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$\delta(E2/M1)$ and $X(E0/E2)$ mixing ratios in ^{134}Ba by means of IBM-2

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Abstract

The ^{134}Ba isotope ($Z = 56$) lies in the transitional region closer to the vibrational range of nuclei. Energy levels B(E2), B(M1) and the mixing ratios $\delta(E2/M1)$ and $X(E0/E2)$ for selected transitions were calculated in the framework of the proton-neutron interacting boson model (IBM-2). All results were compared with experimental data. Some experimental $X(E0/E2)$ ratios were calculated from available experimental data. Majorana parameters were found to have a great effect on the calculated energy levels of the 2_3^+ and 2_4^+, which indicate that both of them have mixed symmetry properties.

Key Words: Energy levels, transition probability, mixing ratios, interacting boson model

1. Introduction

The ^{134}Ba nucleus belongs to a transitional region of $A \approx 130$ whose characteristics have been explored with alternative models [1–3]. ^{134}Ba has been introduced as a possible candidate for critical symmetry E(5) of transition between vibrational U(5) nuclei and γ-unstable O(6) nuclei, by Casten et al. [4] and Da-Li et al. [5]. Recently, there has been an attempt to explore the structure of this isotope as a part of general study of nuclei near Z=52–62 [6]. Kumar et al. [7] investigated the level structure of $^{122-134}$ Ba isotopes, in the framework of IBM-1 and they concluded that the energy of 2_1^+ is increased in faster rate compared with the energy of 2_2^+ at $N = 72$ to 78.

The calculations of Gercekli̇oğlu [8] on transfer strength of the excited 0^+ states in ^{130,132,134}Ba isotopes, found that most of collective states are located at 2.159 MeV in the experimental data and at 1.71 MeV in model predictions. He used the Hamiltonian which includes monopole paring, quadrupole-quadrupole and spin-quadrupole interactions, to produced three 0^+ states in ^{134}Ba and gives a reasonable explanation for abundance of the 0^+ excitation in the low transfer strength of 0^+ states.

2. The model

In the present work, the IBM-2 states that the low lying collective state of even-even nuclei can be described by the interaction of s and $d-$bosons, carrying angular momentum $l=0$ and $l=2$, respectively. The IBM-2 Hamiltonian is written [9]

$$H=\varepsilon_d(n_{d\nu}+n_{d\pi})+\kappa(Q_\nu\cdot Q_\pi)+V_{\nu\nu}+V_{\pi\pi}+M_{\nu\pi}, \tag{1}$$

where the quadruple operator Q_ρ is

$$Q_\rho=[d_\rho^\dagger s_\rho+s_\rho^\dagger d_\rho]^{(2)}+\chi_\rho[d_\rho^\dagger d_\rho]^{(2)}, \quad \rho=\pi \text{ or } \nu, \tag{2}$$

and the Majorana term is given by

$$M_{\nu\pi}=\frac{1}{2}\xi_2([s_\nu^\dagger d_\pi^\dagger-d_\nu^\dagger s_\pi^\dagger]^{(2)}\cdot[s_\nu d_\pi-d_\nu s_\pi]^{(2)})-\sum_{k=1,3}\xi_k([d_\nu^\dagger d_\pi^\dagger]^{(k)}\cdot[d_\nu d_\pi]^{(k)}). \tag{3}$$

The terms $V_{\nu\nu}$ and $V_{\pi\pi}$, which correspond to the interaction between identical-bosons, are sometime included in order to improve the fit to the experimental energy spectra and they are expressed in the form

$$V_{\rho\rho}=\frac{1}{2}\sum_{L=0,2,4}C_L^\rho([d_\rho^+d_\rho^+]^{(L)}\cdot[\tilde{d}_\rho\tilde{d}_\rho]). \tag{4}$$

3. The IBM-2 parameters and energy levels

The NPBOS code [10] is used to obtain the calculated excitation energies by diagonalizing the Hamiltonian in equation (1). Isotope ^{134}Ba has $N_\pi=3$ and $N_\nu=2$, while rest of the parameters are treated as free parameters. Their values were estimated by fitting them to experimental level energies, as shown in Table 1. The parameters in the work of Turkan [11] have been used as starting parameters, with slight modification to fit the experimental data.

Table 1. The IBM-2 parameters compared with the parameters of Turkan [11].

Parameter	This work	Turkan
ε	0.868	0.800
κ	-0.18	-0.09
χ_π	-0.92	-1.2
χ_ν	$+0.92$	$+0.70$
CL_π	0.15, 0.15, 0.01	0.0, 0.0, 0.0
CL_ν	0.2, 0.12, 0.09	0.0, 0.0, 0.0
ξ_2	0.099	-
ξ_1=ξ_3	0.019	-

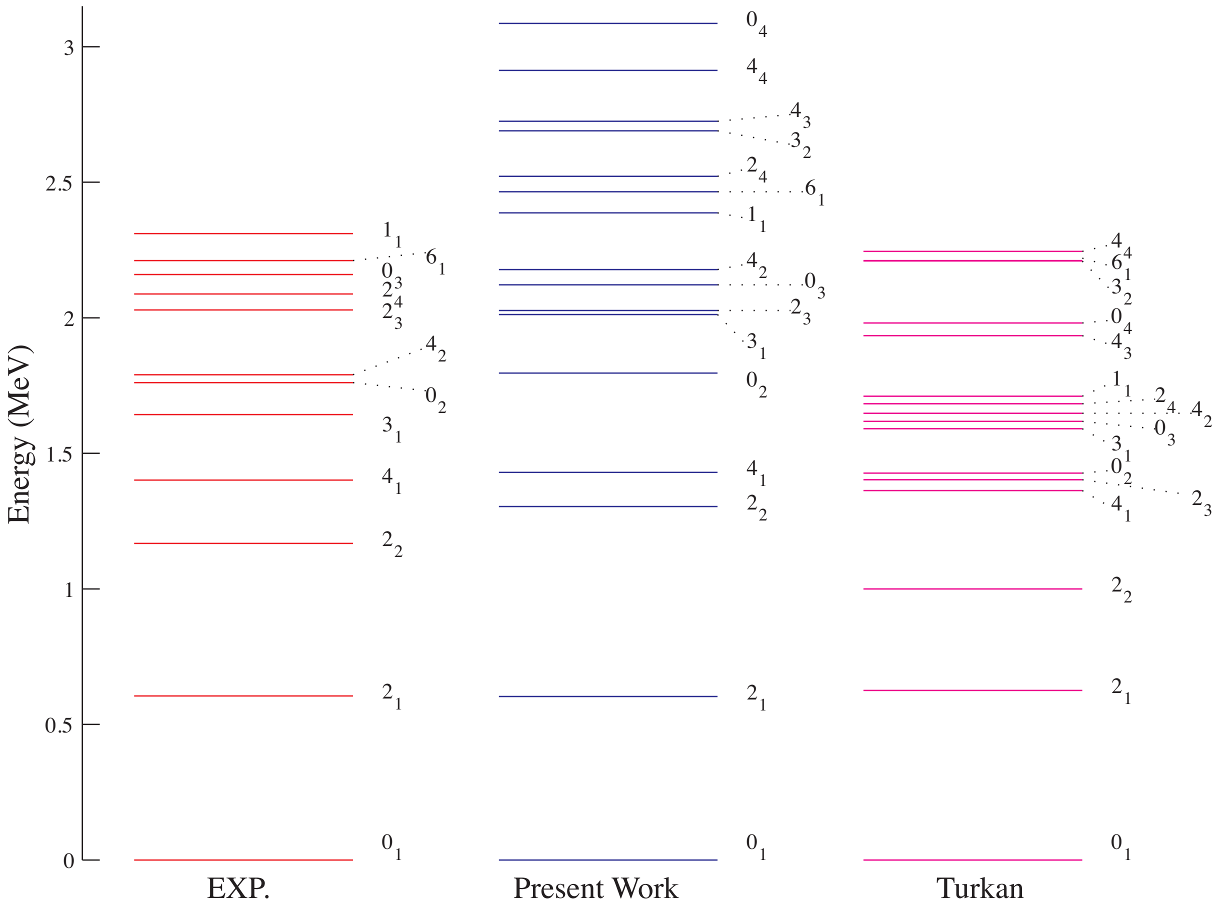

Figure 1. A comparison between the experimental energy levels from IBM-2 calculations for ^{134}Ba [12] in the present work and in Turkan [11].

The calculated energy levels are compared with the available experimental data [12] and shown in Figure 1. The energy level ratios were calculated and compared with experimental value and the prediction of E(5) symmetry[4] along with the results of Turkan [11]. This additional test for the IBM-2 parameters is presented in Table 2. The calculations we made are much closer to the experimental ratios as shown in the table.

Table 2. The energy ratios for states in ^{134}Ba are compared with experimental data [12], E(5) predictions [4], and from the work of Turkan [11].

$E_{4_1^+}/E_{2_1^+}$				$E_{2_2^+}/E_{2_1^+}$				$E_{0_2^+}/E_{2_1^+}$			
Exp.	This work	E(5)	Turkan	Exp.	This work	E(5)	Turkan	Exp.	This work	E(5)	Turkan
2.31	2.37	2.2	2.18	1.93	2.16	2.2	1.6	2.91	2.98	3.03	2.28

It is found that the present calculations fit very well most states in the scheme, except the case of γ-band members (2_2^+, 3_1^+ and 4_2^+ states), which were pushed higher.

In order to investigate the effect of Majorana interaction parameters ($\xi_1 = \xi_3 = 0.019$ MeV and $\xi_2 = 0.099$ MeV) on the energies of 2_2^+, 3_1^+, 2_3^+ and 4_2^+ states, the calculated energy is plotted as a function of ξ_2, all the other parameters were kept at their best-fit values, and presented the results of this part of calculation in Figure 2. The best fit value of ξ_2 used in the present work is given as doted vertical line.

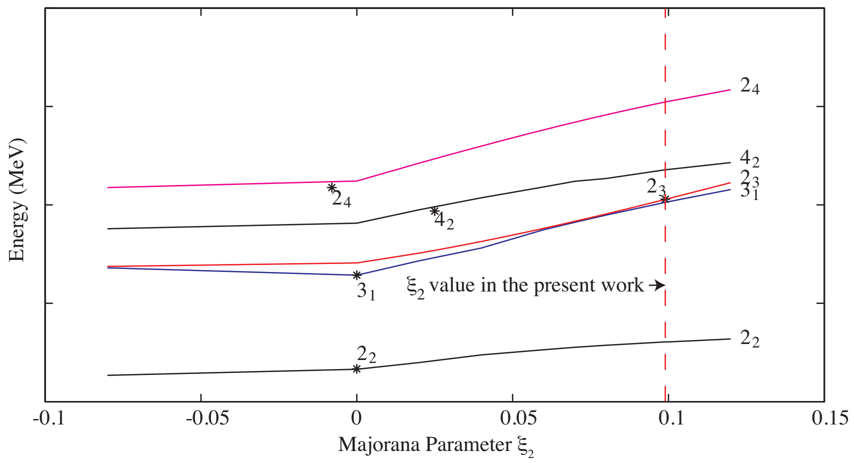

Figure 2. Variation of the 2_2^+, 3_1^+, 2_3^+ and 4_2^+ energies as a function of Majorana parameters ξ_2. The black asterisks are the experimental energy levels [12].

In order to produce the curves in Figure 2 we allowed ξ_2 to vary and kept $\xi_1 = \xi_3$ constant at 0.019. One can see that the energies of 2_3^+ and 2_4^+ states exhibit rapid response to the changes in the parameters compared to the others. This means that these states are good candidates for mixed symmetry states [13]. However, there are effects on the energies of 2_2^+, 3_1^+ and 4_2^+, as can be seen from the Figure 2. This is a good search method to clarify the mixed symmetry states. As hinted in Figure 2, we were unable to find one value of this parameter that fitted all the experimental values. Fazekas et al. [2] have suggested that the two states at 2.029 and 2.088 MeV should share the properties of the mixed symmetry state in this isotope.

4. Electromagnetic transitions

4.1. Electric quadrupole

The reduced electric quadrupole transitions probability $B(E2)$ were calculated using the operator

$$T^{(E2)} = e_\pi Q_\pi + e_\nu Q_\nu, \tag{5}$$

where Q_ρ is the same as in equation (2) and e_π and e_v are boson effective charges dependent on the boson number N_ρ; they can assume any value to fit the experimental results.

The two effective charges are taken to be $e_\pi = 0.08$ and $e_v = 0.20eb$, per the relation between them, as suggested by Hamilton et al. [14], for this mass region of nuclei. The results of the calculations are presented in Table 3. Looking through the table, one can easily recognize that our calculations reproduce the experimental data quite well.

Casten et al. [4] examined the $B(E2)$ ratios using IBM-1, and total number of bosons ($N = 5$) with $E(5)$ symmetry. Transition ratios $R_1 = \frac{B(E2;4_1^+\to 2_1^+)}{B(E2;2_1^+\to 0_1^+)}$, $R_2 = \frac{B(E2;2_2^+\to 2_1^+)}{B(E2;2_1^+\to 0_1^+)}$ and $R_3 = \frac{B(E2;0_2^+\to 2_1^+)}{B(E2;2_1^+\to 0_1^+)}$. have been estimated in the U(5), O(6) and SU(3) dynamical symmetry limits of IBM-1 and are: $R_1 = 2$, 1.4 and 1.6, respectively; $R_2 = 2$, 0.80 and 0.02, respectively; and $R_3 = 2$, 0.0 and 0.0, respectively. In the present work we computed the same ratios using IBM-2 calculations and results are listed in Table 4. The comparison of the quantities predicted in the present work showed a very good agreement with experimental data.

Table 3. The absolute $B(E2)$ values for ^{134}Ba compared with the available experimental data [12] and the calculations of Turkan [11].

$I_i^+ \rightarrow I_f^+$	$B(E2)e^2b^2$		
	Exp.	Present work	Turkan
$2_1^+ \rightarrow 0_1^+$	0.134(2)	0.126	0.132
$4_1^+ \rightarrow 2_1^+$	0.161(18)	0.165	0.187
$4_1^+ \rightarrow 2_2^+$	0.00066(66)	0.00024	0.108
$6_1^+ \rightarrow 4_1^+$	-	0.161	0.161
$2_2^+ \rightarrow 0_1^+$	0.0017(5)	0.0032	0.003
$2_2^+ \rightarrow 2_1^+$	0.141(41)	0.1036	0.203
$2_3^+ \rightarrow 0_1^+$	0.0018(6)	0.0073	0.009
$2_3^+ \rightarrow 2_1^+$	0.0045(20)	0.0023	0.279
$2_4^+ \rightarrow 2_1^+$	-	0.0037	0.109
$2_4^+ \rightarrow 0_1^+$	-	0.00015	0.0001
$0_2^+ \rightarrow 2_1^+$	-	0.0475	0.098
$0_3^+ \rightarrow 2_1^+$	-	0.00575	0.0006
$3_1^+ \rightarrow 2_1^+$	0.00098(34)	0.0097	0.003
$3_1^+ \rightarrow 2_2^+$	0.018(5)	0.145	0.439

Table 4. The B(E2) ratios compared with the experimental data and the predictions of $E(5)$ [4] along with Turkan [11].

$R_1 = \frac{B(E2;4_1^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)}$				$R_2 = \frac{B(E2;2_2^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)}$				$R_3 = \frac{B(E2;0_2^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)}$			
Exp.	This work	$E(5)$	Turkan	Exp.	This . work.	$E(5)$	Turkan	Exp.	This work	$E(5)$	Turkan
1.20	1.31	1.68	1.42	1.05	1.43	1.68	1.53	-	0.38	0.86	0.36

4.2. Magnetic dipole and $\delta(E2/M1)$ mixing ratio

The M1 transition operator can be written as;

$$T^{(M1)} = \left[\frac{3}{4\pi}\right]^{1/2} (g_\pi L_\pi^{(1)} + g_\nu L_\nu^{(1)}). \tag{6}$$

Here, g_π and g_ν are the boson g-factors in units of μ_N (which must be estimated), and $L^{(1)} = \sqrt{10}(d^+ x\tilde{d})^{(1)}$. The reduced $E2$ and $M1$ matrix elements were combined in the calculation of the mixing ratio $\delta(E2/M1)$ using the relation [15]

$$\delta(E2/M1) = 0.835 E_\gamma[MeV] \times \frac{T(E2)}{T(M1)}. \tag{7}$$

Sambataro et al. [16] suggested a total g-factor, which can be used to compute the 2_1^+ level g-factor:

$$g(2_1^+) = (g_\pi N_\pi + g_\nu N_\nu)/(N_\pi + N_\nu). \tag{8}$$

The value of the measured magnetic moment $\mu = 2g(2_1) = 0.86(10)\mu_N$ and the experimental mixing ratio $\delta(\frac{E2}{M1}; 2_2^+ \rightarrow 2_1^+) = -7.4(9)$ [15] were used to produce a suitable estimation for the boson gyromagnetic factors. These values are $g_\pi = 0.49\mu_N$ and $g_\nu = 0.39\mu_N$. The results of the calculations are listed in Table 5. These results exhibit disagreement in some cases, with one case showing disagreement in sign. However, it is a ratio between very small quantities and any change in the dominator that will have a great influence on the ratio. The large calculated value for $3_1^+ \rightarrow 2_1^+$ is not due to a dominate E2 transition, but may be under the effect of very small $M1$ component in the transition. Moreover, the large predicted value for transition $2_4^+ \rightarrow 2_1^+$ compared with experimental value may be related to high predicted energy level value of the IBM-2; $E(2_4^+) =$ 2.523 MeV, while the experimental value is 2.089 MeV. We are unable to bring the energy value of this state close to experimental value simply by changing the Majorana parameters.

Table 5. The calculated mixing ratios for the selected transitions in ^{134}Ba, according to available experimental data [15].

$I_i^+ \rightarrow I_f^+$	Transition Energy E_γ (MeV)	Mixing Ratio $\delta(E2/M1)$	
		Present work	Experiment
$2_2^+ \rightarrow 2_1^+$	0.563	-7.40	-7.4
$2_3^+ \rightarrow 2_1^+$	1.424	-1.24	-0.31
$2_4^+ \rightarrow 2_1^+$	1.483	$+25.32$	+0.02(5)
$3_1^+ \rightarrow 2_1^+$	1.038	$+31.10$	+0.83(14)
$3_1^+ \rightarrow 2_2^+$	0.475	-6.46	-6.0
$4_2^+ \rightarrow 4_1^+$	0.569	$+3.269$	+0.28(3)

4.3. Electric monopole and $X(E0/E2)$ ratio

The $E0$ transition occurs between two states of the same spin and parity by transferring the energy and zero units of angular momentum, and it has no competing gamma ray. The $E0$ transition is present when there is a change in the surface of the nucleus. For example, in nuclear models where the surface is assumed fixed, $E0$ transitions are strictly forbidden, such as in shell and IBM-1 models. Electric monopole transitions are completely under the penetration effect of atomic electrons on the nucleus, and can occur not only in $0^+ \rightarrow 0^+$ transition but also, in competition with gamma multipole transition, and depending on transition selection rules that may compete in any $\Delta I = 0$ decay such as a $2^+ \rightarrow 2^+$ or any $I_i = I_f$ states in the scheme. When the transition energy greater than $2m_oc^2$, monopole pair production is also possible.

The $E0$ reduced transition probability is written [17]

$$B(E0; I_i - I_f) = e^2 R^4 \rho^2(E0),\ I_i = I_f, \tag{9}$$

where e is the electron effective charge, $R = 1.2A^{1/3}$ [fm] is the nuclear radius and $\rho(E0)$ is the transition matrix elements. There are only limited cases of $\rho(E0)$ that can be measured directly. In most cases we have to determine the intensity ratio of $E0$ to the competing $E2$ transition, $X\left(\frac{E0}{E2}\right)$ [17], which is expressed

$$X\left(\frac{E0}{E2}\right) = \frac{B(E0; I_i - I_f)}{B(E2; I_i - I_{f'})}, \tag{10}$$

where $I_f = I_{f'}$ for $I_i = I_f \neq 0$, and $I_{f'} = 2$ for $I_i = I_f = 0$.

The $E0$ operator may be found by setting $l = 0$ on the IBM-2 operator, and is written

$$\rho_{if}(E0) = \frac{Z}{R^2} \sum \tilde{\beta}_{0\rho} \left\langle f \left| d_\rho^+ \times d_\rho \right| i \right\rangle, \tag{11}$$

The two parameters $\tilde{\beta}_{0\pi}$, $\tilde{\beta}_{0\nu}$ in equation (11) must be estimated.

The $E0$ transition also occurs in cases where the levels have the same spin and parity ($I_i = I_f \neq 0$). This means that the $E0$ transition competes with $E2$ and $M1$ components in those transitions. The evaluation of $E0$ strength requires an accurate knowledge of gamma-ray multipoles present in the transition. The experimental $X(E0/E2)$ values may be calculated from [18]:

$$X(E0/E2) = 2.54 \times 10^9 A^{4/3} \frac{E^5(MeV)}{\Omega_k} \alpha(E2) q^2 \quad , \tag{12}$$

where q^2 is defined by Lang et al. [15] as

$$q^2 = \frac{(1+\delta^2)\alpha_{\text{exp.}} - \alpha(M1)}{\alpha(E2)\delta^2} - 1 \quad , \tag{13}$$

where δ is the multipole mixing ratio of the transition with $\Delta I = 0, I_i = I_f \neq 0$, and the $\alpha's$ denote the experimental internal conversion coefficients. These measured values [19] are listed in Table 6 together with the calculated values of IBM-2.

Table 6. The $X(E0/E2)$ values compared with the available experimental data [19].

Initial level (MeV)	Transition energy (MeV)	$I_i^+ \rightarrow I_f^+$	$X(E0/E2)$	
			Exp.	Present work
1.168	0.563	$2_2^+ \rightarrow 2_1^+$	0.69*	0.696
1.766	1.766	$0_2^+ \rightarrow 0_1^+$	2.8(2)	2.801
2.029	1.424	$2_3^+ \rightarrow 2_1^+$	0.31*	0.104
2.029	0.861	$2_3^+ \rightarrow 2_1^+$	-	0.234
(2.159)	2.159	$0_3^+ \rightarrow 0_1^+$	-	5.314
(2.159)	0.393	$0_3 \rightarrow 0_2$	-	19.782
2.088	1.483	$2_4^+ \rightarrow 2_1^+$	21.31*	0.89
2.088	0.920	$2_4^+ \rightarrow 2_2^+$	-	8.376
2.337	2.337	$0_4^+ \rightarrow 0_1^+$	0.041(7)	0.0679
2.337	0.571	$0_4^+ \rightarrow 0_2^+$	-	16.779
2.379	2.379	$0_5^+ \rightarrow 0_1^+$	0.61(11)	8.349
2.488	2.488	$0_6^+ \rightarrow 0_1^+$	6.4(16)	10.165

*The experimental values were calculated using equations (12 and 13) and ref. [15]

The parameters in equation (11) can be predicted from the isotope shift [20], and since such data are not available for Ba isotopes, we calculate these parameters by fitting procedure into two experimental values in table 6. The parameters which were subsequently used to evaluate the X-values were; $\tilde{\beta}_{0\pi} = 0.051eb$ and $\tilde{\beta}_{0\nu} = -0.021eb$.

From the table, one can overall see a reasonable agreement with the experimental data. However, some values are in a poor agreement with the data.

5. Conclusions

In the present work, the energies of low lying collective levels $E2$ and $M1$ reduced transition probabilities for ^{134}Ba were calculated in the framework of IBM-2. The calculated energy levels of low lying states were well reproduce, though some discrepancies remain, especially in the high energy states. The $B(E2)$ between the ground band states, the quasi-γ and quasi-β band states are also described. We examined the existence of mixed symmetry states in ^{134}Ba, as swell as Majorana parameters. An equal emphasis was placed on the reproduction of $\delta(E2/M1)$ and $X(E0/E2)$ ratios. These ratios are important for nuclear structure and the model predictions due to their sensitivity for the nuclear shape. We conclude that more experimental work is needed to clarify the band structure and investigate an acceptable degree of agreement between the predictions of the models and the experimental data.

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