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Scale dimension as the fifth dimension of spacetime

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Abstract

The scale dimension discovered in the theory of infinite nesting of matter is studied from the perspective of physical implementation of well-studied four-and n-dimensional geometric objects. Adding the scale dimension to Minkowski four-dimensional space means the necessity to use the five-dimensional spacetime.

Key Words: Scale dimension, multi-dimensional space, theory of infinite nesting of matter

1. Introduction

The theory of infinite nesting of matter is a study involving the entire hierarchy of space systems and it requires necessarily introducing scale dimension into the everyday life of science. The purpose of this article is to analyze the properties of the new dimension and its relationship with geometric theory of n-dimensional spaces.

The number of basic dimensions in physics is determined by the number of degrees of freedom or independent variables that set the location of a physical body and its elements, considered as points in a given frame of reference. The number of dimensions or degrees of freedom gives the dimensionality of the involved spacetime. By adding the scale dimension to four-dimensional spacetime we obtain a five-dimensional manifold which includes the usual spacetime. Respecting the order of historical understanding this can be written as (3 +1 +1)-space, where in the first place the spatial dimensions are reflected, and then the dimensions of time and scale. In terms of geometry it is convenient to consider the axes of the all dimensions perpendicular to each other.

In contrast to geometry, physics does not deal with mathematical points but with specific material objects. Infinite division of these objects in parts does not lead to the emergence of mathematical points but instead new objects of smaller size. Although these objects seem more and more similar to points, they contain an infinite number of matter carriers and the smallest field quanta. If the motion inside of material objects is regarded as motion along the scale axis, then this axis is a special dimension of space. During motion along the scale axis, transition between scale levels of matter takes place and we can see that the objects of lower levels become parts of the objects of higher levels of matter. As a result the scale dimension can be represented as the dimension characterizing the complexity of space systems, in the sense of their composition of lower order systems.

2. The proposition

The theory of infinite nesting of matter and the idea of the scale dimension acquired their modern form owing to the works by Robert Oldershaw [1] (self-similar cosmological model), Sergey Sukhonos [2, 3] (in their work on scale similarity of space systems) and Sergey Fedosin [4, 5] (investigating similarity of matter levels).

Spatial relations are perceived by the observer through the shape of objects and their configuration relative to each other; and temporal relations are determined as the changes of shapes and configurations, that is, the angular and linear shifts. Owing to the fact that objects at different levels of matter evolve and change relatively independently from each other, and the speed of processes and the characteristic sizes and masses of objects differ significantly, the scale dimension is considered as a new dimension of spacetime. At the same time shapes and configurations of many three-dimensional objects at different levels of matter in many respects are similar to each other. This is fixed by relations of similarity for various physical quantities and SP-symmetry, which conveys the similarity of physical laws at different levels of matter [4].

In areas of research that use large-scale transformations and spatial dimensions the scale dimension can be studied by geometrical methods. In four-dimensional spacetime, the fourth dimension not time t but the product ct, where c is the speed of light. That part of spacetime that is at some time point perpendicular to the time axis defines the hyperplane in the form of three-dimensional geometrical space at a given time. If in the three-dimensional space an object moves to future upcoming events through the point of the present time, in the four-dimensional spacetime the movement from time 1 to time 2 is seen as a trace increasing in size from point 1 to point 2 that is associated with the object. The possibility of observing such a trace in the coordinate system means that the motion can be described as a set of three-dimensional images of an object at different times.

The introduction of the scale dimension takes into account that at different levels of matter the rates of time flow, regarded as the speeds of typical processes for similar objects, differ from each other. However, it is possible to introduce the total coordinate time based, for example, on periodic processes in an electromagnetic wave. In this case, the hyperplanes which are perpendicular to the scale axis will determine the single four-dimensional spacetime at different levels of matter.

By analogy with the four-dimensional Minkowski spacetime, we can now introduce the five-dimensional coordinate system and write down for each world event its vector as $X^i = (ct, \boldsymbol{r}, w)$, where $\boldsymbol{r}$ is a three-dimensional radius-vector, and w is a coordinate along the scale axis.

Scale dimension allows us to understand and prove the possibility of existence of real four-dimensional bodies in five-dimensional spacetime. For constructing such bodies the method of induction may be used, based on the following arguments:

One edge is a line segment bounded by two points (vertices);

Three edges in a plane give a triangle with three vertices, in the form of a face;

Six edges with four vertices in three-dimensional space give a tetrahedron (triangular pyramid) with four faces;

Ten edges with five vertices, with ten faces and the five tetrahedrons in four-dimensional space represent an object called pentachoron (4-simplex).

For a n-dimensional tetrahedron or a simplex with increasing n-dimensionality of space to one, the number of vertices equals $k = n + 1$. Each edge is a connection between two arbitrary vertices of the tetrahedron, so that the number of edges depends on the number of vertices according to the formula for combinations of two with the k vertices. To determine the number of faces of n-dimensional tetrahedron in the form of triangles we need to find combinations of three with the k vertices. In general, the formula for r combinations of k elements is expressed by means of factorials and has the form:

$$C_k^r = \frac{k!}{r!\,(k-r)!}. \tag{1}$$

Equation (1) allows us to find the number of edges, faces and r-polyhedrons as part of n-dimensional tetrahedron.

For a n-dimensional cube (hypercube), we can assume that its number of vertices depends on the dimensionality of space n according to the formula $k = 2^n$, where dimensionality of the point space is zero, linear space has dimensionality 1, flat space has dimensionality 2, etc. Consequently, a tesseract (a tetracube) as a hypercube in four-dimensional space must have 16 vertices and, according to equation (1), 120 edges. If we remove from this number the internal edges between the vertices of different faces, as well as the diagonal edges lying in the plane of faces, then only 32 external edges would remain, which are responsible for the convexity of a tesseract as a geometric figure. Besides, a tesseract has 8 external three-dimensional cubic faces and 24 external two-dimensional square faces which limit it in the four-dimensional space. The common name for n-dimensional bodies with flat sides is the polytope, for four-dimensional bodies the term polychoron is accepted, whereas for three-dimensional bodies the term polyhedron is used, and for two-dimensional bodies the term polygon is used.

In space of n dimensions it is possible to construct a variety of figures and find their properties, including the n-dimensional angles, areas and volumes. In particular, the volume of the n-dimensional sphere can be expressed by means of its radius R and the gamma function, according to the relation [6]

$$V_n = \frac{\pi^{n/2} R^n}{\Gamma\left(\frac{n}{2}+1\right)}. \tag{2}$$

This formula was known already in 19th century writings of Swiss mathematician Ludwig Schläfli [7].

Currently, the branch of mathematics which studies geometric objects in space of n dimensions is developed so well that all possible objects are divided into classes according to symmetry properties [8]. There are online resources, demonstrating the projection of multidimensional bodies on a plane or in three-dimensional space [9, 10], obtained by computer modeling.

From the mathematical point of view, any space of smaller dimensionality, which is part of the space of higher dimensionality, is a hyperplane on which it is possible to make projections of points of n-dimensional body. If a simplex or a tesseract is oriented strictly along the fourth spatial dimension, then at each point of a three-dimensional hyperplane perpendicular to the axis of the fourth dimension, there will be a tetrahedron, or a cube, respectively. Similarly, if we dissect a tetrahedron or a cube perpendicular to their bases, then on the section plane (two-dimensional hyperplane) there will be a triangle, the size of which depends on the cut-point, or a square. While the four-dimensional body is passing through the three-dimensional space one would expect the sudden appearance and disappearance of the projections of this body and changing of their size. As in everyday life such examples are not commonly found, it is seen that the fourth spatial dimension does not reveal itself directly.

From the above it follows that the scale axis, discovered in the hierarchy of space systems is a physical embodiment of the fourth spatial dimension [11]. A set of three-dimensional objects at different levels of matter represents a four-dimensional body oriented along the scale axis, so that each cross section of this axis represents a three-dimensional hyperplane, which may contain corresponding three-dimensional objects.

3. Conclusion

In contrast to the mathematical idealization, the objects of a natural system, due to their limited total number, are located on the scale axis with unequal probability, but unevenly grouped into separate levels of matter. Physical properties of the substance of objects are changed at different levels of matter, a concept that is usually ignored in geometry. At those levels of matter, where gravitational forces prevail, the dominating space objects have spherical shape.

Any set of objects, consisting of spherical galaxies, globular star clusters, individual stars, planets, nucleons, etc., form a definite single four-dimensional spherical body, stretching along the scale axis as along the fourth spatial coordinate. A four-dimensional body is discovered not by its projections as the result of its passing through our three-dimensional world, but in the opposite situation, when the observer himself is moving relative to this body and examining its components in various hyperplanes. Thus, the scale dimension confirms the possibility of real existence of four-dimensional geometric objects in nature. This enables the application of mathematical methods for n-dimensional spaces for the study of composite physical objects at different scale levels of matter.

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