

## Empirical rule for the pair break mechanism in three-quasiparticle rotational bands

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**Abstract:** An empirical rule on the basis of odd–even mass difference is suggested for the breaking of a proton pair or a neutron pair in odd- $A$  nuclei to form a lower lying 3-quasiparticle state. If  $\Delta_p$  is smaller than  $\Delta_n$ , the proton pair breaks; on the other hand, if  $\Delta_n$  is smaller than  $\Delta_p$ , the neutron pair breaks down and forms a lower lying 3-quasiparticle state in an odd- $A$  nuclide. This rule appears not only purely based on the proton pairing/neutron pairing energy balancing condition but also on particular excitation energy, which is  $\geq 1$  MeV in the case of the rare-earth region. The extension of this rule to the configuration dependent pairing energies available in the literature is discussed as well as its applications for deciding on the breaking of proton/neutron pairs at higher excitation energies.

**Key words:** Rotational bands, quasiparticle, pairing gap, binding energy

### 1. Introduction

In odd- $A$  nuclei, the states of next higher seniority following the 1-quasiparticle (1qp) excitations are the 3-quasiparticle (3qp) excitations. At excitation energy  $\geq 1$  MeV, which is approximately the energy gap  $2\Delta$  in the rare-earth region, a proton or a neutron pair can break up and form a 3qp state. Two kinds of 3qp states are possible: those having all 3 particles of the same kind ( $nnn$  and  $ppp$ ) and others having a combination of 2 kinds of particles ( $npp$  and  $npn$ ). In a deformed nucleus, coupling of 3-qps in Nilsson states having  $K$  values, say,  $K_1$ ,  $K_2$ , and  $K_3$ , leads to a quadruplet with resultant  $K = |K_1 \pm K_2 \pm K_3|$ . These 4 intrinsic states split up due to residual interaction among the 3 nucleons; the residual  $n-p$  interaction plays the major role in this splitting [1,2]. The empirically observed energy gap and also the existence of spherical shapes for closed shell nuclei confirm the significant role of the pairing correlations in the lower lying excited states of the deformed nuclei. The pairing force couples 2 particles in the time-reversed single-particle states and a rotation has an opposite effect on the particles forming a pair. The Coriolis force acts in opposite directions and tends to decouple the pairing correlations.

### 2. Methodology

There exist various finite difference formulae such as third-order finite difference formulae (TOFDF) and fourth-order finite difference formulae (FOFDF) for calculations of the proton ( $\Delta_p$ ) and the neutron pairing gap ( $\Delta_n$ ) [3,4]. In order to calculate the proton ( $\Delta_p$ ) and the neutron ( $\Delta_n$ ) pairing gap, we use difference equations derived from Taylor series expansion of masses in the neighborhood of the mass of interest [4]. Since the

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development of this approach depends on the smooth mass surface behavior, special care must be exercised to exclude various departures from this smoothness. Some the important contributions to this departure are:

- (a) Pairing gaps and residual interactions;
- (b) Shell effects;
- (c) Nucleon mass granularity for light nuclei ( $A < 10$ ).

Using the above assumptions of smooth mass surface behavior and by excluding the above departures, ground state mass  $M(Z, N)$  can be written as

$$M(Z, N) = m(Z, N) + D(Z, N), \quad (1)$$

where  $m(Z, N)$  defines smooth mass surface and  $D(Z, N)$  is the correction term as given below:

$$D(Z, N) = \left\{ \begin{array}{ll} \Delta_n + \Delta_p - \delta & (Odd - Odd) \\ \Delta_n & (Odd - N) \\ \Delta_p & (Odd - P) \\ 0 & (Even - Even) \end{array} \right\} \quad (2)$$

The Taylor series expansion about the mass of interest  $M(Z, N_0)$  is given by

$$M(Z, N) = m(Z, N_0) + (N - N_0) \frac{\partial m}{\partial N}(Z, N_0) + \frac{(N - N_0)^2}{2!} \frac{\partial^2 m}{\partial^2 N}(Z, N_0) + \frac{(N - N_0)^3}{3!} \frac{\partial^3 m}{\partial^3 N}(Z, N_0) + \dots + D(Z, N) \quad (3)$$

where  $M(Z, N)$  is the mass corresponding to the nuclei having  $Z$  protons and  $N$  neutrons in the neighborhood of the mass of interest  $M(Z, N_0)$  having  $Z$  protons and  $N_0$  neutrons.

Using the Taylor series expansion carried out in Eq. (3) and by retaining the terms up to second order, we end up with following system of equations:

$$M(Z, N + 1) = m(Z, N) + \frac{\partial m}{\partial N}(Z, N) + \frac{1}{2} \frac{\partial^2 m}{\partial^2 N}(Z, N) + \Delta_n \quad (4)$$

$$M(Z, N) = m(Z, N) \quad (5)$$

$$M(Z, N - 1) = m(Z, N) - \frac{\partial m}{\partial N}(Z, N) + \frac{1}{2} \frac{\partial^2 m}{\partial^2 N}(Z, N) + \Delta_n \quad (6)$$

$$M(Z, N - 2) = m(Z, N) - 2 \frac{\partial m}{\partial N}(Z, N) + 2 \frac{\partial^2 m}{\partial^2 N}(Z, N) \quad (7)$$

The solution of these simultaneous nonlinear equations will lead us to the FOFDF for neutron pairing gap  $\Delta_n$  as:

$$\Delta_n(N) = \frac{(-1)^N}{4} [M(Z, N - 2) - 3M(Z, N - 1) + 3M(Z, N) - M(Z, N + 1)] \quad (8)$$

Or, alternatively in terms of binding energies, Eq. (8) can be written as:

$$\Delta_n(N) = \frac{(-1)^N}{4} [B(Z, N - 2) - 3B(Z, N - 1) + 3B(Z, N) - B(Z, N + 1)] \quad (9)$$

In a similar way, we can formulate FOFDF to calculate the proton pairing gap  $\Delta_p$  as:

$$\Delta_p(P) = \frac{(-1)^Z}{4} [B(Z-2, N) - 3B(Z-1, N) + 3B(Z, N) - B(Z+1, N)] \quad (10)$$

In the present paper, we calculated the proton ( $\Delta_p$ ) and neutron ( $\Delta_n$ ) pairing gaps using 4-point formulae and devised an empirical rule for pair breaking of a proton pair or a neutron pair in odd-A nuclei. According to this rule, if  $\Delta_p$  is smaller than  $\Delta_n$ , the proton pair breaks; on the other hand, if  $\Delta_n$  is smaller than  $\Delta_p$ , the neutron pair breaks down and forms a lower lying 3qp state in an odd-A nuclide. The generalization of this empirical rule to the configuration dependent pairing energy calculations available in the literature [5,6] and its applications for deciding on the breaking of proton/neutron pairs at higher excitation energies are also discussed.

### 3. Results and discussion

In order to test the validity of this rule, we extracted 47 lower lying 3-qp band-heads from our compilation [7] and calculated the proton/neutron pairing energies ( $\Delta_p/\Delta_n$ ), uncertainties in these pairing energies ( $\delta(\Delta_p)/\delta(\Delta_n)$ ), difference in pairing energies ( $|\Delta_p - \Delta_n|$ ), and the uncertainties in the difference of pairing energies ( $\delta(\Delta_p) - \delta(\Delta_n)$ ) as given in Tables 1–3. The pointwise applications of the present rule for all these 47 lower lying 3qp structures are given below:

- (1) In Table 1, we present the bands for which breaking of a proton/neutron pair is in accordance with our empirical rule and the uncertainties ( $\delta(\Delta_p)/\delta(\Delta_n)$ ) in  $\Delta_p$  and  $\Delta_n$  are small as compared to the difference in pairing energies ( $|\Delta_p - \Delta_n|$ ). The 16 cases presented in this table confirm the validity of our rule for the pair break mechanism. These lower lying 16 3qp configurations have been taken from the experimental measurements [8–23]. On the basis of present calculations, we also suggest that the present rule of pair break mechanism will be useful to experimentalists for configuration assignment to lower lying 3qp bands particularly in the situations where experimental information for distinction among the competing configurations is not sufficient.
- (2) In Table 2, we present 15 cases for which our rule is true but  $\delta(\Delta_p)/\delta(\Delta_n)$  in  $\Delta_p$  and  $\Delta_n$  is comparable or larger than  $|\Delta_p - \Delta_n|$ . In most of these cases (**marked in bold**), the binding energies appear from systematics [24], which leads to large uncertainties in the pairing energies. Although the  $\delta(\Delta_p)/\delta(\Delta_n)$  in  $\Delta_p$  and  $\Delta_n$  is comparable to the  $|\Delta_p - \Delta_n|$ , the validity of the present rule in all these cases further strengthens the applicability of this rule for deciding about the breaking of a particular pair. These lower lying 3qp structures have been taken from different experimental measurements [25–39].
- (3) On the basis of these calculations, we also suggest that, if high- $-j$  orbits are not involved in given 3qp configurations and if the  $|\Delta_p - \Delta_n|$  is  $\geq 300$  keV (approx.), then the present rule of pair break mechanism will also be valid at higher excitation energies. In Table 3, we present 4 such different nuclides in which the difference in the proton/neutron pairing energies is  $\geq 300$  keV (approx.) and breaking of proton/neutron pairs up to higher excitation energies is in accordance with our rule. Out of all these cases,  $^{177}\text{Lu}$  [15,40–43],  $^{175}\text{Lu}$  [14,44],  $^{173}\text{Ta}$  [19,45], and  $^{179}\text{Re}$  [36] have 15, 4, 4, and 5 different 3qp band-heads, respectively. All these 27 cases presented in Table 3 obey our rule for the pair break mechanism, even

**Table 1.** The bands that obey our empirical rule of pair break mechanism and for which the uncertainties in pairing energies are small as compared to the difference in pairing energies. The  $\Delta_p/\Delta_n, \delta(\Delta_p)/\delta(\Delta_n), |\Delta_p - \Delta_n|$  and  $|\delta(\Delta_p - \Delta_n)|$  are the proton/neutron pairing energies, uncertainties in pairing energies, difference in the proton/neutron pairing energies and uncertainties in the difference of proton/neutron pairing energies, respectively. The bold values of the pairing energies are the cases for which binding energies appear from systematics.

S. no.	Nucleus	Configuration	$\Delta_n$	$\Delta_n$	Pair break		$ \Delta_p - \Delta_n $	Ref.
			$\delta(\Delta_n)$ (keV)	$\delta(\Delta_n)$ (keV)	Exp.	Th.	$ \delta(\Delta_p - \Delta_n) $ (keV)	
1.	$^{153}_{63}Eu$	$\pi:5/2[413] \nu:3/2[651] \otimes 11/2[505]$ or $\pi:5/2[413] \nu:3/2[402] \otimes 11/2[505]$ or a mixture of both	1125.6 (7.9)	1088.0 (6.0)	$\nu$	$\nu$	37.6 (9.9)	[8]
2.	$^{153}_{65}Tb$	Competing configurations are: $A_p \otimes AF$ or $B_p \otimes AE$ $A_p = 7/2[523](\alpha = -1/2)$ $B_p = 7/2[523]$ $(\alpha = +1/2)$ $A = 3/2[651](\alpha = +1/2)$ $F = 3/2[521](\alpha = -1/2)$ $E = 3/2[521](\alpha = +1/2)$	1480.4 (11.1)	820.8 (64.4)	$\nu$	$\nu$	659.6 (65.4)	[9]
3.	$^{163}_{67}Ho$	$\pi:7/2[523] \nu:5/2[642] \otimes 5/2[523]$	972.5 (6.0)	806.5 (7.1)	$\nu$	$\nu$	166.0 (9.3)	[10]
4.	$^{165}_{69}Tm$	$\pi:7/2[404] \nu:5/2[642] \otimes 5/2[523]$	1061.3 (9.9)	921.8 (23.9)	$\nu$	$\nu$	139.5 (25.9)	[11]
5.	$^{159}_{71}Lu$	$\pi:7/2[523](\alpha = -1/2)$ $\nu:3/2[651](\alpha = +1/2) \otimes 3/2[521]$ $(\alpha = -1/2)$	1511.3 (152.0)	<b>145.0</b> <b>(265.1)</b>	$\nu$	$\nu$	1366.3 (305.6)	[12]
6.	$^{171}_{71}Lu$	$\pi:7/2[404] \nu:7/2[633] \otimes 1/2[521]$	979.4 (50.3)	726.5 (20.1)	$\nu$	$\nu$	252.9 (54.1)	[13]
7.	$^{175}_{71}Lu$	$\pi:7/2[404] \nu:7/2[514] \otimes 5/2[512]$	914.2 (6.9)	571.1 (5.4)	$\nu$	$\nu$	343.1 (8.8)	[14]
8.	$^{177}_{71}Lu$	$\pi:7/2[404] \nu:7/2[514] \otimes 9/2[624]$	860.0 (50.2)	457.8 (5.4)	$\nu$	$\nu$	402.2 (50.5)	[15]
9.	$^{173}_{72}Hf$	$\pi:1/2[521] \otimes 5/2[512] \nu:7/2[633]$	<b>730.4</b> <b>(128.1)</b>	<b>1562.0</b> <b>(229.1)</b>	$\pi$	$\pi$	831.6 (262.5)	[16]
10.	$^{177}_{72}Hf$	$\pi:7/2[404] \otimes 9/2[514] \nu:7/2[514]$	666.3 (6.2)	758.0 (5.6)	$\pi$	$\pi$	91.7 (8.4)	[17]
11.	$^{179}_{72}Hf$	$\pi:7/2[404] \otimes 9/2[514] \nu:9/2[624]$	609.4 (5.8)	704.1 (5.2)	$\pi$	$\pi$	94.7 (7.8)	[18]
12.	$^{173}_{73}Ta$	Competing configurations are: $\pi:5/2[402] \nu:5/2[512] \otimes 7/2[633]$ $\pi:7/2[404] \nu:7/2[633] \otimes 1/2[521]$ }	<b>1663.8</b> <b>(381.3)</b>	37.5 (373.5)	$\nu$	$\nu$	1626.3 (533.8)	[19]
13.	$^{177}_{73}Ta$	$\pi:9/2[514] \nu:7/2[514] \otimes 1/2[521]$	960.2 (8.2)	679.5 (173.3)	$\nu$	$\nu$	280.7 (173.4)	[20]
14.	$^{181}_{73}Ta$	$\pi:7/2[404] \nu:1/2[510] \otimes 9/2[624]$	806.0 (7.7)	611.4 (7.9)	$\nu$	$\nu$	194.6 (11.0)	[21]
15.	$^{187}_{75}Re$	$\pi:5/2[402] \nu:3/2[512] \otimes 11/2[615]$	905.5 (16.6)	666.9 (5.6)	$\nu$	$\nu$	238.6 (17.6)	[22]
16.	$^{171}_{77}Ir$	$\pi: h_{11/2}$ $\nu:3/2[651](\alpha = +1/2) \otimes 3/2[521]$ $(\alpha = +1/2)$	<b>1456.0</b> <b>(249.1)</b>	<b>400.0</b> <b>(455.5)</b>	$\nu$	$\nu$	1056 (519.2)	[23]

**Table 2.** Same as Table 1, but for the bands that obey our empirical rule of pair break mechanism and have uncertainties ( $\delta(\Delta_p)/\delta(\Delta_n)$ ) in the proton/neutron pairing energies comparable or larger than  $|\Delta_p - \Delta_n|$ .

S. no.	Nucleus	Configuration	$\Delta_n$ $\delta(\Delta_n)$ (keV)	$\Delta_p$ $\delta(\Delta_p)$ (keV)	Pair break		$ \Delta_p - \Delta_n $ $ \delta(\Delta_p - \Delta_n) $ (keV)	Ref.
					Exp.	Th.		
1.	$^{157}_{67}\text{Ho}$	For $\alpha = +1/2$ signature: $A_p \otimes AX$ For $\alpha = -1/2$ signature: $A_p \otimes AY$ $A_p = 7/2[523](\alpha = -1/2)A = 3/2[651]$ ( $\alpha = +1/2$ ) $X = 11/2[505](\alpha = +1/2)Y = 11/2[505]$ ( $\alpha = -1/2$ )	1018.5 (65.5)	539.8 (404.6)	v	v	478.7 (409.8)	[25]
2.	$^{157}_{68}\text{Er}$	For $\alpha = +1/2$ signature: $\pi:7/2[523](\alpha = -1/2) \otimes 7/2[404](\alpha = +1/2)$ $\nu:3/2[651](\alpha = +1/2)$ For $\alpha = -1/2$ signature: $\pi:7/2[523](\alpha = -1/2) \otimes 7/2[404](\alpha = -1/2)$ $\nu:3/2[651](\alpha = +1/2)$	793.5 (424.7)	1327.5 (424.6)	$\pi$	$\pi$	534.0 (600.5)	[26]
3.	$^{157}_{69}\text{Tm}$	For $\alpha = -1/2$ signature: $\pi:7/2[523](\alpha = +1/2)$ $\nu:3/2[651](\alpha = +1/2) \otimes 3/2[521](\alpha = +1/2)$ For $\alpha = +1/2$ signature: $\pi:7/2[523](\alpha = -1/2)$ $\nu:3/2[651](\alpha = +1/2) \otimes 3/2[521](\alpha = +1/2)$	1628.8 (434.0)	1238.3 (188.4)	v	v	390.5 (473.1)	[27]
4.	$^{165}_{70}\text{Yb}$	$\pi:7/2[523] \otimes 7/2[404] \nu:5/2[523]$	907.0 (162.6)	<b>1171.3</b> <b>(143.1)</b>	$\pi$	$\pi$	264.3 (216.5)	[28]
5.	$^{163}_{71}\text{Lu}$	Competing configurations are: $\pi:7/2[404], \nu:AB$ $\pi:5/2[402], \nu:AB$ $\pi:1/2[411], \nu:AB$ $A=5/2[642](\alpha = +1/2), B = 5/2[642]$ ( $\alpha = -1/2$ )	<b>1242.5</b> <b>(375.0)</b>	<b>932.5</b> <b>(446.4)</b>	v	v	310.0 (583.0)	[29]
6.	$^{165}_{71}\text{Lu}$	$\pi:9/2[514], \nu:AE$ $A = 5/2[642](\alpha = +1/2), E = 5/2[523]$ ( $\alpha = +1/2$ )	<b>1317.3</b> <b>(326.2)</b>	<b>805.0</b> <b>(316.2)</b>	v	v	512.3 (454.4)	[30]
7.	$^{169}_{72}\text{Hf}$	Tentative configuration is: $\pi:5/2[402] \otimes 1/2[411] \nu:5/2[642]$	<b>870.8</b> <b>(229.8)</b>	<b>1062.5</b> <b>(317.0)</b>	$\pi$	$\pi$	191.7 (391.6)	[31]
8.	$^{171}_{72}\text{Hf}$	$\pi:7/2[404] \otimes 5/2[402] \nu:7/2[633]$	<b>815.3</b> <b>(276.5)</b>	<b>982.5</b> <b>(298.2)</b>	$\pi$	$\pi$	167.2 (406.7)	[32]
9.	$^{175}_{72}\text{Hf}$	$\pi:7/2[404] \otimes 5/2[402] \nu:7/2[633]$	<b>704.6</b> <b>(100.1)</b>	<b>847.8</b> <b>(100.1)</b>	$\pi$	$\pi$	143.2 (141.6)	[33]
10.	$^{169}_{75}\text{Re}$	Tentative configurations are $\pi:9/2[514]$ $\nu:3/2[651](\alpha = +1/2) \otimes 3/2[521](\alpha = +1/2)$	<b>1389.5</b> <b>(510.6)</b>	<b>910.0</b> <b>(617.3)</b>	v	v	479.5 (801.0)	[34]
11.	$^{175}_{75}\text{Re}$	Competing configurations are: $\pi:9/2[514] \nu:5/2[512] \otimes 7/2[633]$ $\pi:5/2[402] \nu:5/2[512] \otimes 7/2[633]$	<b>1242.5</b> <b>(620.8)</b>	<b>152.5</b> <b>(753.3)</b>	v	v	1090.0 (976.2)	[35]
12.	$^{179}_{75}\text{Re}$	Tentative configuration is: $\pi:9/2[514] \nu:7/2[514] \otimes 1/2[521]$	<b>1051.0</b> <b>(211.9)</b>	<b>727.5</b> <b>(295.8)</b>	v	v	323.5 (363.9)	[36]
13.	$^{181}_{75}\text{Re}$	$\pi:5/2[402] \nu:9/2[624] \otimes 7/2[514]$	890.0 (29.7)	793.5 (116.6)	v	v	96.5 (120.3)	[37]
14.	$^{181}_{77}\text{Ir}$	$\pi:1/2[541] \nu:7/2[514] \otimes 7/2[633]$	<b>1025.0</b> <b>(364.0)</b>	<b>942.5</b> <b>(572.0)</b>	v	v	82.5 (678.0)	[38]
15.	$^{185}_{78}\text{Pt}$	$\pi:1/2[541] \otimes 1/2[660] \nu:9/2[624]$	<b>680.0</b> <b>(394.2)</b>	<b>1090.0</b> <b>(373.0)</b>	$\pi$	$\pi$	410.0 (542.7)	[39]

**Table 3.** The bands for which the difference in pairing energies ( $|\Delta_p - \Delta_n|$ ) is  $\geq 300$  keV (approx.) and which obey present empirical rule of pair break mechanism at higher excitation energies.

S. no.	Nucleus	Configuration	$E_{\text{exp.}}$ (keV)	$\Delta_n$ $\delta(\Delta_n)$ (keV)	$\Delta_n$ $\delta(\Delta_n)$ (keV)	Pair break		$ \Delta_p - \Delta_n $ $ \delta(\Delta_p - \Delta_n) $ (keV)	Ref.
						Exp.	Th.		
1.	$^{177}_{71}\text{Lu}_{106}$	$\pi:7/2[404]v:7/2[514]\otimes 9/2[624]$	970	859.9 (50.2)	457.8 (5.4)	v	v	402.2 (50.5)	[15]
		$\pi:7/2[404]v:7/2[514]\otimes 9/2[624]$	1049.5			v	v		[40]
		$\pi:9/2[514]v:7/2[514]\otimes 9/2[624]$	1230.4			v	v		[41]
		$\pi:9/2[514]v:7/2[514]\otimes 9/2[624]$	1241.5			v	v		[41]
		$\pi:9/2[514]v:7/2[514]\otimes 9/2[624]$	1325			v	v		[15]
		$\pi:9/2[514]v:7/2[514]\otimes 9/2[624]$	1336.5			v	v		[41]
		$\pi:7/2[404]v:7/2[514]\otimes 1/2[510]$	1356.5			v	v		[42]
		$\pi:9/2[514]v:7/2[514]\otimes 1/2[510]$	1437.9			v	v		[43]
		$\pi:7/2[404]v:9/2[624]\otimes 1/2[510]$				v	v		[43]
		$\pi:7/2[404]v:7/2[514]\otimes 1/2[521]$				v	v		[42]
		$\pi:7/2[404]v:7/2[514]\otimes 1/2[521]$	1632.8			v	v		[42]
		$\pi:7/2[404]v:7/2[514]\otimes 1/2[510]$	1502.6			v	v		[42]
		$\pi:1/2[411]v:7/2[514]\otimes 1/2[510]$	1617.0			v	v		[41]
		$\pi:1/2[411]v:7/2[514]\otimes 1/2[510]$	1717.5			v	v		[41]
		$\pi:5/2[402]v:7/2[514]\otimes 1/2[510]$	1728.6			v	v		[41]
$\pi:5/2[402]v:7/2[514]\otimes 1/2[510]$	1882.0	v	v	[41]					
2.	$^{175}_{71}\text{Lu}_{104}$	$\pi:7/2[404]v:7/2[514]\otimes 5/2[512]$	1391	914.2 (6.9)	571.1 (5.4)	v	v	343.2 (8.8)	[14]
		$\pi:7/2[404]v:7/2[514]\otimes 5/2[512]$	1511			v	v		[44]
		$\pi:7/2[404]v:7/2[514]\otimes 1/2[521]$	1590			v	v		[44]
		$\pi:7/2[404]v:7/2[514]\otimes 1/2[521]$	1732			v	v		[44]
3.	$^{173}_{73}\text{Ta}_{100}$	$\pi:5/2[402]v:5/2[512]\otimes 7/2[633]$	1479.7	1663.8 (381.3)	37.5 (373.5)	v	v	1626.3 (533.8)	[19]
		$\pi:7/2[404]v:7/2[633]\otimes 1/2[521]$				v	v		[19]
		$\pi:1/2[541]v:1/2[521]\otimes 7/2[633]$				v	v		[19]
		$\pi:9/2[514]v:7/2[514]\otimes 5/2[512]$				v	v		[45]
4.	$^{179}_{75}\text{Re}_{104}$	$\pi:9/2[514]v:7/2[514]\otimes 1/2[521]$	1297.6	1051.0 (211.9)	727.5 (258.8)	v	v	323.5 (363.9)	[36]
		$\pi:5/2[402]v:7/2[514]\otimes 7/2[633]$	1771.8			v	v		[36]
		$\pi:9/2[514]v:7/2[514]\otimes 7/2[633]$	1771.8+x			v	v		[36]
		$\pi:5/2[402]v:7/2[514]\otimes 5/2[512]$	1813.7			v	v		[36]
		$\pi:9/2[514]v:9/2[624]\otimes 1/2[521]$	1826.4			v	v		[36]

at higher excitation energies. It is quite interesting to note that in the case of  $^{177}\text{Lu}$  the difference between proton/neutron pairing energies is about 402.2 keV (with  $\Delta_p < \Delta_n$ ) and all the band-heads have  $\pi\nu\nu$  configurations, which is in accordance with our rule of pair break mechanism.

- (4) In Table 4, we present extension of this empirical rule to configuration dependent pairing energies. The configuration dependent pairing energies for a given nuclide presented in this table have been taken from the literature [5,6], not calculated using the 4-point formula given in Eqs. (9) and (10). In this table, we present 2 nuclides,  $^{177}\text{Ta}$  and  $^{183}\text{Re}$ , having 16 and 9 different 3qp band-heads, respectively. It is interesting to note that all 25 of these 3qp band-heads obey our rule of pair break mechanism, which further strengthens the explanation given above in point (3).

It is natural to expect that in odd-Z nuclei a neutron pair breaks and in odd-N nuclei a proton pair breaks. However, this natural expectation is not obeyed in all the observed cases. There are many examples (**marked as bold in Table 4**) that have odd-Z and/or odd-N and are not in accordance with the natural expectation but satisfy the present empirical rule of pair break mechanism.

(5) There are 16 cases for which it is not possible to test the validity of our rule because of the following reasons:

- a) The binding energies appear from systematics and hence uncertainties ( $\delta(\Delta_p)/\delta(\Delta_n)$ ) in  $\Delta_p$  and  $\Delta_n$  are large as compared to the difference in pairing energies.
- b) The configuration assignments are tentative.
- c) The lowest lying 3qp configurations are not pure.

**Table 4.** The bands for which configuration dependent pairing energies are available in the literature and breaking of proton/neutron pair is in accordance with our empirical rule of pair break mechanism.

S. no.	Nucleus	Configuration	$\Delta_p$ (keV)	$\Delta_n$ (keV)	Pair Break		Ref.
					Exp.	Th.	
1.	$^{177}_{73}\text{Ta}$	$\pi:7/2[404]v:1/2[521]\otimes 5/2[512]^*$	866	540		v	[5]
		$\pi:1/2[541]\otimes 5/2[402]\otimes 7/2[404]^*$	<b>598</b>	<b>742</b>		$\pi$	
		$\pi:7/2[404]v:1/2[521]\otimes 7/2[514]^*$	866	531		v	
		$\pi:5/2[402]v:5/2[512]\otimes 7/2[514]^*$	860	533		v	
		$\pi:1/2[541]\otimes 7/2[404]\otimes 9/2[514]$	<b>598</b>	<b>742</b>	$\pi$	$\pi$	
		$\pi:7/2[404]v:5/2[512]\otimes 7/2[514]^*$	866	533		v	
		$\pi:5/2[402]v:7/2[514]\otimes 7/2[514]$	860	538	v	v	
		$\pi:7/2[404]v:5/2[512]\otimes 7/2[514]^*$	866	551		v	
		$\pi:5/2[402]\otimes 7/2[404]\otimes 9/2[514]$	<b>597</b>	<b>742</b>	$\pi$	$\pi$	
		$\pi:9/2[514]v:5/2[512]\otimes 7/2[514]^*$	861	533		v	
		$\pi:7/2[523]v:7/2[663]\otimes 7/2[514]$	866	538	v	v	
		$\pi:5/2[402]v:7/2[514]\otimes 9/2[624]^*$	860	550		v	
		$\pi:9/2[514]v:7/2[633]\otimes 7/2[514]$	861	538	v	v	
		$\pi:7/2[404]v:7/2[514]\otimes 9/2[624]^*$	866	550		v	
		$\pi:9/2[514]v:7/2[514]\otimes 9/2[624]$	861	550	v	v	
$\pi:9/2[514]v:9/2[624]\otimes 7/2[633]$	861	552	v	v			
2.	$^{183}_{75}\text{Re}$	$\pi:5/2[402]v:1/2[510]\otimes 9/2[624]$	711	497	v	v	[6]
		$\pi:5/2[402]v:3/2[512]\otimes 9/2[624]$	709	500	v	v	
		$\pi:5/2[402]v:11/2[615]\otimes 9/2[624]$	710	521	v	v	
		$\pi:9/2[514]v:7/2[503]\otimes 9/2[624]$	723	543	v	v	
		$\pi:5/2[402]\otimes 9/2[514]\otimes 1/2[541]$	<b>552</b>	<b>667</b>	$\pi$	$\pi$	
		$\pi:5/2[402]v:7/2[503]\otimes 9/2[624]$	710	537	v	v	
		$\pi:9/2[514]v:1/2[510]\otimes 9/2[624]$	730	511	v	v	
		$\pi:9/2[514]v:11/2[615]\otimes 9/2[624]$	728	538	v	v	
		$\pi:11/2[505]v:11/2[615]\otimes 9/2[624]$	726	547	v	v	

\*bands not observed experimentally

- d) High- $j$  orbits are involved in given 3qp configurations and hence there is competition between Coriolis (anti-pairing effects) and pairing correlation.

#### 4. Conclusions

In summary, we conclude that in the case of 3qp bands a proton pair will break if proton pairing energy ( $\Delta_p$ ) is less than neutron pairing energy ( $\Delta_n$ ) and a neutron pair will break if neutron pairing energy ( $\Delta_n$ ) is less than proton pairing energy ( $\Delta_p$ ) to form a 3qp state at an excitation energy  $\geq 1$  MeV. This rule works reasonably well in all cases for which:

- a) Configuration assignments are known.
- b) There is no significant configuration mixing.
- c) The uncertainties in the binding energies are small as compared to the difference between the pairing energies.

This rule will be useful to experimentalists for configuration assignments to lower lying 3qp bands especially in those cases where experimental information for distinction among the competing configurations is not sufficient.

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