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Spin assignment and behavior of superdeformed bands in A∼150 mass region

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Abstract: The smoothed experimental dynamical moment of inertia $J^{(2)}$ values were fitted with a theoretical version using the Harris three parameter formula in even powers of angular frequency ω, derived for results from the cranking model. The expansion parameters were adjusted by using a computer simulated search program. The best expansion parameters from the fit were used to assign the spins of the superdeformed (SD) rotational bands (RB) by integrating the calculated $J^{(2)}$. The data set includes 23 RB's in 11 SD nuclei, which show no evidence of either irregular behavior near the bottom of the bands or abrupt angular momentum at low rotational frequency in the mass region ranging from $A = 142$ to $A = 154$. We used the differences of angular momenta at constant frequency as effective alignment. The relative properties of superdeformed rotational bands (SDRB's) are analyzed in terms of the effective alignment of the valence nucleons. The effective alignment is a powerful tool to assign the configurations, to select the identical bands as well as to predict new SD bands from other combination of the orbitals. The $\Delta I = 2$ energy staggering observed in 3 of our selected SDRB's are also described from a smooth reference representing the finite difference approximation to the fourth derivative of the γ-ray transition energies.

Key words: Superdeformed rotational bands, nuclear effective alignment, identical bands, nuclear staggering

1. Introduction

Superdeformed rotational bands (SDRB's) in nuclei occur when there are large gaps in the energy level spectrum of the various single-particle orbitals. In the region around mass number $A \sim 150$, the gap corresponds to deformed prolate nuclear shape whose axis lengths are in the ratio $2 : 1 : 1$. The first SD bands found in the region $A \sim 150$ was in ^{152}Dy [1], but several more cases are now known [2–4]. The spectroscopic properties of the different bands in this region can generally be understood from the occupation of the highest spin orbitals $N = 6$ and $N = 7$ protons and neutrons.

For all SD bands, gamma ray energies are unfortunately the only spectroscopic information universally available. The spin assignments for SD bands represent the most difficult and unsolved problem. Several theoretical procedures for assigning spins and studying the structure properties of SD nuclei have been proposed [5–19].

One of the most interesting discoveries in the $A \sim 150$ region was the existence of SD bands in both ^{151}Tb (SD-2) and ^{152}Dy (SD-1) with identical gamma ray energies [20]. The energies are the same to within $2\ KeV$ over the whole range of the bands (16 transitions). The dynamical moments of inertia for both bands

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are also identical. The population of high spin orbitals is the same for both bands, $\pi\, 6^4$ and $\nu\ 7^2$. Also, it has been demonstrated that some SDRB's with nuclear spins differing by two may split into two branches [21–24]. This phenomenon is called $\Delta I = 2$ staggering or $\Delta I = 4$ bifurcation. The amplitudes of bifurcation are very small. In order to investigate the structure of SDRB's we will base our interpretation on the relative properties of the bands. Application are made to some SDRB's in the Gd / Tb / Dy nuclei.

The motivation of the present paper is to highlight some theoretical aspects that are used to describe the properties of SD nuclei, in particular to introduce a method to assign and to concern the origin of $\Delta I = 2$ staggering and identical bands in SDRB's in A$\sim$150 mass region.

The paper is organized as follows. In section 2 three parameters formula described by Harris for rotational bands is presented and is used to predict the spins of the band heads and to examine the main properties of the SD rotational bands. Understanding the configurations of SDRB's by using the relative alignment is presented in section 3. In section 4 the relative alignment is used to select pairs of identical SD bands. Section 5 is devoted to explore the $\Delta I = 2$ staggering in $A \sim 50$ region. In section 6 we present calculations and obtained results for SD bands in Gd/Tb/Dy nuclei.

2. Spin Assignment of SDRB's

For the SD bands, gamma ray energies are unfortunately the only spectroscopic information universally available. Spin assignment for RB is one of the most difficult and still unsolved problems in the study of nuclear superdeformation. This is due to the difficulty of establishing the de-excitation of a SD band into known yrast states. Obviously, unless the direct transition from the SD band to the yrast band is measured, it is impossible to be sure how many units of angular momentum have been carried away during the de-excitation, therefore, a $\pm 2\,\hbar$ uncertainly is expected in general. Several related fitting procedures to assign the spins of SDRB's in terms of the observed gamma ray energies have been proposed [19]. The most important approach used to assign spin is the Harris parameterizations [25], grounded on extension of the cranking model.

Cranking analysis leads to the level energies E as a function of rotational frequency ω in the following Harris formula for third order cranking:

$$E = \frac{1}{2}\alpha\omega^2 + \frac{3}{4}\beta\omega^4 + \frac{5}{6}\gamma\omega^6. \tag{1}$$

The standard way to analyze SD bands is to consider the dynamical moment of inertia $J^{(2)}$ because it does not require any knowledge of the spin value which is not determined experimentally.

The corresponding expression for $J^{(2)}$ is given by

$$\begin{aligned} J^{(2)} &= \tfrac{1}{\omega}\tfrac{dE}{d\omega} \\ &= \alpha + 3\,\beta\ \omega^2 + 5\ \gamma\omega^4. \end{aligned} \tag{2}$$

The expansion parameters α, β, γ, which result from fitting $J^{(2)}$ with the experimental values, are used to determine the spin from the expression

$$\begin{aligned} I &= \textstyle\int dw J^{(2)} \\ &= \alpha\omega\ + \beta\omega^3 + \gamma\ \omega^5. \end{aligned} \tag{3}$$

The corresponding expansion for the kinematic moment of inertia $J^{(1)}$ is given from

$$\begin{aligned} J^{(2)} &= \tfrac{dI}{d\omega} \\ &= \omega \tfrac{dJ^{(1)}}{d\omega} + J^{(1)}, \end{aligned} \tag{4}$$

which turns out to be

$$J^{(1)} = \alpha + \beta\,\omega^2 + \gamma\,\omega^4. \tag{5}$$

3. Understanding of SDRB's using relative alignment

The relative alignment of two bands is defined as the difference in the spin of the two bands of constant rotational frequency ω.

Recalling definition of the dynamical moment of inertia for odd and even mass nuclei,

$$J_o^{(2)} = \frac{dI_o}{d\omega} \tag{6}$$

$$J_e^{(2)} = \frac{dI_e}{d\omega}, \tag{7}$$

the odd-even difference in dynamical moment of inertia $\delta J_{oe}^{(2)}$ is given by

$$\begin{aligned} \delta J_{oe}^{(2)} &= J_o^{(2)} - J_e^{(2)} \\ &= \tfrac{d(I_o - I_e)}{d\omega} \\ &= \tfrac{di}{d\omega}, \end{aligned} \tag{8}$$

where i is the relative alignment of the two bands. Combining equations (6)–(8) results in

$$\frac{\delta J_{oe}^{(2)}}{J_o^{(2)}} = \frac{di}{dI_o}, \tag{9}$$

which implies that the fractional change in the dynamical moment of inertia of the odd-A nucleus relative to its even-even neighbor is simply the slope of the i versus I_o curve. In case where the fractional change in dynamical moment of inertia is dependent of spin or rotational frequency, the relative alignment becomes a linear function of spin.

4. Identical bands in superdeformed nuclei

In our analysis we employ the simple approximation in which the independent particle motion of one or more valence particles with angular momentum j is coupled to rotating deformed core with angular moment R and moment of inertia $J^{(1)}$, forming the total angular momentum $I = R + j$. If the coupling of the odd particle to the core is much stronger than the perturbation of the single particle motion by the Coriolis interaction, the odd particle will follow the core deformation adiabatically. This strong coupling limit is expected to work particularly well for SD nuclei where the splitting of the Nilsson levels is large and the Coriolis interaction is small.

The energy spectrum of an odd-A nucleus with axial symmetry in first order perturbation theory can be given by the relation

$$JE(I) = E_o(1/2) + \frac{1}{2J^{(1)}}[I(I+1) + a(-1)^{I+1/2}(I+1/2)\delta_{k,1/2}]. \tag{10}$$

γ-ray energy for $\Delta I = 2$ in band transition takes the form

$$\begin{aligned} E^o_\gamma(I) &= E(I) - E(I-2) \\ &= \tfrac{1}{2J^{(1)}}[4I - 2 + 2a(-1)^{I+1/2}\delta_{k,1/2}]. \end{aligned} \tag{11}$$

Here, k is the projection of j onto the symmetry axis and a is the decoupling parameter.

From expression (11), one can obtain special relations between the γ-ray energies of the nucleus and the core as follows:

i. For strong coupled bands (CB) $(a = 0, k \neq 1/2)$ The signature splitting disappears and the transition energies follow the simple rule

$$\begin{aligned} E^o_\gamma(R \pm 1/2) &= \tfrac{1}{2J^{(1)}}[4\,R \pm 2 - 2] \\ &= \tfrac{3}{4}\,E^c_\gamma(R) + \tfrac{1}{4}E^c_\gamma(R \pm 2) \end{aligned} \tag{12}$$

or

$$\frac{1}{2}\,[E^o_\gamma(R + 1/2) + E^o_\gamma(R - 1/2)] = E^c_\gamma(R)$$

ii. For $k = 1/2$ band

$$E^o_\gamma(R \pm 1/2) = \frac{1}{2J^{(1)}}[\,4\,R \pm 2 \mp 2\,a - 2\,]. \tag{13}$$

Transitions from both signature form degenerate doublets with $a = 1$ case giving identical to those of the core, *i.e.* twin bands (TB)

$$\begin{aligned} E^o_\gamma(\,R \pm 1/2\,) &= \tfrac{1}{2\,J^{(1)}}[\,4\,R - 2\,] \\ &= E^c_\gamma(R), \end{aligned} \tag{14}$$

while the $a = -1$ case has energies midway between those of adjacent transition in the core, *i.e.* indirect twin bands (ITB)

$$\begin{aligned} E^o_\gamma(R \pm 1/2) &= \tfrac{1}{2J^{(1)}}[\,4\,R \pm 4 - 2\,] \\ &= \tfrac{1}{2}\,[E^c_\gamma(R) + E^c_\gamma(R \pm 1/2\,)]. \end{aligned} \tag{15}$$

If the decoupling parameter a takes an another values $(a \neq 0, a \neq \pm 1)$, the previous rules can be generalized in the form

$$E^o_\gamma(I) = x\,E^c_\gamma(R) + (\,1 - x\,)\,E^c_\gamma(\,R + 2\,). \tag{16}$$

5. $\Delta I = 2$ Staggering in SDRB's

It has been found that some SD nuclear bands show unexpected $\Delta I = 2$ staggering effects in the γ-ray energies [21–24]. The curve found by smoothly interpolating the band energy of the spin sequence $I = I_o + 4n$

$(n = 0, 1, 2, ...)$ is somewhat displaced from the corresponding curve of the sequence $I = I_o + 4\,n + 2$. The magnitude of the displacement in the gamma transition energies is found to be in the range of some hundred eV to a few KeV.

To explore more clearly the $\Delta I = 2$ staggering, for each band the derivation of the γ-ray energies from a smooth reference, $\Delta^4 E_\gamma$, was determined by calculating the finite difference approximation to the fourth derivative of the γ-ray energies at a given spin, $d^4E_\gamma/\,dI^4$. This smooth reference is given by [21] the relation

$$\begin{aligned}\Delta^4 E_\gamma(I) = \;& \tfrac{1}{16}\,[\,E_\gamma(\,I - 4\,) - 4\,E_\gamma(I-2\,) + 6\,E_\gamma(I) \\ & -4\,E_\gamma(I+2\,) + E_\gamma(I+4\,)].\end{aligned} \tag{17}$$

This formula includes five consecutive transition energies E_γ and is denoted by a five-point formula. We say that $\Delta I = 2$ staggering is observed if the parameter $\Delta^4 E_\gamma(I)$ exhibits alternating signs with increasing spin or rotational frequency ω.

6. Calculations and analysis

In our calculations, all γ-ray transition energies were assumed to be $I_o, I_o + 2\,, I_o + 4\,, ...\ (\Delta\, I = 2)$. The spins of the 23 SDRB's in the $A : 150$ mass region are predicted by fitting the experimental dynamical moment of inertia $J^{(2)}_{exp}(i)$ with the expression calculated from the Harris three parameters formula. The optimized expansion parameters in question were adjusted by using a computer simulated search program in order to obtain a minimum root mean square deviation between the calculated and the experimental dynamical moment of inertia. The spin of the band head is taken as the nearest integer or half integer of the fitted I_o. The Table summarizes the Harris parameters α, β, γ obtained by best fitting procedure and the correct band head lowest level spin I_o and also the lowest gamma transition energies $E_\gamma(I_o + 2 \rightarrow I_o)$ for all the above selected SDRB's.

The systematic behavior of dynamical moment of inertia $J^{(2)}$ seems to be very useful to the understanding of the properties of the SD bands. The extracted $J^{(2)}$ at the assigned spin values are calculated as a function of rotational frequency $\hbar\omega$ and are illustrated in Figures 1 and 2. From these figures, it is seen that the agreement between calculated (solid lines) and the values extracted from the observed data (solid circle with error bars) is excellent. Because of the large single particle SD gaps at $Z = 66$ and $N = 86$, the nucleus ^{152}Dy is expected to be a very good doubly magic SD core. Moreover, the pairing correlation in the SD band of ^{152}Dy is very weak, which leads to a rigid like rotational pattern. The new data on neighboring nuclei in the region $A \sim 150$ allow a test of the stability of this core with respect to the addition of a valence particle or hole.

In our analysis we will use the differences of calculated spins at constant frequency which is a measure of the contribution from different Nilsson orbitals as effective alignment i_{eff}. The contribution of i_{eff} comes from the alignment of the orbital being occupied when going from even A to odd $(A{+}1)$. In our calculations, we considered the third and fourth $N{=}6$ proton orbitals $\pi\,6_3$, $\pi\,6_4$ and the second $N = 7$ neutron orbitals $\nu 7_2$. The orbitals, being filled with increasing particle number for Gd/Dy superdeformed bands, are mainly down-sloping, thus leading to increased deformation. The large slopes on $i(\omega)$ in yrast SD bands of $N = 85$, 86 nuclei seen in Figure 3 are due to the occupation of the $\pi 6_3$ and $\nu 7_2$ orbitals, while in ^{152}Dy the $\pi 6_4$ level is also occupied and this leads to a more constant $i(\omega)$. A plot of $i(\omega)$ for the excited SD band SD-2 in ^{151}Tb gives a curve that is practically constant and which clearly follows the $i(\omega)$ curve traced out by the yrast SD band in ^{152}Dy, but which is very different from the yrast SD band in ^{151}Tb. It is concluded from these comparisons that the excited SD band in ^{151}Tb has the same N orbitals occupied as the yrast SD band in ^{152}Dy.

Table. Spin proposition I_0 and adopted best parameters α, β, γ Harris parameterizations for the selected SD rotational bands in the $A \sim 150$ mass regions.

Bands	α ($\hbar^2 MeV^{-1}$)	β ($\hbar^2 MeV^{-3}$)	γ ($\hbar^2 MeV^{-5}$)	E_γ KeV	I_0 ($\hbar$)
^{142}Sm (SD-1)	67.9476	-2.5313	-0.44976	679.7	25
^{144}Gd (SD-2)	64.1064	7.7816	-6.24926	774.5	25
^{148}Gd (SD-1)	93.0826	-30.8826	16.18054	699.9	29
(SD-5)	91.6343	-8.75047	3.75808	853.70	38
(SD-6)	100.9703	-37.6449	20.85396	802.2	38
^{150}Gd (SD-1)	140.0079	-73.531	36.93994	815.0	30
(SD-5)	93.1604	-30.54297	18.77032	712.5	28
(SD-7)	95.7162	-33.03713	19.33374	733.2	29
(SD-8)	94.5086	-29.58147	16.1051	711.2	28
(SD-9)	93.3919	-34.66237	20.16158	800.6	31
(SD-10)	97.8442	-32.6218	18.38964	827.8	33
^{152}Dy (SD-1)	92.5951	-14.51377	9.6349	602.4	24
(SD-3)	80.0752	-0.0067	-7.63074	793.0	36
(SD-5)	72.9304	10.04747	-5.15676	642.1	26
(SD-6)	92.4278	-8.6054	9.10208	761.5	32
^{154}Dy (SD-1)	91.5166	-14.04857	11.37638	701.7	30
odd-A Nuclei					
^{147}Gd (SD-2)	84.0128	-10.1879	-1.73307	730.21	30.5
^{149}Gd (SD-1)	85.4885	-8.7058	-0.52232	617.8	23.5
(SD-2)	112.247	-22.81526	-10.9783	858.5	31.5
^{151}Tb (SD-1)	93.1017	-5.9703	-2.79292	726.5	28.5
(SD-2)	87.8335	-4.6605	-2.4597	602.1	24.5
odd-odd Nuclei					
^{151}Tb (SD-1)	79.7532	-4.1087	-0.1712	596.8	24

From plots in Figure 3, one sees that all $N = 86$ isotones have identical super shell structure, but generally different alignment configurations. These nuclei are more sensitive to the high-j alignment, therefore, even for the same nucleus its excited bands may not be the same as its yrast band. The excited band of ^{151}Tb (^{150}Gd) has a proton hole. The SD shell configurations for $N = 86$ isotones relative to the core ^{152}Dy $(\pi 6_4 \nu_2)$ are:

$$
\begin{array}{rcll}
^{150}Gd \text{ (yrast)} & : & \pi(3)^{\bar{0}}[(4)^{10}(5)^{12}] & (i_{13/2})^2 \\
\text{(excited)} & : & \pi(3)^{\bar{1}}[(4)^{10}(5)^{12}] & (i_{13/2})^3 \\
^{151}Tb \text{ (yrast)} & : & \pi(3)^{\bar{0}}[(4)^{10}(5)^{12}] & (i_{13/2})^3 \\
\text{(excited)} & : & \pi(3)^{\bar{1}}[(4)^{10}(5)^{12}] & (i_{13/2})^4 \\
^{152}Dy \text{ (yrast)} & : & \pi(3)^{\bar{0}}[(4)^{10}(5)^{12}] & (i_{13/2})^4 \\
\text{(excited)} & : & \pi(3)^{\bar{1}}[(4)^{10}(5)^{12}] & (i_{13/2})^5.
\end{array}
$$

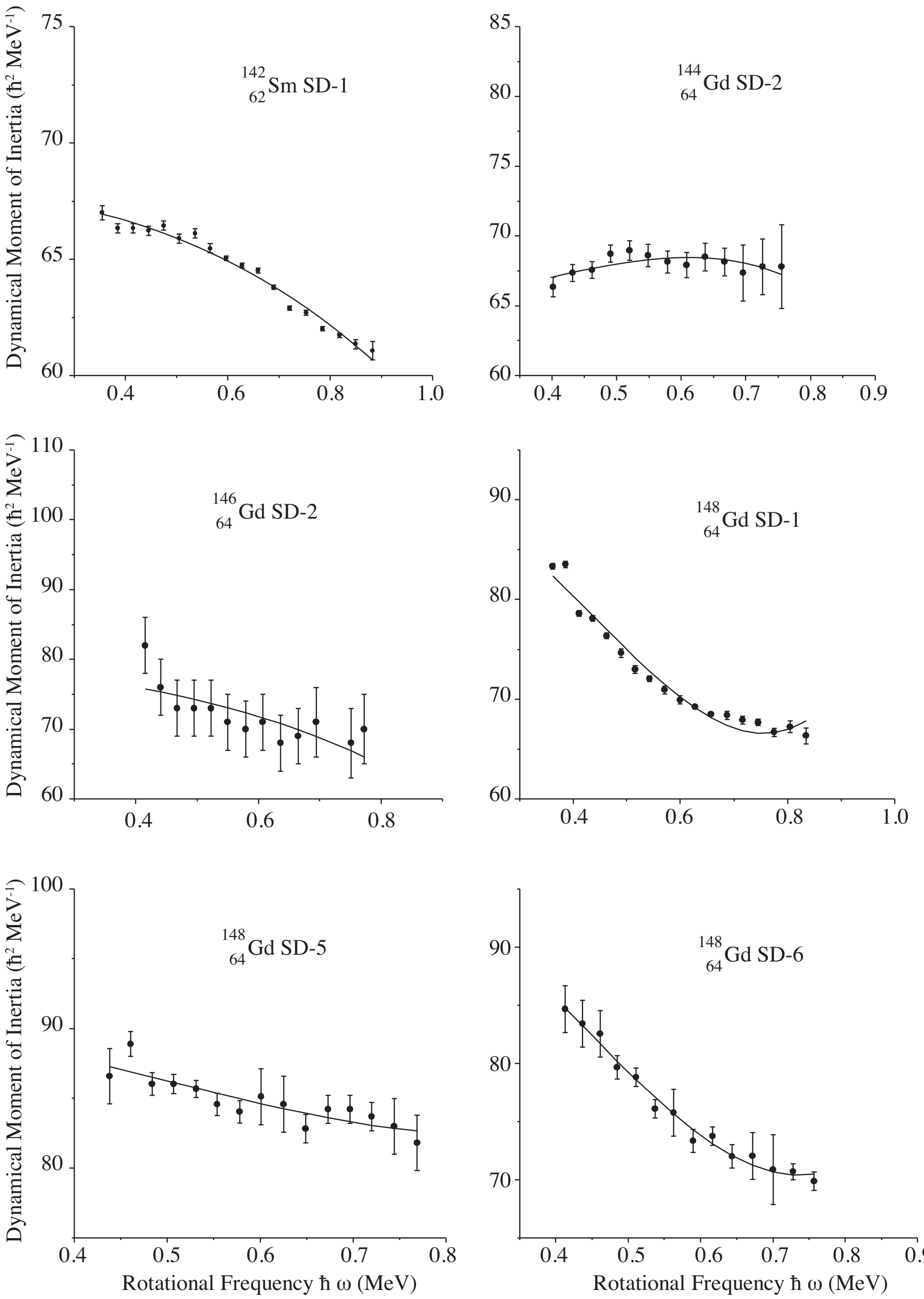


Figure 1. Dynamical moment of inertia $J^{(2)}$ as a function of rotational frequency $\hbar\omega$ for even-even SD nuclei in region $A \sim 150$. The solid curve represents the calculated results extracted from Harris parameterization with best fit parameters. The experimental solid circles with error bars are presented for comparison.

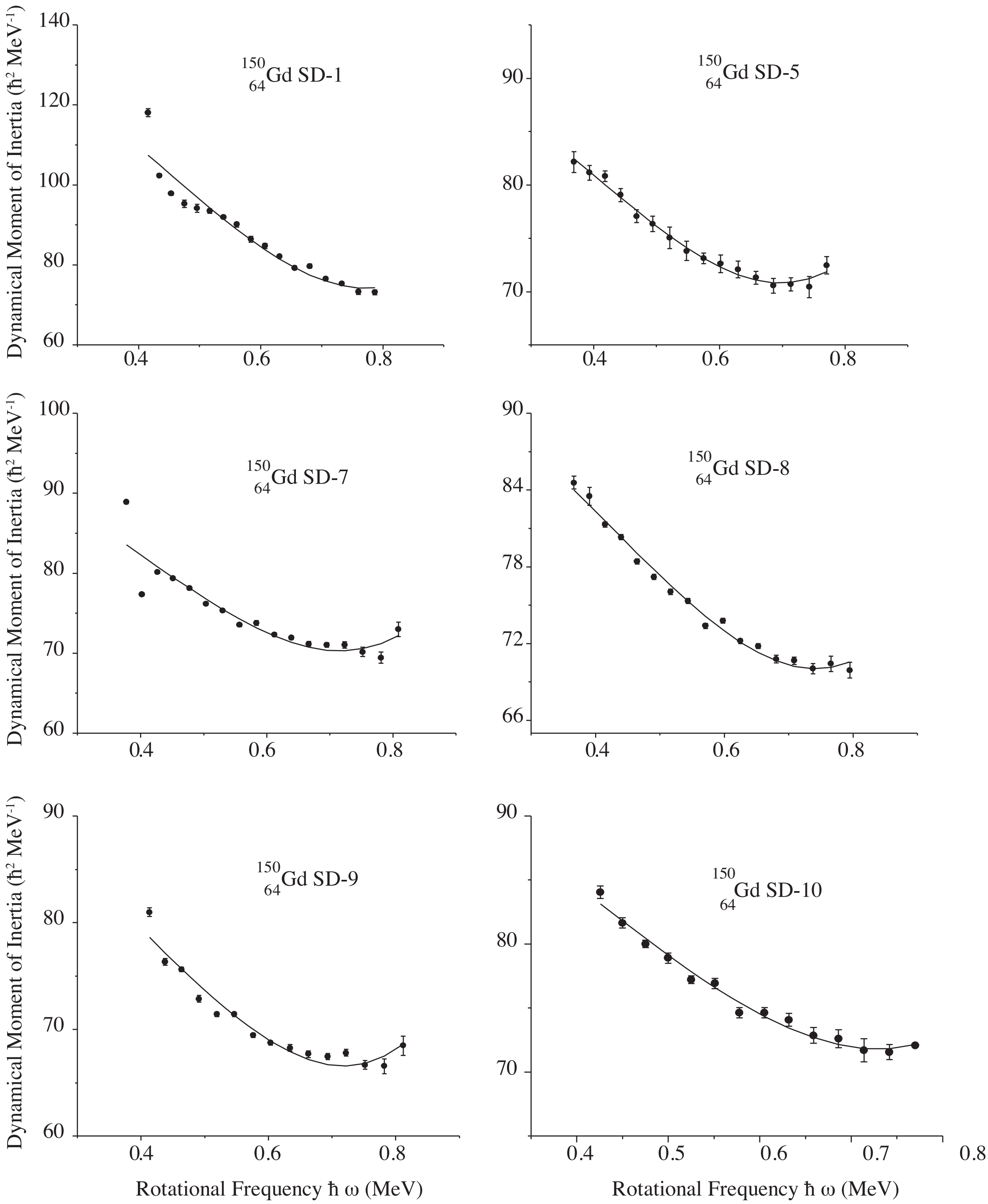
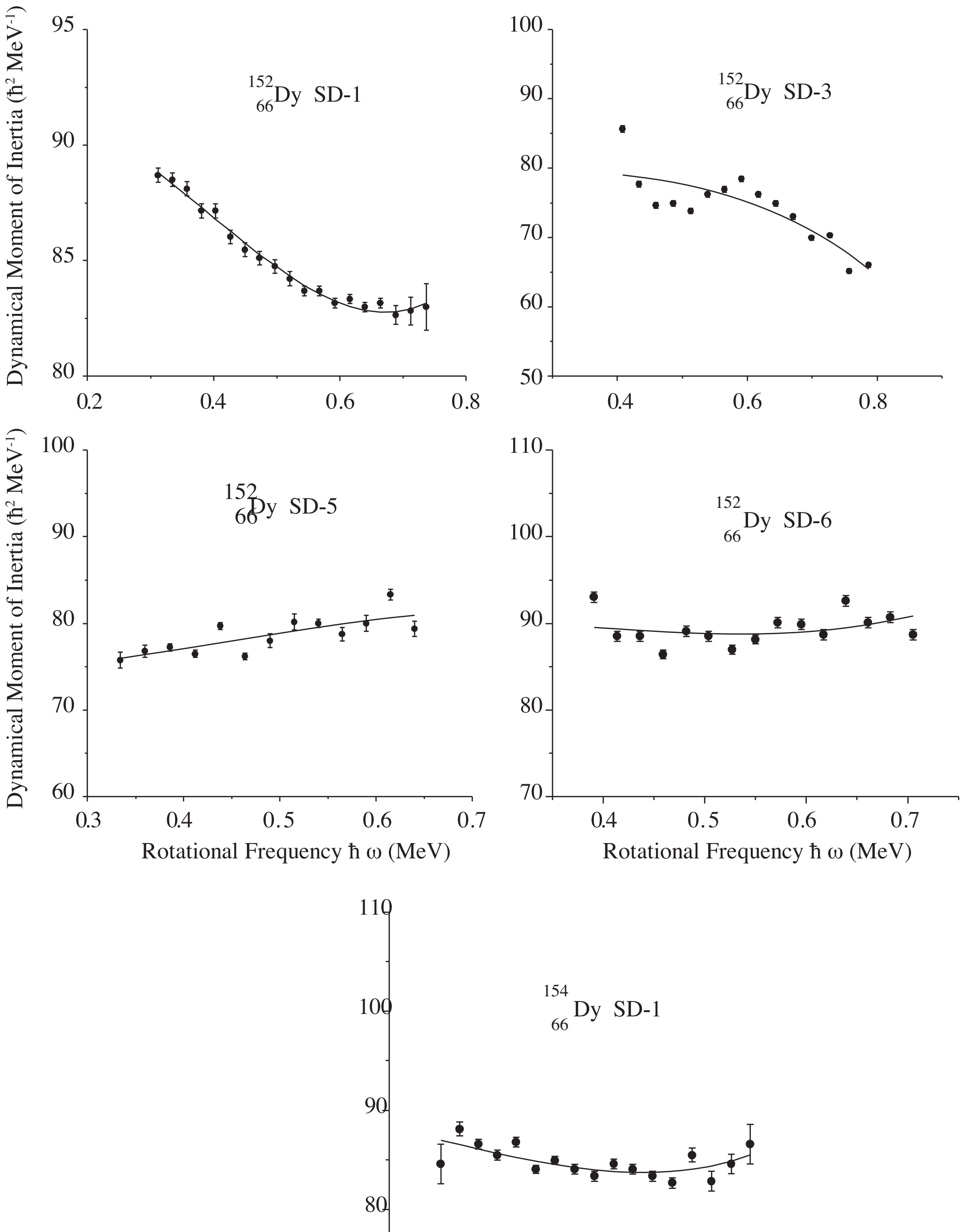


Figure 1. Continued.



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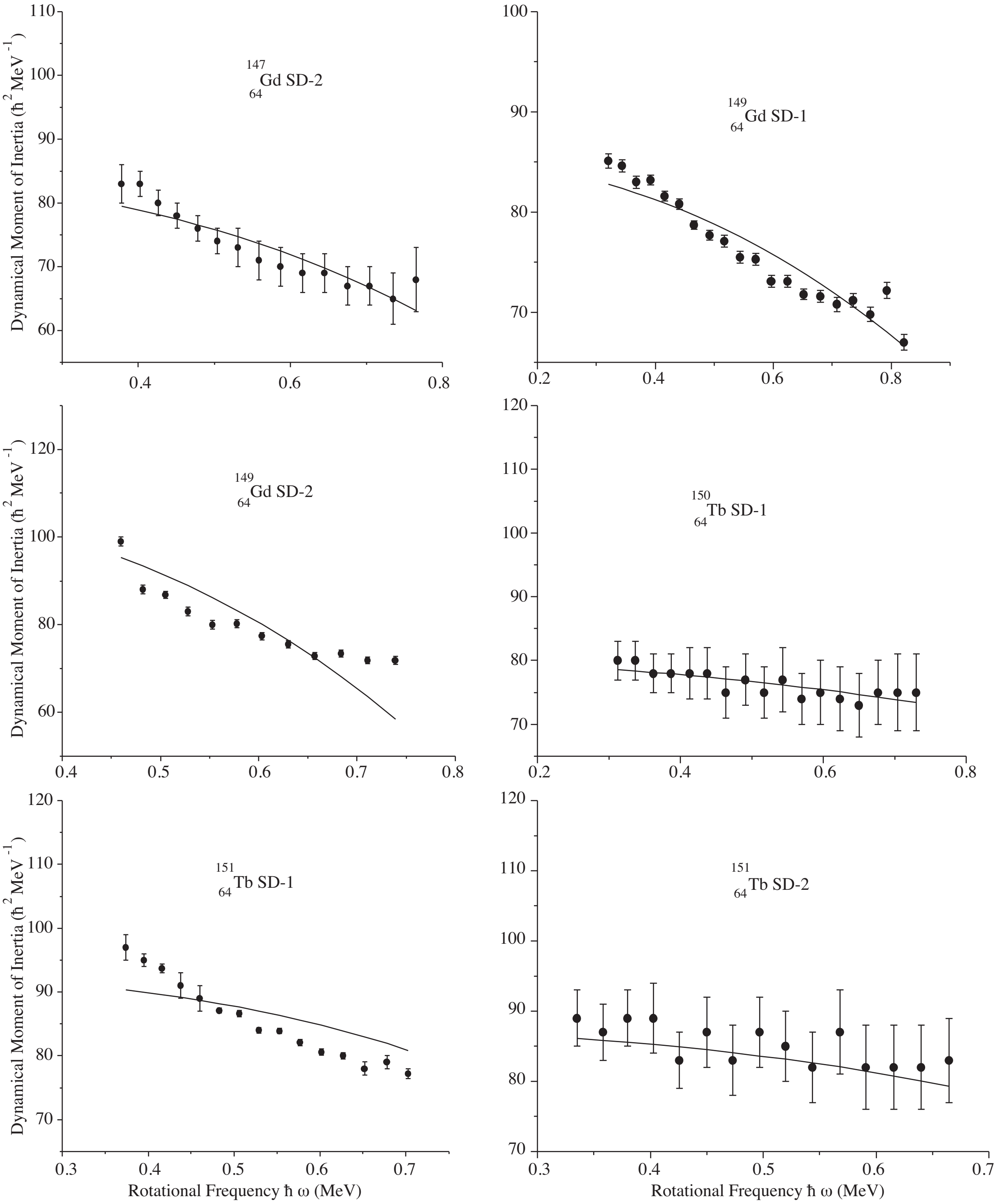

Figure 2. Dynamical moment of inertia $J^{(2)}$ versus rotational frequency $\hbar\omega$ for odd-A SD nuclei in region $A \sim 150$ and for the odd-odd nucleus ^{151}Tb.

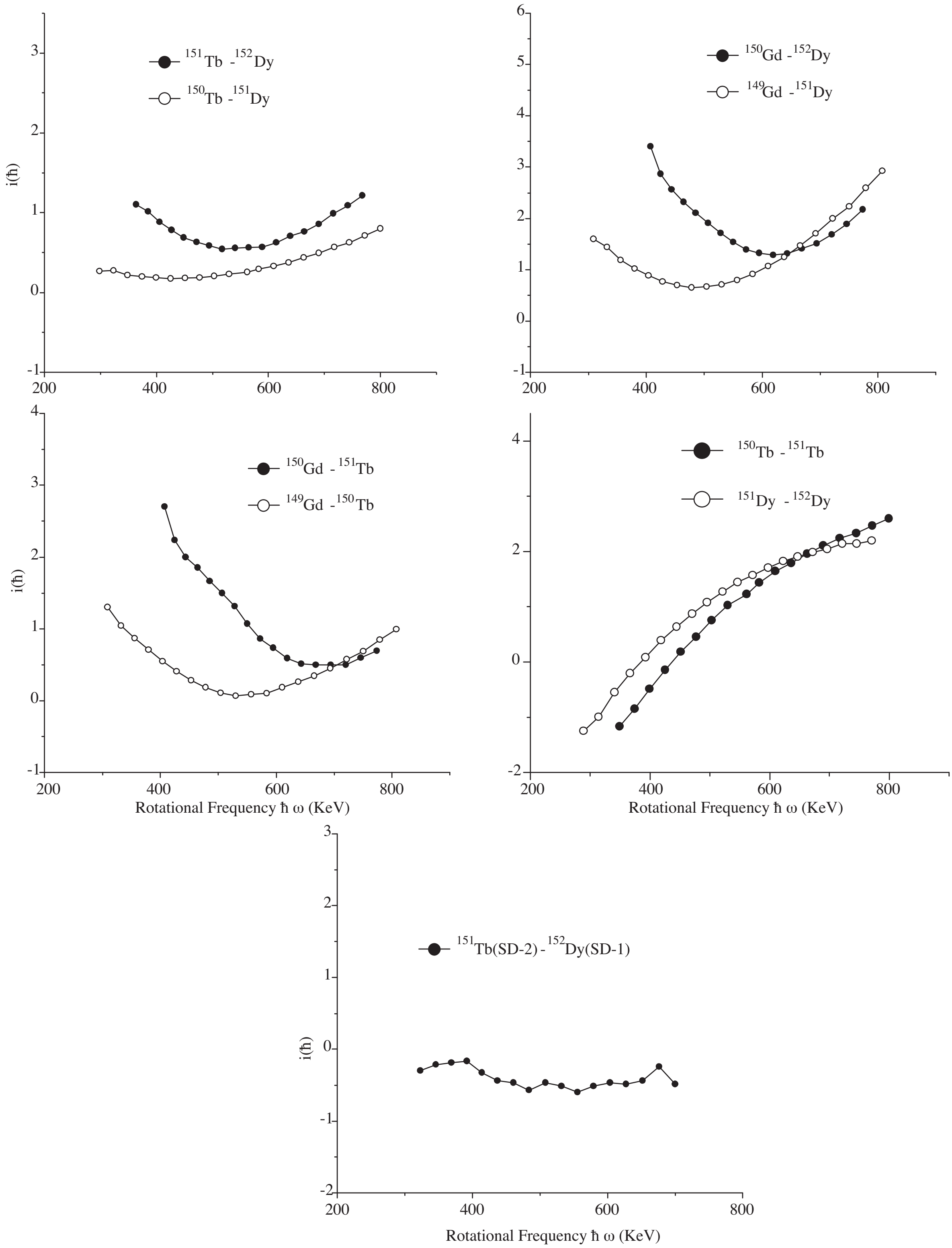

Figure 3. Effective alignment i versus rotational frequency $\hbar\omega$ for $N = 86$ SDRB's in Gd/Tb/Dy nuclei for three different orbitals.

The neutron configuration is $\nu(4)^{14}\ [(5)^{14}(6)^{16}]\ (j_{15/2})^2$.

Identical bands are two bands which have essentially identical transition energies (difference in energies up to 2.0–3.0 keV) and thus essentially identical. The excited SD band ($\beta_2 \approx 0.6$) in ^{151}Tb has a band of levels whose transition energies are essentially equal to those of the yrast SD band in ^{152}Dy. The observed difference is much smaller than the approximate 1% variation one would expect in the limit of a rigid body moment of inertia because of their mass difference. More dramatic is the difference compared to the $10-15\%$ increase in moment of inertia expected and previously observed in neighboring odd-A nuclei compared to an even-even one because of the reduction of the pairing correlations by the odd particle. In these bands the role of the 1/2 + [301] orbital is believed to be important. This orbital is at the Fermi surface for the N=86 isotones and it is thought that the excited bands in these nuclei are based on a proton excitation 1/2 + [301] orbital to the low-lying intruder state 3/2 + [651].

The remarkable similarities between the excited SD bands and the previously observed yrast SD bands in the Z + 1 isotones are further illustrated when direct comparisons of the γ-ray energies are made. Figure 4 shows the difference between the γ-ray energies observed in the identical bands in the pairs of nuclei ^{151}Tb (SD-2) / ^{152}Dy (SD-1). It can be seen that on average the deviation is less than 1.5 KeV over the whole energy range. This identical band has been associated with 1/2 + [301] hole in the ^{152}Dy core.

Another result from the present work is the observation of $\Delta I = 2$ staggering effects in the γ-ray energies in ^{148}Gd (SD-1), ^{148}Gd (SD-6) and ^{149}Gd (SD-1). The two sequences for spins $I = 4j$, $I = 4j + 1$ ($j = 0, 1, 2, ...$) and $I = 4j + 2$ ($j = 0, 1, 2, ...$) are bifurcated. A few theories have been advanced to explain the $\Delta I = 2$ staggering [21–24]. For SD-1 in ^{149}Gd, deviation of the γ-ray energies from a smooth reference ΔE_γ was determined by calculating the fourth derivative of the γ-ray energies $\Delta E_\gamma(I)$ of a given spin. The yrast bands in ^{148}Gd and ^{149}Gd and the excited band in ^{148}Gd show an unexpected staggering, $\Delta^4 E_\gamma$, in the

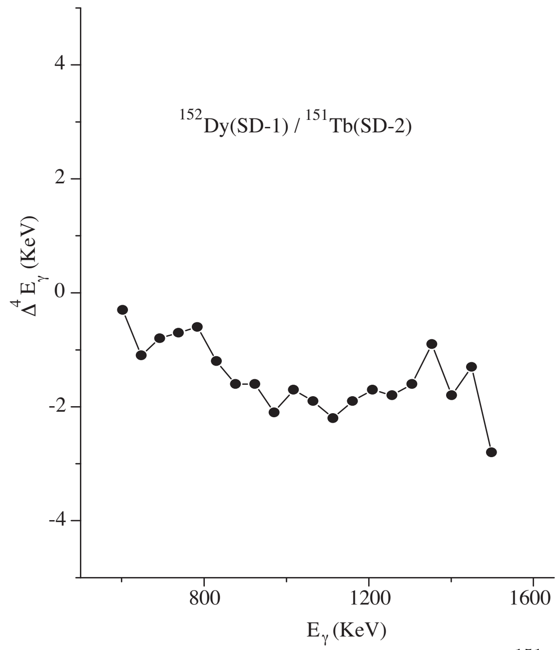

Figure 4. The difference in γ-ray energies ΔE_γ between the excited band in ^{151}Tb and the yrast band in ^{152}Dy.

γ-ray energies at a given spin. Plots of $\Delta^4 E_\gamma$ versus rotational frequency $\hbar\omega$ are shown in Figure 5. Until now, only several SD bands have been identified to exist the transition from SD levels to ND levels [26–28]. For ^{152}Dy (SD-1) our band head spin is $I_0 = 24^+$, which is consistent with the experimental value [28].

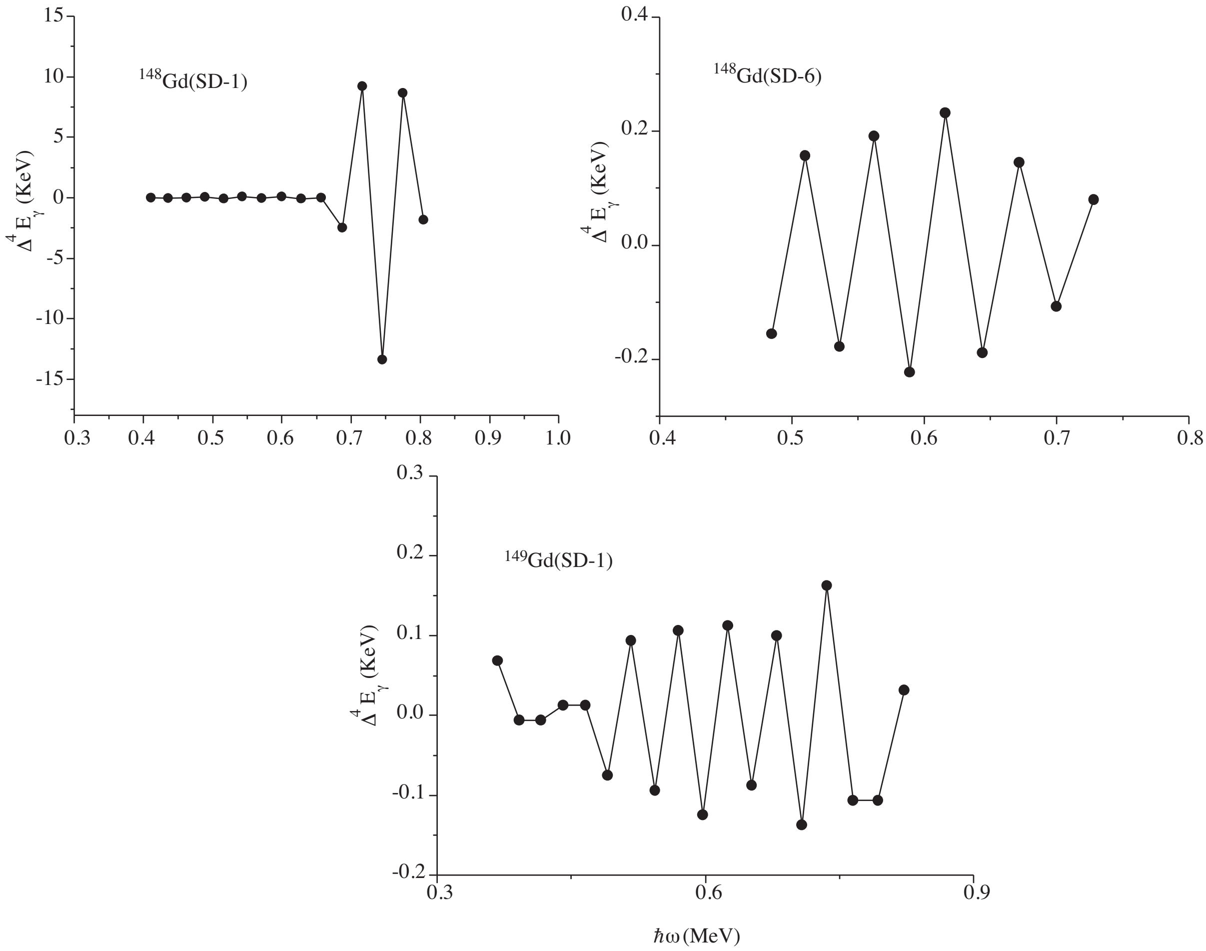

Figure 5. The $\Delta I = 2$ energy staggering obtained by the five point formula $\Delta^4 E_\gamma$ versus rotational frequency $\hbar\omega$ for ^{148}Gd (SD-1), ^{148}Gd (SD-6), and ^{149}Gd (SD-1).

The main conclusions of the present paper are summarized as follows:

i. Transition energies of many nuclear SD bands in the $A \sim 150$ mass region have been calculated. The corresponding rotational frequencies and dynamical moments of inertia are also tabulated.

ii. Optimized model parameters for each nucleus have been adjusted by using a computer simulation search program to fit the calculated dynamical moments of inertia with the corresponding experimental values.

iii. The values of the band head spins of our selected SD band from the present paper are excellent with all the spin assignments of other approaches.

iv. The appearance of identical bands and $\Delta I = 2$ nuclear staggering effects in the transition energies in some SD nuclei have been examined.

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