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Improved Hilbert moment thermal QCD sum rules for B_s meson and stability with respect to moment parameter

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Abstract: In this paper, we deal with the temperature dependence of the leptonic decay constant and mass of B_s meson in the framework of the Hilbert moment QCD sum rule. In our calculations, we improve the thermal QCD sum rules, taking into account the thermal spectral density and the perturbative 2-loop order corrections to the correlation function. Moreover, we investigate the stability of the results with respect to the Hilbert moment parameter. Our numerical calculations demonstrate that the mass and decay constant are insensitive to the variation of temperature up to $T \cong 100\,\text{MeV}$; however, after this value, they start to decrease with increasing temperature. We observe that the results are stable for different values of the Hilbert moment parameter, n. At deconfinement or critical temperature, the decay constant and mass approach to roughly 16% and 78% of their values at zero temperature, respectively. The obtained results at zero temperature are in good agreement with the existing experimental data as well as predictions of the other nonperturbative models.

Key words: Thermal QCD sum rule, leptonic decay constant, pseudoscalar mesons

1. Introduction

The investigation of properties of the pseudoscalar mesons with 1 heavy and 1 light quark can give information for understanding the nature of CP violation. As is known, the pseudoscalar meson decay width in the lowest order is related to the Cabibbo–Kobayashi–Maskava (CKM) matrix element and is given by:

$$\Gamma\left(P \to lv\right) = \frac{G_F^2}{8\pi} f_P^2 m_l^2 m_p \left(1 - \frac{m_l^2}{m_P^2}\right) \left|V_{q_1 q_2}\right|^2, \tag{1}$$

where m_P and m_l are the pseudoscalar meson and lepton masses, respectively; $V_{q_1 q_2}$ is the CKM matrix element; G_F is the Fermi coupling constant; and f_P is the leptonic decay constant. Since theoretical values of f_P are known, the CKM matrix element can be determined.

In this paper, the thermal QCD sum rules obtained in [1,2] are improved by taking into account the thermal spectral density and perturbative 2-loop order corrections to the correlation function. The determination of properties of the decay constants, masses, coupling constants, and form factors of mesons in medium is one of the most important research areas in particle physics. A large number of experimental and theoretical studies have been performed in the literature during the last 2 decades in this respect. In order to understand the hadronic properties and explain the related experimental results, we need to evaluate the hadronic matrix

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elements of the operators in QCD beyond the perturbation theory. Some of the nonperturbative approaches are QCD sum rules, lattice theory, heavy quark effective theory, chiral perturbation theory, and phenomenological quark models. The QCD sum rule approach [3] is one of the most powerful theoretical tools in meson studies [3–5] and this method was extended to finite temperatures in [6]. This method is one of the reliable and powerful approaches in understanding the thermal properties of light-light [7–9], heavy-light [2,10,11], and heavy-heavy [12–16] mesons.

To obtain the thermal sum rules, we need to calculate the correlation function in 2 different ways: from the QCD side by using quark degrees of freedom and from the phenomenological side by using the hadronic parameters. In the QCD side, the correlation function is evaluated with the help of operator product expansion (OPE). The thermal version of QCD sum rules has some new features compared to the one in vacuum [17–21]. The Lorentz invariance is broken in medium with the choice of the thermal rest frame. In comparison to the vacuum QCD sum rules, additional operators also appear in thermal OPE. In the phenomenological side, the correlation function is obtained by inserting a complete set of intermediate hadronic states having the same quantum numbers as the interpolating current $J(x)$. Matching then these 2 representations, the thermal sum rules for the leptonic decay constant and mass of hadrons are obtained.

The decay constant of B_s meson with quantum numbers $I(J^P) = 0(0^-)$ is defined by the following matrix element: $\langle 0|\bar{s}\gamma_\mu\gamma_5 b|\rangle = if_{B_s}q_\mu$. The properties of this meson were deeply investigated in [1,2]. In this paper, we reanalyze the temperature dependences of the leptonic decay constant and mass of the B_s meson by choosing its interpolating current as $J(x) = (m_b + m_s)\ :\bar{s}(x)\,i\gamma_5 b(x):$. In our calculations, we take into account the thermal spectral density and the perturbative 2-loop order corrections to the correlation function and improve the thermal QCD sum rules obtained in [1,2]. As a result, we show that the final calculations with additional contributions are significantly important in the investigation of the temperature behavior of the leptonic decay constants and the results are stable for different values of Hilbert moment parameter$n$.

2. Improved thermal QCD sum rules for B_s meson

To calculate the B_s meson mass and its leptonic decay constant at a finite temperature, we start with the following 2-point thermal correlator:

$$\Pi(q,T) = i\int d^4x e^{iq\cdot x} Tr\left(\rho T\left(J(x)J^{+}(0)\right)\right), \tag{2}$$

where T indicates the time ordered product and $\rho = e^{-\beta H}/Tre^{-\beta H}$ is the thermal density matrix of QCD at temperature $T = 1/\beta$.

The correlation function can be written in terms of a dispersion integral [2]:

$$\Pi(q^2) = \int_{\infty}^{0} ds\frac{\rho(s)}{s-q^2} + \text{ subtractions} \tag{3}$$

where $\rho(q,T) = \frac{1}{\pi}\Pi(q,T)\tanh\left(\frac{\beta q_0}{2}\right)$ is called the thermal spectral density at fixed $|\mathbf{q}|$.

In order to remove the subtraction terms, we use the Hilbert moment method. The Hilbert moment transformation is expressed as follows:

$$M_nF\left(Q^2\right)=\frac{(-1)^{n+1}}{(n+1)!}\left(\frac{d}{dQ^2}\right)^{n+1}F\left(Q^2\right)\Big|_{Q^2=0}. \tag{4}$$

To obtain the QCD side, we need to evaluate the correlation function via the OPE in which the short distance and long distance contributions are separated. The short distance contribution is calculated with the perturbation theory. For this aim, we need to calculate the thermal spectral density in the lowest order of the perturbation theory. Using the thermal fermion propagator at real-time formalism in Eq. (2) and carrying out the integration over k_0, we obtain the imaginary part of the correlation function in the following form:

$$\begin{aligned}&Im\Pi\left(\mathrm{q,T}\right)=-N_c\int\frac{d\mathbf{k}}{8\pi^2}\frac{1}{\omega_1\omega_2}\left\{\left(m_b^2-\omega_1q_0-m_bm_s\right)\right.\\&\times\left[(1-n_1-n_2+2n_1n_2)\,\delta\left(q_0-\omega_1-\omega_2\right)-(n_1+n_2-2n_1n_2)\,\delta\left(q_0-\omega_1+\omega_2\right)\right]\\&+\left(m_b^2+\omega_1q_0-m_bm_s\right)\\&\left.\times\left[(1-n_1-n_2+2n_1n_2)\,\delta\left(q_0+\omega_1+\omega_2\right)-(n_1+n_2-2n_1n_2)\,\delta\left(q_0+\omega_1-\omega_2\right)\right]\right\},\end{aligned} \tag{5}$$

where m_b and m_s are quark masses, $\omega_1=\sqrt{\mathbf{k}^2+m_b^2}$, $\omega_2=\sqrt{\mathbf{k}^2+m_s^2}$, $n(x)=[\exp(\beta x)+1]^{-1}$ is the Fermi distribution function, $n_1=n(\omega_1)$, and $n_2=n(\omega_2)$. The terms including and not including the Fermi distribution functions express the medium and the vacuum contributions, respectively. The delta-functions in the different terms of Eq. (5) control the regions of nonvanishing parts of spectral density, which define the position of the branch cuts. As seen, the term including $\delta\left(q_0-\omega_1-\omega_2\right)$ contributes when $q_0=\omega_1+\omega_2$ and the term including $\delta\left(q_0-\omega_1+\omega_2\right)$ contributes when $q_0=\omega_1-\omega_2$. Taking into account the mentioned features above and using

$$(n_1+n_2-2n_1n_2)\tanh\frac{\beta q_0}{2}=n_2-n_1 \text{ for } q_0=\omega_1-\omega_2, \tag{6}$$

$$(1-n_1-n_2+2n_1n_2)\tanh\frac{\beta q_0}{2}=1-n_2-n_1 \text{ for } q_0=\omega_1+\omega_2, \tag{7}$$

the annihilation and scattering parts of the thermal spectral density are obtained in the following forms [20]:

$$\rho_a\left(s,T\right)=\rho_0\left(s\right)\left[1-n\left(\frac{\sqrt{s}}{2}\left(1+\frac{m_b^2-m_s^2}{s}\right)\right)-n\left(\frac{\sqrt{s}}{2}\left(1-\frac{m_b^2-m_s^2}{s}\right)\right)\right], \tag{8}$$

for $(m_b+m_s)^2\leq s\leq\infty$,

$$\rho_s\left(s,T\right)=\rho_0\left(s\right)\left[n\left(\frac{\sqrt{s}}{2}\left(1+\frac{m_b^2-m_s^2}{s}\right)\right)-n\left(\frac{\sqrt{s}}{2}\left(1-\frac{m_b^2-m_s^2}{s}\right)\right)\right], \tag{9}$$

for $0\leq s\leq(m_b-m_s)^2$. Here, $\rho_0\left(s\right)$ is the spectral density in the lowest order of the perturbation theory at zero temperature:

$$\rho_0\left(s\right)=\frac{3\left(m_b+m_s\right)^2}{8\pi^2 s}v\left(s\right)q^2\left(s\right), \tag{10}$$

where $q\left(s\right)=s-\left(m_b-m_s\right)^2$ and $v\left(s\right)=\left(1-4m_sm_b/q\left(s\right)\right)^{1/2}$ [22,23].

In our calculations, we also take into account the perturbative 2-loop order α_s correction to the spectral density, which is given in [4]:

$$\begin{aligned}\rho_{\alpha_s}(s,T) &= \tfrac{4\alpha_s}{3\pi}\rho_0(s)\left[\tfrac{3}{8}\left(7-v^2\right)+\sum_{i=1}^{2}\left(v+v^{-1}\right)\left[Li_2\left(\alpha_1\alpha_2\right)-Li_2\left(-\alpha_i\right)-\ln\alpha_i\ln\beta_i\right]\right. \\ &\left. +A_i\ln\alpha_i+B_i\ln\beta_i\right]\end{aligned}, \tag{11}$$

where $Li_2(x)$ is the Spence function, $Li_2(x)=-\int_0^x dt\ t^{-1}\ln(1-t)$. Furthermore, A_i, B_i, α_i, and β_i are expressed in the following forms:

$$A_1=\frac{3}{4}\frac{3m_s+m_b}{m_s+m_b}-\frac{19+2v^2+3v^4}{32v}-\frac{m_s\left(m_s-m_b\right)}{q^2(s)\,v\,(1+v)}\left(1+v+\frac{2v}{1+\alpha_1}\right), \tag{12}$$

$$B_1=2+\frac{2\left(m_s^2-m_b^2\right)}{q(s)\,v},\quad \alpha_1=\frac{m_s}{m_b}\frac{1-v}{1+v},\quad \beta_1=\frac{\sqrt{1+\alpha_1}\,(1+v)^2}{4v}, \tag{13}$$

where $v\equiv v(s)$. The expressions of A_2, B_2, α_2, and β_2 are obtained from Eqs. (12) and (13) by the interchanging of the strange and bottom quark masses.

Matching the OPE and the hadron representations of the correlation function, and using the quark-hadron duality, the improved sum rule is obtained as:

$$\frac{f_{B_s}^2 m_{B_s}^4}{Q^2+m_{B_s}^2}=\int_{(m_b+m_s)^2}^{s_0(T)} ds\frac{\rho_a(s,T)+\rho_{\alpha_s}(s,T)}{s+Q^2}+\int_0^{(m_b-m_s)^2} ds\,\frac{\rho_s(s,T)}{s+Q^2}+\Pi^{np}\left(Q^2\right), \tag{14}$$

where $Q^2=-q^2$. We apply the Hilbert moment transformation to both sides of this sum rule and we get:

$$m_{B_s}^2(T)=F_n(T)/F_{n+1}(T), \tag{15}$$

$$f_{B_s}^2(T)=m_{B_s}^{2n}(T)\,F_n(T), \tag{16}$$

where $F_n(T)$ function has the following form:

$$F_n(T)=\int_{(m_b+m_s)^2}^{s_0(T)} ds\,\frac{\rho_a(s,T)+\rho_{\alpha_s}(s,T)}{s^{n+2}}+\int_0^{(m_b-m_s)^2} ds\,\frac{\rho_s(s,T)}{s^{n+2}}+M_n\Pi^{np}. \tag{17}$$

Here, $s_0(T)$ is the temperature-dependent continuum threshold. In Eq. (17), $M_n\Pi^{np}$ shows the nonperturbative part of the QCD side in the Hilbert moment transformed scheme, which is given by:

$$\begin{aligned}M_n\Pi^{np} &= -\tfrac{\langle\bar{q}q\rangle}{m_b^{2n+1}}K(\varepsilon)+\tfrac{\left\langle\alpha_s G^2\right\rangle}{12\pi}\tfrac{1}{m_b^{2n+2}}L(\varepsilon)+\tfrac{1}{4}M_0^2\langle\bar{q}q\rangle\tfrac{(n+1)(n+2)}{m_b^{2n+3}}N(\varepsilon) \\ &+\tfrac{4}{81}\pi\alpha_s\rho\langle\bar{q}q\rangle^2\tfrac{1}{m_b^{2n+4}}(n+2)\left(n^2+10n+9\right)\end{aligned}, \tag{18}$$

where $K(\varepsilon)$, $L(\varepsilon)$, and $N(\varepsilon)$ express the contributions of operators with various dimensions and are given by:

$$K(\varepsilon)=1-\frac{1}{2}\varepsilon(n-1)+\frac{1}{2}\varepsilon^2(n+2)(n+1)-\frac{1}{2}\varepsilon^3(n+3)\left(1+\frac{1}{3}(n+2)(n-2)\right),$$

$$L(\varepsilon) = 1 + \varepsilon\left(3n^2 + 7n + 5\right) + \varepsilon\left[\left(6 + 9n + 3n^2\right)\ln\,\varepsilon + 3\sum_{r=1}^{n+1}\frac{1}{r}\left(n+2-r\right)\left(n+1-r\right)\right], \qquad (19)$$

$$N(\varepsilon) = 1 - \frac{1}{3}\varepsilon\left(n-3\right),$$

where $\varepsilon = \frac{m_s}{m_b}$.

For the quark and gluon condensates at finite temperature, we use the results obtained in chiral perturbation theory [24,25]. The temperature dependency of these condensates are written as:

$$\langle\bar{q}q\rangle = \langle 0|\bar{q}q|0\rangle\left[1 - 0.4\left(\frac{T}{Tc}\right)^4 - 0.6\left(\frac{T}{Tc}\right)^8\right], \qquad (20)$$

$$\left\langle\alpha_s G^2\right\rangle = \left\langle 0|\alpha_s G^2|0\right\rangle\left[1 - \left(\frac{T}{T_c}\right)^8\right]. \qquad (21)$$

For the numerical evaluation of the improved sum rule, we use $m_s = 120$ MeV, $m_b = 4.4\,\mathrm{GeV}$ for quark masses and $\langle 0\,|\bar{q}q|\,0\rangle = -(0.015 \pm 0.001)\,\mathrm{GeV}^3$ and $\left\langle 0\left|\frac{\alpha_s}{\pi}G^2\right|0\right\rangle = (0.024 \pm 0.012)\,\mathrm{GeV}^4$ for quark and gluon condensates at zero temperature. The sum rules also include 2 auxiliary parameters: continuum threshold s_0 and Hilbert moment parameter n. The continuum threshold is not completely arbitrary and it is related to the energy of the first excited state with the same quantum numbers of the interpolating current. The continuum threshold is also temperature-dependent and is given as [10]:

$$s_0(T) = s_0(0)\frac{\langle\bar{q}q\rangle}{\langle 0|\bar{q}q|0\rangle}\left[1 - \left(\frac{(m_b + m_s)^2}{s_0(0)}\right)\right] + (m_b + m_s)^2. \qquad (22)$$

Therefore, we look for regions of these parameters such that the dependencies of the mass and decay constant on these parameters are weak. We use the interval $s_0(0) = (34 - 35)\,\mathrm{GeV}^2$ for the continuum threshold and $n = 1$, $n = 4$, and $n = 5$ for the Hilbert moment parameters. Finally, we plot the temperature dependency of the leptonic decay constant and the mass of the B_s meson in Figures 1 and 2. As shown in these graphs, the decay constant and the mass remain insensitive to the variation of the temperature up to $T \sim 100\,\mathrm{MeV}$; however, after this point, they start to diminish. At deconfinement or critical temperature, the decay constant and the mass approach to roughly 16% and 78% of their values at zero temperature, respectively. As seen, the mass and the leptonic decay constant are stable for different Hilbert moment parameters, n.

Our investigations show that the vacuum values of the mass and the decay constant of B_s meson are $m_{B_s} = 5.367\,\mathrm{GeV}$ and $f_{B_s} = 0.24\,\mathrm{GeV}$. These results are in good agreement with the existing experimental data $m_{B_s} = 5.36677 \pm 0.00024\,\mathrm{GeV}$[26], with predictions of other nonperturbative models [13–16] (for more details, see [2]) and with lattice QCD calculations $m_{B_s} = 5.385 \pm 0.044\,\mathrm{GeV}$, $f_{B_s} = 0.253 \pm 0.015\,\mathrm{GeV}$ [27]. We also make an error analysis to understand the sensitivity of obtained results to uncertainties on the continuum threshold and quark and gluon condensates. Our investigations show that the decay constant and the mass values are insensitive to these uncertainties; to demonstrate this situation, we give the decay constant and the mass values in the Table at $T = 150\,\mathrm{MeV}$. Our results for the leptonic decay constants as well as their behavior with respect to the temperature can be verified in future experiments.

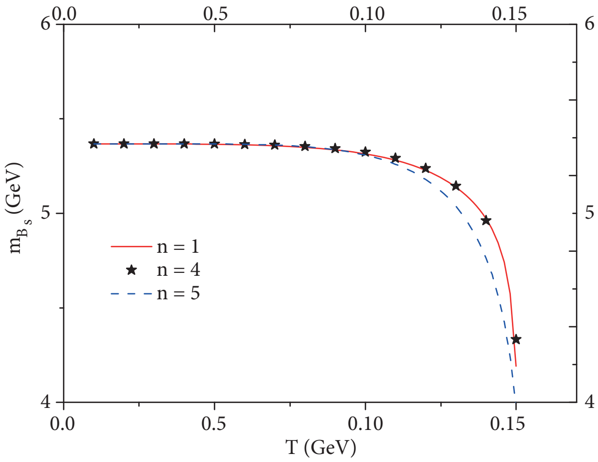

Figure 1. The temperature dependence of the mass of B_s meson for values of $s_0(0) = 35\,\text{GeV}^2$, $\langle 0\,|\bar{q}q|\,0\rangle = -0.014\,\text{GeV}^3$, $\langle 0\,|\frac{\alpha_s}{\pi}G^2|\,0\rangle = 0.012\,\text{GeV}^4$, $n = 1$, $n = 4$, and $n = 5$.

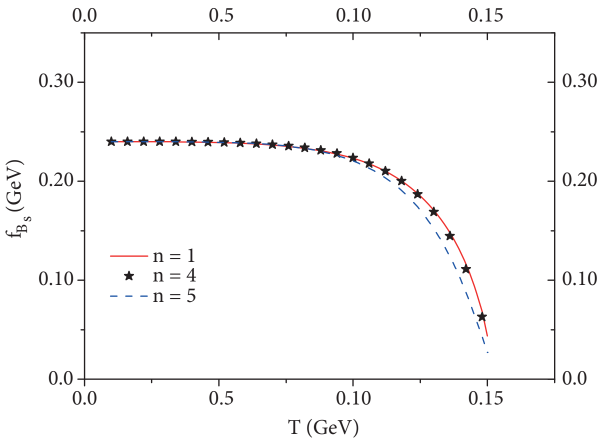

Figure 2. The temperature dependence of the leptonic decay constant of B_s meson for values of $s_0(0) = 35\,\text{GeV}^2$, $\langle 0\,|\bar{q}q|\,0\rangle = -0.014\,\text{GeV}^3$, $\langle 0\,|\frac{\alpha_s}{\pi}G^2|\,0\rangle = 0.012\,\text{GeV}^4$, $n = 1$, $n = 4$, and $n = 5$.

Table. The calculated mass and decay constant of B_s meson at $T = 150\,\text{MeV}$.

$s_0(0)$(GeV)	$\langle 0\,|\frac{\alpha_s}{\pi}G^2|\,0\rangle$ (GeV^4)	$\langle 0\,|\bar{q}q|\,0\rangle$ (GeV^3) (GeV^3)	m_{B_s} (GeV)	f_{B_s} (GeV)
34	0.024 ± 0.012	0.015 ± 0.001	5.10 ± 0.03	0.082 ± 0.002
35	0.024 ± 0.012	0.015 ± 0.001	4.15 ± 0.05	0.042 ± 0.002

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