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DKP equation under scalar and vector Kratzer potentials

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Abstract: Approximate analytical solutions of Duffin-Kemmer-Petiau (DKP) equation under scalar and vector Kratzer terms are obtained via an elegant ansatz after successive transformations. The results, after application of proper fits, can be used in the study of relativistic spinless particles in various branches of physics such as cosmology and theoretical nuclear physics including meson spectroscopy.

Key words: DKP equation, Kratzer potential, ansatz approach **PACS:** 03.65.Ca; 03.65.Pm, 03.65.Nk.

1. Introduction

The Duffin-Kemmer-Petiau (DKP) equation appeared more than seventy years ago and introduced a basis which enabled theoretical physicists to investigate both spin-0 and spin-1 particles on the basis of a single equation in the relativistic regime [1–4]. This equation for spin-zero bosons under a vector potential possesses the same mathematical structure of its well known partner, the Klein-Gordon (KG) equation. Consequently the physical community thought the equations are completely equivalent. We now know, however, that the "equivalence" exhibits violations [5–14] and the former is a more appealing for the study of interactions in comparison with its alternatives, i.e. KG and Proca equations which are more well known [15–20].

The DKP equation has a range of applications from sub-atomic to large-scale physics [21–25] due to its similarity with the KG equation and can be investigated by the well-known techniques of quantum mechanics [26–29] under vector potential [30–40]. We intend to explore the equation under a scalar potential.

In the present work, we briefly review the DKP equation under scalar and vector potentials. Next, using elegant successive transformations, obtain approximate analytical solutions of the equation under the most applied interaction of quantum mechanics, i.e. the Kratzer interaction. As a final point, we wish to address the interesting papers of [41–44] in which DKP was used to investigate related topics in the annals of particle physics. The interested reader might find instructive points about the physical consequences within them.

2. DKP equation

The DKP Hamiltonian for scalar and vector interactions is

$$\left(\beta \cdot \vec{p}c + mc^2 + U_s + \beta^0 U_v^o\right) \psi(\vec{r}) = \beta^0 E\psi(\vec{r}), \tag{1}$$

where

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$$\psi(\vec{r}) = \begin{pmatrix} \psi_{upper} \\ i\psi_{lower} \end{pmatrix}, \tag{2}$$

and the upper and lower components, respectively, are

$$\begin{aligned} \psi_{upper} &\equiv \begin{pmatrix} \varphi \\ \phi \end{pmatrix}, \\ \psi_{lower} &\equiv \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}. \end{aligned} \tag{3}$$

Also, β^0 is the usual 5×5matrix and U_s, U_v^o, respectively, represent the scalar and vector interactions. The equation in (3+0)-dimensions is written [1–4]

$$\begin{aligned} &\left(mc^2 + U_s\right)\varphi = \left(E - U_v^o\right)\phi + \hbar c\vec{\nabla}\cdot\vec{A}, \\ &\vec{\nabla}\varphi = \left(mc^2 + U_s\right)\vec{A}, \\ &\left(mc^2 + U_s\right)\phi = \left(E - U_v^o\right)\varphi, \end{aligned} \tag{4}$$

where $\vec{A} = (A_1, A_2, A_3)$. In equation (3) ψ is a simultaneous eigenfunction of J^2andJ_3, i.e.

$$\begin{aligned} J^2\begin{pmatrix} \psi_{upper} \\ \psi_{lower} \end{pmatrix} &= \begin{pmatrix} L^2\psi_{upper} \\ (L+S)^2\psi_{lower} \end{pmatrix} = J(J+1)\begin{pmatrix} \psi_{upper} \\ \psi_{lower} \end{pmatrix}, \\ J_3\begin{pmatrix} \psi_{upper} \\ \psi_{lower} \end{pmatrix} &= \begin{pmatrix} L_3\psi_{upper} \\ (L_3+s_3)\psi_{lower} \end{pmatrix} = M\begin{pmatrix} \psi_{upper} \\ \psi_{lower} \end{pmatrix}, \end{aligned} \tag{5}$$

and the general solution is considered as

$$\psi_{JM}(r) = \begin{pmatrix} f_{nJ}(r)Y_{JM}(\Omega) \\ g_{nJ}(r)Y_{JM}(\Omega) \\ i\sum_L h_{nJL}(r)Y_{JL1}^M(\Omega) \end{pmatrix}, \tag{6}$$

where spherical harmonics $Y_{JM}(\Omega)$ are of order J, $Y_{JL1}^M(\Omega)$are the normalized vector spherical harmonics and $f_{nJ}, g_{nJ}$and h_{nJL}denote the radial wavefunctions. The above equations yield the coupled differential equations [30–38]

$$\begin{aligned} &\left(E_{n,J} - U_v^0\right)F_{n,J}(r) = \left(mc^2 + U_s\right)G_{n,J}(r), \quad \left(\tfrac{dF_{n,J}(r)}{dr} - \tfrac{J+1}{r}F_{n,J}(r)\right) = -\tfrac{1}{\alpha_J}\left(mc^2 + U_s\right)H_{1,n,J}(r), \\ &\left(\tfrac{dF_{n,J}(r)}{dr} + \tfrac{J}{r}F_{n,J}(r)\right) = \tfrac{1}{\zeta_J}\left(mc^2 + U_s\right)H_{-1,n,J}(r), -\alpha_J\left(\tfrac{dH_{1,n,J}(r)}{dr} + \tfrac{J+1}{r}H_{1,n,J}(r)\right) \\ &+\zeta\left(\tfrac{dH_{-1,n,J}(r)}{dr} - \tfrac{J}{r}H_{-1,n,J}(r)\right) = \tfrac{1}{\hbar c}\left(\left(mc^2 + U_s\right)F_{n,J}(r) - \left(E_{n,J} - U_v^0\right)G_{n,J}(r)\right), \end{aligned} \tag{7}$$

which give [40]

$$\begin{aligned} &\tfrac{d^2F_{n,J}(r)}{dr^2}\left[1 + \tfrac{\zeta_J^2}{\alpha_J^2}\right] - \tfrac{dF_{n,J}(r)}{dr}\left[\tfrac{U_s'}{(m+U_s)}\left(1 + \tfrac{\zeta_J^2}{\alpha_J^2}\right)\right] \\ &+F_{n,J}(r)\left[\begin{array}{l} -\tfrac{J(J+1)}{r^2}\left(1 + \tfrac{\zeta_J^2}{\alpha_J^2}\right) + \tfrac{U_s'}{(m+U_s)}\left(\tfrac{J+1}{r} - \tfrac{\zeta_J^2}{\alpha_J^2}\tfrac{J}{r}\right) \\ -\tfrac{1}{\alpha_J^2}\left(\left(m + U_s\right)^2 - \left(E_{n,J} - U_v^0\right)^2\right) \end{array}\right] = 0, \end{aligned} \tag{8}$$

where $f_{nJ}(r) = F(r)/r$, $g_{nJ}(r) = G(r)/r$, $h_{nJJ\pm1} = H_{\pm1}/r$ and $\zeta_J = \sqrt{J/(2J+1)}$. When $U_s = 0$, we recover the well-known formula [30–38]

$$\left(\frac{d^2}{dr^2} - \frac{J\left(J+1\right)}{r^2} + \left(E_{n,J} - U_v^0\right)^2 - m^2\right) F_{n,J}(r) = 0. \tag{9}$$

Meanwhile, the Kratzer potential is amongst the most attractive physical potentials as it contains a degeneracy-removing inverse square term besides the common Coulomb term. It appears in a wide class of physical and chemical branches including the atomic and molecular physics providing quite motivating results [45-50]. We consider the following Kratzer vector and scalar potentials:

$$U_s = \frac{a_s}{r} + \frac{b_s}{r^2}, \tag{10a}$$

$$U_v^0 = \frac{a_v}{r} + \frac{b_v}{r^2}. \tag{10b}$$

Substitution of equations (10a) and (10b) in equation (8) gives

$$\begin{gathered}\frac{d^2F_{n,J}(r)}{dr^2}\left[1+\frac{\zeta_J^2}{\alpha_J^2}\right] - \frac{dF_{n,J}(r)}{dr}\left[\frac{-\frac{a_s}{r^2}-\frac{2b_s}{r^3}}{\left(m+\frac{a_s}{r}+\frac{b_s}{r^2}\right)}\left(1+\frac{\zeta_J^2}{\alpha_J^2}\right)\right] \\ +F_{n,J}(r)\left[\begin{array}{l} -\frac{J(J+1)}{r^2}\left(1+\frac{\zeta_J^2}{\alpha_J^2}\right) + \frac{-\frac{a_s}{r^2}-\frac{2b_s}{r^3}}{\left(m+\frac{a_s}{r}+\frac{b_s}{r^2}\right)}\left(\frac{J+1}{r} - \frac{\zeta_J^2}{\alpha_J^2}\frac{J}{r}\right) \\ -\frac{1}{\alpha_J^2}\left(\left(m+\frac{a_s}{r}+\frac{b_s}{r^2}\right)^2 - \left(E_{n,J} - \left(\frac{a_v}{r}+\frac{b_v}{r^2}\right)\right)^2\right) \end{array}\right] = 0.\end{gathered} \tag{11}$$

By choosing

$$F_{n,J}(r) = \sqrt{m + \frac{a_s}{r} + \frac{b_s}{r^2}}\ \varphi_{n,J}(r) \tag{12}$$

brings (7) to the form

$$\begin{aligned} &\frac{d^2\varphi_{n,J}(r)}{dr^2} + [\frac{a_s}{m}\frac{1}{r(r^2+\frac{a_s}{m}r+\frac{b_s}{m})} + \frac{3b_s}{m}\frac{1}{r^2(r^2+\frac{a_s}{m}r+\frac{b_s}{m})} - \frac{3}{4}\frac{a_s^2}{m^2}\frac{1}{(r^2+\frac{a_s}{m}r+\frac{b_s}{m})^2} - \frac{3b_s^2}{m^2}\frac{1}{r^2(r^2+\frac{a_s}{m}r+\frac{b_s}{m})^2} \\ &-\frac{3a_sb_s}{m^2}\frac{1}{r(r^2+\frac{a_s}{m}r+\frac{b_s}{m})^2} - \frac{J(J+1)}{r^2} - \frac{Ca_s}{mA}\frac{1}{r(r^2+\frac{a_s}{m}r+\frac{b_s}{m})} - \frac{2Cb_s}{Am}\frac{1}{r^2(r^2+\frac{a_s}{m}r+\frac{b_s}{m})} - \frac{1}{A\alpha_J^2}(m^2 - E_{n,J}^2) \\ &+(-\frac{2ma_s}{A\alpha_J^2} - \frac{2E_{n,J}a_v}{A\alpha_J^2})\frac{1}{r} + (\frac{-a_s^2-2mb_s+a_v^2-2E_{n,J}b_v}{A\alpha_J^2})\frac{1}{r^2} + (\frac{2a_vb_v-2a_sb_s}{A\alpha_J^2})\frac{1}{r^3} + (\frac{b_v^2-b_s^2}{A\alpha_J^2})\frac{1}{r^4}]\varphi_{n,J}(r) = 0. \end{aligned} \tag{13}$$

After decomposition of fractions, we arrive at

$$\begin{aligned} &[\frac{d^2}{dr^2} + (\frac{a_sf}{m} + \frac{3b_sf_1}{m} - \frac{3b_s^2f_4}{m^2} - \frac{3a_sb_sd_3}{m^2} - \frac{Ca_sf}{mA} - \frac{2Cb_sf_1}{Am} - \frac{1}{A\alpha_J^2}(2ma_s + 2E_{n,J}a_v))\frac{1}{r} \\ &+(\frac{3b_sh_1}{m} - \frac{3b_s^2h_4}{m^2} - J(J+1) - \frac{2Cb_sh_1}{Am} + \frac{1}{A\alpha_J^2}(-a_s^2 - 2mb_s + a_v^2 - 2E_{n,J}b_v))\frac{1}{r^2} \\ &+(\frac{2a_vb_v-2a_sb_s}{A\alpha_J^2})\frac{1}{r^3} + (\frac{b_v^2-b_s^2}{A\alpha_J^2})\frac{1}{r^4} + (\frac{a_sh}{m} + \frac{3b_sd_1}{m} - \frac{3}{4}\frac{a_s^2f_2}{m^2} - \frac{3b_s^2d_4}{m^2} - \frac{3a_sb_sg_3}{m^2} - \frac{Ca_sh}{mA} - \frac{2Cb_sd_1}{mA})\frac{1}{r+r_1} \\ &(-\frac{3}{4}\frac{a_s^2h_2}{m^2} - \frac{3b_s^2g_4}{m^2} - \frac{3a_sb_sh_3}{m^2})\frac{1}{(r+r_1)^2} + (\frac{a_sg}{m} + \frac{3b_sg_1}{m} - \frac{3a_s^2g_2}{4m^2} - \frac{3b_s^2s_1}{m^2} - \frac{3a_sb_sf_3}{m^2} - \frac{Ca_sg}{mA} - \frac{2Cb_sg_1}{mA})\frac{1}{r+r_2} \\ &+(-\frac{3}{4}\frac{a_s^2d_2}{m^2} - \frac{3b_s^2s_2}{m^2} - \frac{3a_sb_ss}{m^2})\frac{1}{(r+r_2)^2} - \frac{1}{A\alpha_J^2}(m^2 - E_{n,J}^2)]\varphi_{n,J}(r) = 0, \end{aligned} \tag{14}$$

where

$$r_1=\tfrac{1}{2}(\tfrac{a_s}{m}+\sqrt{\tfrac{a_s^2}{m^2}-4\tfrac{b_s}{m}}),\ \ r_2=\tfrac{1}{2}(\tfrac{a_s}{m}-\sqrt{\tfrac{a_s^2}{m^2}-4\tfrac{b_s}{m}}),\ \ f=\tfrac{1}{r_1r_2},\ \ h=\tfrac{1}{r_1(r_1-r_2)}$$
$$g=\tfrac{-1}{r_2(r_1-r_2)},\ \ f_1=-\tfrac{r_1+r_2}{(r_1r_2)^2},\ \ h_1=\tfrac{1}{r_1r_2},\ \ g_1=\tfrac{1}{r_2^2(r_1-r_2)},\ \ d_1=-\tfrac{1}{r_1^2(r_1-r_2)}$$
$$f_2=\tfrac{2}{(r_1-r_2)^3},\ \ h_2=\tfrac{1}{(r_2-r_1)^2},\ \ g_2=-\tfrac{2}{(r_1-r_2)^3},\ \ d_2=\tfrac{1}{(r_2-r_1)^2}$$
$$f_3=-\tfrac{-3r_2+r_1}{r_2^2(r_1-r_2)^3},\ \ h_3=-\tfrac{1}{r_1(r_1-r_2)^2},\ \ g_3=-\tfrac{3r_1-r_2}{r_1^2(r_1-r_2)^3},\ \ d_3=\tfrac{1}{(r_1r_2)^2},\ \ s=-\tfrac{1}{r_2(r_1-r_2)^2} \tag{15}$$
$$f_4=-\tfrac{2(r_2+r_1)}{(r_1r_2)^3},\ \ h_4=\tfrac{1}{(r_1r_2)^2},\ \ g_4=\tfrac{1}{r_1^2(r_1-r_2)^2},\ \ d_4=\tfrac{2(2r_1-r_2)}{r_1^3(r_1-r_2)^3}$$
$$s_1=\tfrac{2(-2r_2+r_1)}{r_2^3(r_1-r_2)^3},\ \ s_2=\tfrac{1}{r_2^2(r_1-r_2)^2}.$$

Let us now consider an ansatz of the form

$$\varphi_{n,J}(r)=f_n(r)\exp(g_J(r)), \tag{16}$$

where

$$f_n(r)=\begin{cases}1 & n=0\\ \prod_{i=1}^{n}(r-\alpha_i^n) & n>0,\end{cases} \tag{17a}$$

$$g_J(r)=\gamma\ln(r+r_1)+\delta\ln(r+r_2)+\beta\ln(r)+\frac{\xi}{r}+\eta r. \tag{17b}$$

If we choose nodeless state ($n=0$), we get

$$\begin{aligned}\varphi''_{0,J}(r)=&[(\tfrac{2\xi\delta}{r_2^2}+\tfrac{2\xi\gamma}{r_1^2}+2\beta\eta+\tfrac{2\beta\gamma}{r_1}+\tfrac{2\beta\delta}{r_2})\tfrac{1}{r}+(\beta^2-\beta-2\eta\xi-\tfrac{2\xi\delta}{r_2}-\tfrac{2\xi\gamma}{r_1})\tfrac{1}{r^2}+(2\xi-2\beta\xi)\tfrac{1}{r^3}\\&+\xi^2\tfrac{1}{r^4}+(2\eta\gamma-\tfrac{2\delta\gamma}{r_1-r_2}-\tfrac{2\xi\gamma}{r_1^2}-\tfrac{2\gamma\beta}{r_1})\tfrac{1}{r+r_1}+(\gamma^2-\gamma)\tfrac{1}{(r+r_1)^2}+(2\eta\delta-\tfrac{2\xi\delta}{r_2^2}+\tfrac{2\delta\gamma}{r_1-r_2}-\tfrac{2\beta\delta}{r_2})\tfrac{1}{r+r_2}\\&+(\delta^2-\delta)\tfrac{1}{(r+r_2)^2}+\eta^2]\varphi_{0,J}(r).\end{aligned} \tag{18}$$

Equating the coefficients on both sides, we find

$$\frac{2\xi\delta}{r_2^2}+\frac{2\xi\gamma}{r_1^2}+2\beta\eta+\frac{2\beta\gamma}{r_1}+\frac{2\beta\delta}{r_2}=-(\frac{a_sf}{m}+\frac{3b_sf_1}{m}-\frac{3b_s^2f_4}{m^2}-\frac{3a_sb_sd_3}{m^2}-\frac{Ca_sf}{mA}-\frac{2Cb_sf_1}{Am}-\frac{1}{A\alpha_J^2}(2ma_s+2E_{n,J}a_v)), \tag{19a}$$

$$\beta^2-\beta-2\eta\xi-\frac{2\xi\delta}{r_2}-\frac{2\xi\gamma}{r_1}=-(\frac{3b_sh_1}{m}-\frac{3b_s^2h_4}{m^2}-J(J+1)-\frac{2Cb_sh_1}{Am}+\frac{1}{A\alpha_J^2}(-a_s^2-2mb_s+a_v^2-2E_{n,J}b_v)), \tag{19b}$$

$$2\xi-2\beta\xi=-(\frac{2a_vb_v-2a_sb_s}{A\alpha_J^2}), \tag{19c}$$

$$\xi^2=-(\frac{b_v^2-b_s^2}{A\alpha_J^2}), \tag{19d}$$

$$2\eta\gamma-\frac{2\delta\gamma}{r_1-r_2}-\frac{2\xi\gamma}{r_1^2}-\frac{2\gamma\beta}{r_1}=-(\frac{a_sh}{m}+\frac{3b_sd_1}{m}-\frac{3}{4}\frac{a_s^2f_2}{m^2}-\frac{3b_s^2d_4}{m^2}-\frac{3a_sb_sg_3}{m^2}-\frac{Ca_sh}{mA}-\frac{2Cb_sd_1}{mA}), \tag{19e}$$

$$\gamma^2-\gamma=-(-\frac{3}{4}\frac{a_s^2h_2}{m^2}-\frac{3b_s^2g_4}{m^2}-\frac{3a_sb_sh_3}{m^2}), \tag{19f}$$

$$2\eta\delta - \frac{2\xi\delta}{r_2^2} + \frac{2\delta\gamma}{r_1 - r_2} - \frac{2\beta\delta}{r_2} = -(\frac{a_s g}{m} + \frac{3b_s g_1}{m} - \frac{3a_s^2 g_2}{4m^2} - \frac{3b_s^2 s_1}{m^2} - \frac{3a_s b_s f_3}{m^2} - \frac{Ca_s g}{mA} - \frac{2Cb_s g_1}{mA}), \tag{19g}$$

$$\delta^2 - \delta = -(-\frac{3}{4}\frac{a_s^2 d_2}{m^2} - \frac{3b_s^2 s_2}{m^2} - \frac{3a_s b_s s}{m^2}), \tag{19h}$$

$$\eta^2 = \frac{1}{A\alpha_J^2}(m^2 - E_{n,J}^2). \tag{19i}$$

From equations (19a)– (19i), for the fixed values of a_s, α_J, m and ζ_J in particular, the system of nine equations (19) determines the sets of variables $b_s,\ a_v,\ b_v,\ E_{0,J},\ \eta,\ \gamma,\ \delta,\ \beta,\ \xi$ as follows:

$$\xi = \pm\sqrt{-(\frac{b_v^2 - b_s^2}{A\alpha_J^2})}, \tag{20a}$$

$$\eta = \pm\sqrt{\frac{1}{A\alpha_J^2}(m^2 - E_{n,J}^2)} \tag{20b}$$

$$\delta = \frac{1}{2}[1 \pm \sqrt{1 - 4(-\frac{3}{4}\frac{a_s^2 d_2}{m^2} - \frac{3b_s^2 s_2}{m^2} - \frac{3a_s b_s s}{m^2})}] \tag{20c}$$

$$\gamma = \frac{1}{2}[1 \pm \sqrt{1 - 4(-\frac{3}{4}\frac{a_s^2 h_2}{m^2} - \frac{3b_s^2 g_4}{m^2} - \frac{3a_s b_s h_3}{m^2})}] \tag{20d}$$

$$\beta = 1 \pm \frac{2a_v b_v - 2a_s b_s}{2A\alpha_J^2\sqrt{-(\frac{b_v^2 - b_s^2}{A\alpha_J^2})}} \tag{20e}$$

$$a_v = \frac{A\alpha_J^2}{2E_{n,J}}(\frac{2\xi\delta}{r_2^2} + \frac{2\xi\gamma}{r_1^2} + 2\beta\eta + \frac{2\beta\gamma}{r_1} + \frac{2\beta\delta}{r_2} + \frac{a_s f}{m} + \frac{3b_s f_1}{m} - \frac{3b_s^2 f_4}{m^2} - \frac{3a_s b_s d_3}{m^2} - \frac{Ca_s f}{mA} - \frac{2Cb_s f_1}{Am} - \frac{2ma_s}{A\alpha_J^2}) \tag{20f}$$

$$b_v = \frac{A\alpha_J^2}{2E_{n,J}}(\beta^2 - \beta - 2\eta\xi - \frac{2\xi\delta}{r_2} - \frac{2\xi\gamma}{r_1} + \frac{3b_s h_1}{m} - \frac{3b_s^2 h_4}{m^2} - J(J+1) - \frac{2Cb_s h_1}{Am} + \frac{1}{A\alpha_J^2}(-a_s^2 - 2mb_s + a_v^2)) \tag{20g}$$

$$E_{n,J} = \sqrt{m^2 - \frac{A\alpha_J^2}{4\gamma^2}\left[\begin{array}{l} -\frac{a_s h}{m} - \frac{3b_s d_1}{m} + \frac{3}{4}\frac{a_s^2 f_2}{m^2} + \frac{3b_s^2 d_4}{m^2} + \frac{3a_s b_s g_3}{m^2} \\ +\frac{Ca_s h}{mA} + \frac{2Cb_s d_1}{mA} + \frac{2\delta\gamma}{r_1 - r_2} + \frac{2\xi\gamma}{r_1^2} + \frac{2\gamma\beta}{r_1} \end{array}\right]^2} \tag{20h}$$

where η *and* ξ should be negative and $\gamma,\ \delta$ *and* β should be positive.

Using equations (20e) and (19e), we obtain a constraint relation:

$$1 - \frac{2a_v b_v - 2a_s b_s}{2A\alpha_J^2\sqrt{\frac{b_s^2 - b_v^2}{A\alpha_J^2}}} = \frac{r_2}{2\delta}\left[\begin{array}{l} 2\eta\delta - \frac{2\xi\delta}{r_2^2} + \frac{2\delta\gamma}{r_1 - r_2} + \frac{a_s g}{m} + \frac{3b_s g_1}{m} - \frac{3a_s^2 g_2}{4m^2} \\ -\frac{3b_s^2 s_1}{m^2} - \frac{3a_s b_s f_3}{m^2} - \frac{Ca_s g}{mA} - \frac{2Cb_s g_1}{mA} \end{array}\right]. \tag{21}$$

3. Conclusion

The motivation behind our study was twofold. The first was the successful predictions of the Kratzer potential in various fields. The second was exploration of the scalar sector of the DKP equation, an area quite open to debate. In our calculations, bearing in mind the physical insight the analytical approach provides to the physics of the problem, we picked up the ansatz methodology. By proposing novel ansatz solutions, we were able to find a quasi-exact solution to the equation under scalar and vector Kratzer terms. Although we obtained the solution for the nodeless mode, the higher states can be simply obtained by the same token via choosing $f_n(r)$ as $(r-\alpha_1^1)$, $(r-\alpha_1^2)(r-\alpha_2^2)$ for the first node, second node, etc. Our results, after application of proper fits, can be used in the study of relativistic spinless particles in various branches of physics including meson spectroscopy.

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