^{\frac{n-3}{n}}\right)\right], \quad (42)$$

$$p = \frac{(\gamma-1)}{\left[\gamma - 3D\xi_0 (nDt+c_1)^{-1}\right]}\left[2nD^2 (nDt+c_1)^{-2} + \left(x_1x_2+x_2x_3-x_2^2-x_3^2\right)(nDt+c_1)^{-\frac{6}{n}} - \right.$$
$$\left. 2m^2 m_1^{-2}(nDt+c_1)^{-\frac{2}{n}} \exp\left(-\frac{2x_1}{D(n-3)}(nDt+c_1)^{\frac{n-3}{n}}\right)\right], \quad (43)$$

$$\xi = \frac{\xi_0}{\left[\gamma - 3D\xi_0 (nDt+c_1)^{-1}\right]}\left[2nD^2 (nDt+c_1)^{-2} + \left(x_1x_2+x_2x_3-x_2^2-x_3^2\right)(nDt+c_1)^{-\frac{6}{n}} \right.$$
$$\left. - 2m^2 m_1^{-2}(nDt+c_1)^{-\frac{2}{n}} \exp\left(-\frac{2x_1}{D(n-3)}(nDt+c_1)^{\frac{n-3}{n}}\right)\right], \quad (44)$$

$$\Lambda = \left[3 - \frac{2n}{\left[\gamma - 3D\xi_0 (nDt+c_1)^{-1}\right]}\right] D^2 (nDt+c_1)^{-2} + \left(x_1x_2+x_2x_3+x_1x_3 - \frac{(x_1x_2+x_2x_3-x_2^2-x_3^2)}{\left[\gamma - 3D\xi_0 (nDt+c_1)^{-1}\right]}\right)$$
$$\times (nDt+c_1)^{-\frac{6}{n}} - \left[3 - \frac{2n}{\left[\gamma - 3D\xi_0 (nDt+c_1)^{-1}\right]}\right] m^2 m_1^{-2} (nDt+c_1)^{-\frac{2}{n}} \exp\left(-\frac{2x_1}{D(n-3)}(nDt+c_1)^{\frac{n-3}{n}}\right)\Bigg], \quad (45)$$

We observe that the energy density ρ, cosmological constant Λ decrease very sharply due to presence of viscous term.

Physical behaviour of the model

At $t = t_0 = \frac{-c_1}{nD}$ it is observed that the spatial volume is zero and the expansion scalar is infinite, which corresponds to the universe beginning its evolution with zero volume with an infinite rate of expansion. The scale factor vanishes at $t = t_0$, hence the model has a point singularity at the initial epoch. The pressure, energy density and shear scalar diverges at the initial singularity. For $n < 3$, the anisotropy of expansion is infinity and will be zero for $n > 3$ at the initial epoch. Hence the universe exhibits power law expansion as the scalar decreases; hence the rate of expansion slows down with increase in time. The energy density and pressure along with shear scalar are infinite at $t = t_0$, clearly indicating the point of singularity at this epoch. The directional Hubble parameter and the generalized Hubble parameter are both infinite at this singularity point. For large time the expansion will be completely exhaust and the model will become isotropic at the ratio $\frac{\sigma}{\theta} \to 0$. Thus the model represent shearing, non rotating and expanding model of the universe with a big bang approaching isotropy at late times. The integral $\int\limits_{t_0}^{t} R(t')dt' = \frac{1}{D(n+1)} (nDt' + c_1)_{t_0}^{t}$ is finite provided $n \neq -1$. Therefore a horizon exists in this model. Further, we observe that for $n = 3$ the spatial volume grows linearly with cosmic time. For $n \leq 1$, we get $-1 < q \leq 0$, which shows that the model represent an accelerating model of the universe. For $n > 1, q > 0$, which implies a decelerating model of the universe. Recent observations of type Ia Supernovae (Perlmutter [55–57], Riess et al .[58, 59], Tonry et al. [60], Knop et al. [61] and John [62]) represent that universe is accelerating with the deceleration parameter lying somewhere in the range $-1 < q \leq 0$. It follows that the solutions obtained in this model are consistent with observations.

4.2. Cosmologies for $n = 0$

Using (17) in (21)–(23), we get the following expressions for scale factors:

$$a_1 = m_1 c_2^{\frac{1}{3}} \exp\left(Dt - \frac{x_1}{3c_2 D} e^{-3Dt}\right), \tag{46}$$

$$a_2 = m_2 c_2^{\frac{1}{3}} \exp\left(Dt - \frac{x_2}{3c_2 D} e^{-3Dt}\right), \tag{47}$$

$$a_3 = m_3 c_2^{\frac{1}{3}} \exp\left(Dt - \frac{x_3}{3c_2 D} e^{-3Dt}\right). \tag{48}$$

The pressure and energy density are given by

$$p - 3\xi D - \Lambda = -3D^2 - \left(x_2^2 + x_3^2 + x_2 x_3\right) c_2^{-2} e^{-6Dt} + m^2 m_1^{-2} c_2^{-\frac{2}{3}} \exp 2\left(\frac{x_1}{3c_2 D} e^{-3Dt} - Dt\right), \tag{49}$$

$$\rho + \Lambda = 3D^2 + \left(x_1 x_2 + x_2 x_3 + x_1 x_3\right) e^{-6Dt} - 3m^2 m_1^{-2} c_2^{-\frac{2}{3}} \exp 2\left(\frac{x_1}{3c_2 D} e^{-3Dt} - Dt\right). \tag{50}$$

Solutions (46)–(50) represent exact solutions of field equations (7)–(11). The other cosmological parameters of the model have the following expressions:

$$H_i = D + x_i c_2^{-1} e^{-3Dt} \,(i = 1, 2, 3), \tag{51}$$

$$\theta = 3H = 3D, \tag{52}$$

$$R^3 = c_2 e^{3Dt}, \tag{53}$$

$$A = \frac{1}{3D^2} \left(x_1^2 + x_2^2 + x_3^2\right) e^{-6Dt} c_2^{-2}, \tag{54}$$

$$2\sigma^2 = \frac{1}{3}\left[(x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_1)^2\right] e^{-6Dt} c_2^{-2}. \tag{55}$$

4.2.1. Model with constant Λ-term and $\xi(t)$

In this case, equations (49) and (50) together with (12) we find the following solutions:

$$\rho = 3D^2 + (x_1x_2 + x_2x_3 + x_1x_3)\, e^{-6Dt} - 3m^2 m_1^{-2} c_2^{-\frac{2}{3}} \exp 2\left(\frac{x_1}{3c_2D} e^{-3Dt} - Dt\right) - \Lambda, \tag{56}$$

$$p = (\gamma - 1)\left[3D^2 + (x_1x_2 + x_2x_3 + x_1x_3)\, e^{-6Dt} - 3m^2 m_1^{-2} c_2^{-\frac{2}{3}} \exp 2\left(\frac{x_1}{3c_2D} e^{-3Dt} - Dt\right) - \Lambda\right], \tag{57}$$

$$\begin{aligned} \xi \;&=\; \gamma D + \left[\left(x_2^2 + x_3^2 + x_2x_3\right) c_2^{-2} + (\gamma - 1)\,(x_1x_2 + x_2x_3 + x_1x_3)\right] \frac{e^{-6Dt}}{3D} + \frac{(2 - 3\gamma)}{3D} \\ &\quad \times m^2 m_1^{-2} c_2^{-\frac{2}{3}} \exp 2\left(\frac{x_1}{3c_2D} e^{-3Dt} - Dt\right) - \frac{\gamma\Lambda}{3D}. \end{aligned} \tag{58}$$

The solutions have no initial singularity. However, at late times, the expressions are physically valid subject to the condition $3D^2 \geq \Lambda$.

4.2.2. Model with variable Λ and constant ξ

Let us assume that $\xi(t) = \xi_0 = \text{const} \cdot \tan t$. Then from equations (49) and (50) together with (12), the solutions for energy density, pressure and cosmological constant are obtained as:

$$\begin{aligned} \rho \;&=\; \frac{1}{\gamma}\Big\{3D\xi_0 + \left[(x_1x_2 + x_2x_3 + x_1x_3) - \left(x_2^2 + x_3^2 + x_2x_3\right) c_2^{-2}\right] e^{-6Dt} \\ &\quad - 2m^2 m_1^{-2} c_2^{-\frac{2}{3}} \exp 2\left(\frac{x_1}{3c_2D} e^{-3Dt} - Dt\right)\Big\}, \end{aligned} \tag{59}$$

$$\begin{aligned} p \;&=\; \frac{(\gamma - 1)}{\gamma}\Big\{3D\xi_0 + \left[(x_1x_2 + x_2x_3 + x_1x_3) - \left(x_2^2 + x_3^2 + x_2x_3\right) c_2^{-2}\right] e^{-6Dt} \\ &\quad - 2m^2 m_1^{-2} c_2^{-\frac{2}{3}} \exp 2\left(\frac{x_1}{3c_2D} e^{-3Dt} - Dt\right)\Big\}, \end{aligned} \tag{60}$$

$$\begin{aligned} \Lambda \;&=\; 3D^2 + \left[(x_1x_2 + x_2x_3 + x_1x_3)\left(1 - \frac{1}{\gamma}\right) + \left(x_2^2 + x_3^2 + x_2x_3\right) \frac{c_2^{-2}}{\gamma} - \frac{3D\xi_0}{\gamma}\right] e^{-6Dt} \\ &\quad - \left(3 - \frac{2}{\gamma}\right) m^2 m_1^{-2} c_2^{-\frac{2}{3}} \exp 2\left(\frac{x_1}{3c_2D} e^{-3Dt} - Dt\right)\Big\}. \end{aligned} \tag{61}$$

All the parameters start with constant values and converge to some non-zero positive constants as $t \to \infty$. At late times the cosmological constant is positive.

4.2.3. Model with variable Λ and $\xi\alpha\rho$

Assuming $\xi=\xi_0\rho$, then equations (49) and (50) together with (12), we get the following solutions:

$$\rho = \frac{1}{(\gamma-3D\xi_0)}\left\{\left[(x_1x_2+x_2x_3+x_1x_3)-\left(x_2^2+x_3^2+x_2x_3\right)c_2^{-2}\right]e^{-6Dt} - 2m^2m_1^{-2}c_2^{-\frac{2}{3}}\exp 2\left(\frac{x_1}{3c_2D}e^{-3Dt}-Dt\right)\right\}, \tag{62}$$

$$p = \frac{(\gamma-1)}{(\gamma-3D\xi_0)}\left\{\left[(x_1x_2+x_2x_3+x_1x_3)-\left(x_2^2+x_3^2+x_2x_3\right)c_2^{-2}\right]e^{-6Dt} - 2m^2m_1^{-2}c_2^{-\frac{2}{3}}\exp 2\left(\frac{x_1}{3c_2D}e^{-3Dt}-Dt\right)\right\}, \tag{63}$$

$$\xi = \frac{\xi_0}{(\gamma-3D\xi_0)}\left\{\left[(x_1x_2+x_2x_3+x_1x_3)-\left(x_2^2+x_3^2+x_2x_3\right)c_2^{-2}\right]e^{-6Dt} - 2m^2m_1^{-2}c_2^{-\frac{2}{3}}\exp 2\left(\frac{x_1}{3c_2D}e^{-3Dt}-Dt\right)\right\}, \tag{64}$$

$$\Lambda = 3D^2+\left[1-\frac{1}{(\gamma-3D\xi_0)}\right](x_1x_2+x_2x_3+x_1x_3)\,e^{-6Dt}+\frac{1}{(\gamma-3D\xi_0)} \times\left(x_2^2+x_3^2+x_2x_3\right)c_2^{-2}e^{-6Dt}+\left[\frac{2}{(\gamma-3D\xi_0)}-3\right]m^2m_1^{-2}c_2^{-\frac{2}{3}}\exp 2\left(\frac{x_1}{3c_2D}e^{-3Dt}-Dt\right). \tag{65}$$

The parameters have constant values at $t=0$. As $t\to\infty\rho,p,\xi$ vanish, whereas Λ converges to $3D^2$.

Physical Behaviour of the Model

It can easily be observed that the spatial volume, all three scale factors, and all other physical, kinematical parameters are constant at $t=0$, this shows that the model is free from the initial singularity. The expansion scalar is constant throughout the evolution. This indicates that the universe starts evolving with constant volume and expands with the exponential rate. It is interesting to note that there is a constant rate of expansion in this model. The negative value of q indicates inflation. As t increases, the scale factors and the spatial volume increases exponentially. While the pressure, energy density, AP and shear scalar decreases. As the cosmic time increases, the scale factors and the volume become infinitely large whereas AP and shear scalar tends to 0. The directional Hubble parameter and the average generalize Hubble parameter will become constant for large time. The model indicates that the universe starts evolving with constant volume and exponentially with constant rate of expansion and will finally approach isotropy at late time.

Anisotropy parameter starts with maximum value $\frac{1}{3D^2}\left(x_1^2+x_2^2+x_3^2\right)c_2^{-2}$ at $t=0$.For $n=0$, we get $q=-1$. Incidentally, this value of deceleration parameter leads to $\frac{dH}{dt}=0$ and implies the greatest value of Hubble's parameter and the fastest rate of expansion for the universe. It follows that the solution obtained in this model are consistent with the recent observation of Ia Supernovae. The ratio $\frac{\sigma^2}{\theta^2}\to 0$, as $t\to\infty$ which implies that the models approach to isotropy at late times. The model represents a shearing, non-rotating and expanding universe.

5. Conclusion

In this paper we have presented two categories of Bianchi type-V cosmological solutions to field equation with viscous fluid in the presence of a cosmological constant in general relativity. The cosmological constant offers a potentially important contribution to the dynamics of the evolution of the universe. Using the power law for an exponential for the average scale factor derived from the variation law of Hubble's parameter, which gives a constant value of the deceleration parameter. In the first category of the model (i.e., for $n \neq 0$), the universe begins expansion from a singular state and all matter and radiation is concentrated at the big bang epoch, the expansion driven by the big bang impulse. The rate of expansion slows down and vanishes as $t \to \infty$. This gives a physically realistic model of the universe with variable Λ in the presence of bulk viscosity. On the other hand, the model is not physically realistic in the presence of constant Λ. Thus only variable Λ is allowed in the physically relevant viscous models. The cosmological constant is observed to have a small, positive values at late times. The model has a point singularity at the initial epoch as the scale factors and the volume vanish at this moment. The model represents a shearing and non-rotating and expanding universe. This approaches isotropy for large values of t.

In the second category, $n = 0$, the universe has no singular state; the universe starts expanding with constant expansion rate for the constant volume where all physical quantities are well behaved. We have also discussed the physical and kinematical properties of the universe.

The solution obtained for the models in both the categories are consistent with the recent observation of type Ia Supernovae. Finally, there is possibility the law of variation for Hubble's Parameter presented in this paper may be useful in studying new solutions of Einstein's field equations for anisotropic Bianchi type-V space time in other alternative theories. Thus, more realistic models may be analyzed by using this technique, which may lead to interesting a different physical behaviour of the evolution of the universe.

References

[1] M. A. H. MacCallum, An Einstein Centenary Survey, ed. by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge. 1979).

[2] G. F. R. Ellis, A. R. King, *Comm. Math. Phys.*, **38**, (1974), 119.

[3] A. R. King, G. F. R. Ellis, *Comm. Math. Phys.*, **31**, (1973), 209.

[4] C. B. Collins, G. F. R. Ellis, *Phys. Rep.*, **56**, (1979), 65.

[5] W. Israel, J. N. Vardalas, *Lett. Nouovo Cin.,* **4**, (1970), 887.

[6] Z. Kilmel, *Post. Astron.,* **19**, (1971), 165.

[7] S. Weinberg, *Astrophys. J.,* **168**, (1975), 175.

[8] L. Landau, E. M. Lifchitz, *Fluid* Mechanics, Addison-Wisley, (Mass. 1962), p.304.

[9] J. D. Barrow, *Phys. Lett. B,* **180**, (1986), 335.

[10] W. Zimdahl, *Phys. Rev. D,* **53**, (1996), 5483.

[11] D. Pavon, J. Bafaluy, D. Jou, *Class. Quantum. Gravity,* **8**, (1991), 347.

[12] R. Maartens, *Class. Quantum. Gravity,* **12**, (1995), 1455.

[13] J. A. S. Lima, A. S. M. Germano, L. R. W. Abramo, *Phys. Rev. D,* **53**, (1993), 4287.

[14] S. K. Tripathy, S. K. Nayak, S. K. Sahu, T. R. Routray, *Astrophys. Space Sci.,* **323**, (2009), 281.

[15] S. K. Tripathy, D. Behera, T. R. Routray, *Astrophys. Space Sci.,* **325**, (2010), 93.

[16] O. Gron, *Astrophys. Space Sci.,* **173**, (1990), 213.

[17] G. L. Murphy, *Phys. Rev. D.,* **8**, (1973), 4231.

[18] D. Pavon, J. Jou, D. Bafaluy, *Class. Quant. Grav.*, **8**, (1991), 357.

[19] T. Padmanabhan and S. M. Chitre, *Phys. Lett. A.,* **120**, (1987), 433.

[20] V. B. Johri, R. Sudaarshan, *Phys. Lett. A.,* **132**, (1988), 316.

[21] R. Maartens, *Class. Quant. Grav.,* **12**, (1995), 1455.

[22] W. Zimdahl, *Phys. Rev. D.,* **53**, (1996), 5483.

[23] N. O. Santos, R. S. Dias, A. Banerjee, *J. Math. Phys.,* **26**, (1985), 878.

[24] A. Pradhan, R. V. Sarayakar, A. Beesham, *Astr. Lett. Commun.*, **35**, (1997), 283.

[25] D. Kalyani, G. P.Singh, In new direction in Relativity and Cosmology, eds. V. de. Sabbata and T. Singh, (Hadronic Press, U.S.A. 1997), p.41.

[26] T. Singh, A. Beesham, W. S. Mbokazi, *Gen. Rel. Grav.,* **30**, (1998), 537.

[27] A. Pradhan, V. K. Yadav, I. Chakraborty, *Int. J. Mod. Phys. D.,* **10**, (2001), 339.

[28] A. Pradhan, V. K. Yadav, N. N. Saste, *Int. J. Mod. Phys. D.*, **11**, (2002a), 857.

[29] A. Pradhan, Iaotemshi, *Int. J. Mod. Phys. D.,* **11**, (2002b), 1419.

[30] A. Banerjee, A. K. Sanyal, *Gen. Rel. Grav.,* **20**, (1988), 103.

[31] A. A. Coley, *Gen. Rel. Grav.,* **22**, (1990), 3.

[32] R. Bali, B. L. Meena, *Pramana-Journal of Physics.,* **62**, (2004), 1007.

[33] A. Pradhan, A. Rai, *Astrophys. Space Sci.,* **291**, (2004), 149.

[34] S. Ram , M. Zeyauddin, C. P. Singh, *Pramana-Journal of Physics.,* **72**, (2009), 415.

[35] B. Ratra, P. J. E. Peebles, *Phys. Rev. D,* **37**, (1988), 3406.

[36] A. D. Dolgov, In the very early universe, eds. G. W. Gibbons, S. W. Hawking and S. T. C. Siklov, (Cambridge University Press. 1983).

[37] A. D. Dolgov, M. V. Sazhin, and Ya. B. Zeldovich, Basics of Modern Cosmology, (Editions Frontiers. 1990).

[38] A. D. Dolgov, *Phys. Rev. D,* **55**, (1997), 5881.

[39] V. Sahni, A. Starbinsky, *Int. J. Mod. Phys. D*, **9**, (2000), 373.

[40] A. G. Riess, et al., *Astron. J.,* **116**, (1998), 1009.

[41] A. G. Riess, et al., *Astrophys. J.,* **560**, (2001), 49.

[42] A. G. Riess, et al., *Astrophys. J.,* **607**, (2004), 665.

[43] A. G. Riess, et al., *Astrophys. J.,* **659**, (2007), 98.

[44] R. K. Knop, et al., *Astrophys. J.,* **598**, (2003), 102 .

[45] D. Kramer, et al., Exact solutions of Einsteins field equations, (Cambridge University Press, Cambridge. 1994).

[46] D. Davidson, *Mon. Not. R.Astron. Soc.,* (1962), 12479.

[47] A. A. Coley, B. O. J. Tupper, *Can. J., Phys.,* **64**, (1986), 204.

[48] M. S. Berman, F. M. Gomide, *Gen. Rel. Grav.,* **20**, (1988), 191.

[49] C. P. Singh, S. Kumar, *Int. J. Mod. Phys. D.,* **15**, (2006), 419.

[50] C. P. Singh, S. Kumar, *Pramana-Journal of Physics.,* **68**, (2007), 707.

[51] C. P. Singh, S. Kumar, *Astrophys. Space Sci., doi.*, 10, (2007), 1007/s10509-007-94111-1.

[52] D. R. K. Reddy, *Astrophys. Space Sci.,* **133**, (1987a), 389.

[53] D. R. K. Reddy, R. Venkateswarlu, *Astrophys. Space Sci.,* **136**, (1987b), 17.

[54] M. S. Berman, *Nuovo. Cim. B.*, **74**, (1983), 182.

[55] S. Perlmutter, et al., *Astrophys. J.,* **483**, (1997), 565.

[56] S. Perlmutter, et al., *Nature.,* **391**, (1998), 51.

[57] S. Perlmutter, et al., *Astrophys. J.,* **517**, (1999), 565.

[58] A. G. Riess, et al., *Astron. J.,* **116**, (1998), 1009.

[59] A. G. Riess, et al., *Astron. J.,* **607**, (2004), 665.

[60] J. L. Tonry, et al. *Astrophys. J.,* **594**, (2003), 1.

[61] R. A. Knop, et al., *Astrophys. J.,* **598**, (2003), 102.

[62] M. V. John, *Astrophys. J.,* **614**, (2004), 1.