

Exact traveling wave solutions of the perturbed Klein–Gordon equation with quadratic nonlinearity in (1+1)-dimension, Part I: Without local inductance and dissipation effect

Zai-yun ZHANG*

School of Mathematics, Hunan Institute of Science and Technology, Yueyang, 414006 Hunan Province, P. R. China

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Abstract: In this paper, the auxiliary ordinary differential equation is employed to solve the perturbed Klein–Gordon equation with quadratic nonlinearity in the (1+1)-dimension without local inductance and dissipation effect. By using this method, we obtain abundant new types of exact traveling wave solutions.

Key words: Perturbed Klein–Gordon equation, traveling wave solutions, auxiliary ordinary differential equation method

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1. Introduction

It is well known that traveling wave solutions of nonlinear partial differential equations (NPDEs) play an important role in the study of nonlinear wave phenomena. The wave phenomena are observed in fluid dynamics, plasma, elastic media, optical fibers, etc. In the recent decade, many methods were developed and proposed for finding the exact solutions of nonlinear evolution equations to NPDEs, such as the modified mapping method and the extended mapping method [1], trigonometric function series method [2], modified trigonometric function series method [3], bifurcation method and qualitative theory of dynamical systems [4], modified G'/G -expansion method [5], dynamic systems approach [6], exp-function method [7], reliable analysis method [8], homotopy perturbation method [9,10], modified variational iteration method [11], variational iteration method [12], He's variational iteration method [13], Riccati equation and Cole–Hopf transformation [14], multiple exp-function method [15,16], transformed rational function method [17], symmetry algebra method (consisting of Lie point symmetries) [18], Wronskian technique [19,20], linear superposition principle [21,22], and so on.

In this paper, we consider exact traveling wave solutions of the perturbed Klein–Gordon equation (KGE) with quadratic nonlinearity in the (1+1)-dimension without local inductance and dissipation effect:

$$u_{tt} - u_{xx} + f(u) = \varepsilon(\alpha u + \beta u_{xt} + \gamma u_{tt}), \quad (1.1)$$

where $f(u) = au - bu^2$; $a, b, \alpha, \beta, \gamma$ are constants; and ε is the perturbation parameter.

Recently, Biswas et al. and Sassaman and Biswas [23,24] investigated the perturbed KGE,

$$u_{tt} - u_{xx} + f(u) = \varepsilon(\alpha u + pu_t + qu_x + \beta u_{xt} + \gamma u_{tt}), \quad (1.2)$$

*Correspondence: zhangzaiyun1226@126.com

and obtained the exact 1-soliton solution. These perturbation terms typically arise in the study of long Josephson junction in the context of the sine-Gordon equation (SGE). Since the SGE can be approximated by the KGE, an exact solution of the perturbed KGE will make sense in the context of the study of the SGE. For the perturbation terms, α represents losses across the junction, p accounts for dissipative losses in Josephson junction theory due to tunneling of normal electrons across the dielectric barrier, q is generated by a small inhomogeneous part of the local inductance, β represents diffusion, and γ is the capacity inhomogeneity. More details are presented in [24]. As we all known, the term pu_t (call dissipation effect) is generated by a variety of dissipative mechanisms. For the dissipation effect, we can see [25] and [26].

When $\varepsilon = 0$, Eq. (1.1) reduces to the KGE with quadratic nonlinearity:

$$u_{tt} - u_{xx} + \alpha u - \beta u^2 = 0.$$

If $f(u) = au - bu^3$, Eq. (1.1) becomes the KGE with cubic nonlinearity:

$$u_{tt} - u_{xx} + \alpha u - \beta u^3 = 0. \quad (1.3)$$

Applying the trigonometric function series method, Zhang [2] studied Eq. (1.3) and obtained the new exact traveling wave solutions as complex linear combinations of kink solitary wave solutions and bell solitary wave solutions. Eq. (1.3) describes the propagation of dislocations within crystals, the Bloch wall motion of magnetic crystals, the propagation of a “splay wave” along a lied membrane, the unitary theory for elementary particles and the propagation of magnetic flux on a Josephson line, etc. More details are presented in [2] and the references therein. Quite recently, based on modified G'/G -expansion method [5], Xiao and Zhang [27] investigated Eq. (1.3) and obtained the exact traveling wave solutions expressed in terms of hyperbolic functions, trigonometric functions, and rational functions.

In the absence of perturbed terms, there is a large literature concerning the exact traveling wave solutions of the KGE and generalized Klein–Gordon equation (gKGE), as we can see in [28–33]. In [30], Sassaman and Biswas obtained the exact 1-soliton solution of 5 different forms of the gKGE by using the solitary wave solution ansatz. In [31], Sassaman and Biswas obtained the exact 1-soliton solution of 5 different forms of the KGE in 1 + 2 dimensions. In [32], Sassaman and Biswas investigated the coupled KGEs in (1+1) and (1+2) dimensions with the cubic law of nonlinearity and arbitrary power law nonlinearity. They then obtained the 1-soliton solution of the coupled system. In [33], Sassaman et al. studied topological and nontopological soliton solutions of 5 different forms of the gKGE in 1+2 dimensions. However, for the presence of a perturbed term, we can see [34–37]. In [34], Sassaman and Biswas obtained the 1-soliton solution to the perturbed KGE by He’s semiinverse variational principle. In [35], Sassaman et al. obtained the topological and nontopological soliton solutions of the perturbed KGE by He’s semiinverse variational principle and carried out the integration of the KGE with 5 types of nonlinearity in the presence of a few perturbation terms. In [36], Sassaman and Biswas obtained the adiabatic variation of the soliton velocity, in the presence of perturbation terms, of the phi-four model and the nonlinear KGEs. There are 3 types of models of the nonlinear KGE with power law nonlinearity. In [37], Esfahani considered the solitary wave solutions of the perturbed KGE by using the sech-ansatz method.

Recently, Parkes et al. [38] used the Jacobi elliptic function expansion method to find double periodic solutions to Eq. (1.3). There have also been some methods developed to solve the KGE-type equations, such as the auxiliary ordinary differential equation method [39,40], the Weierstrass elliptic function method [41], and the elliptic equation rational expansion method [42]. However, these methods can only obtain the periodic

solutions and their limiting solutions; they cannot obtain most of those solutions that are obtained by using the auxiliary equation method. In our contribution, we are interested in exact traveling wave solutions of perturbed KGE Eq. (1.1) with quadratic nonlinearity in the (1+1)-dimension without local inductance and dissipation effect. In this paper, we will use the auxiliary ordinary differential equation method to construct exact solutions to Eq. (1.1). By using this method, we obtain abundant new types of exact traveling wave solutions.

Remark 1.1. The Wronskian technique[19,20] provides a direct but powerful approach for constructing rational solutions, positon solutions, and complexiton solutions to the Boussinesq equation. Within Wronskian formulations, soliton solutions and rational solutions are usually expressed as some kind of logarithmic derivatives of Wronskian-type determinants and the determinants involved are made of eigenfunctions satisfying linear differential equations. Clearly, Wronskian formulations connect nonlinear problems with linear problems, and thus soliton equations can be solved by means of linear theories. More details were presented in [20].

Remark 1.2. In [21] and [22], though the linear superposition principle does not apply to nonlinear differential equations in general, Ma and his colleagues emphasized that Hirota bilinear equations still can possess the linear superposition principle among exponential wave solutions. This guarantees the existence of linear subspaces of solutions and amends the diversity of exact solutions by other direct methods. Generally speaking, the linear superposition principle does not apply to nonlinear differential equations, but it can hold for some special kinds of wave solutions to Hirota bilinear equations, such as, for example exponential waves. More details were presented in [21] and [22].

2. Description of the auxiliary ordinary differential equation method

Before proceeding to our analysis, we illustrate the basic idea behind this method as follows.

A given NPDE,

$$H(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \dots) = 0,$$

can be converted to an ordinary differential equation,

$$F(u, u_\xi, u_{\xi\xi}, \dots) = 0, \tag{2.1}$$

where $u(x, t) = u(\xi)$, $\xi = x - \omega t$. Eq. (2.1) is then integrated as long as all terms contain derivatives where integration constants are considered zeros.

To determine $u(\xi)$ explicitly, we follow the below main steps.

Supposing that $u(\xi)$ can be expanded as follows:

$$u(\xi) = \sum_{i=0}^n a_i z^i(\xi), \xi = x - \omega t, \tag{2.2}$$

and $z(\xi)$ is the solutions of the auxiliary ordinary differential equation (AODE)

$$\left(\frac{dz}{d\xi}\right)^2 = Az^2(\xi) + Bz^3(\xi) + Cz^4(\xi), \tag{2.3}$$

where a_i, A, B, C , and ω are all real constants to be determined, n is a positive integer that can be determined. Moreover, the solutions $z(\xi)$ of the AODE (2.3) are given by Remark 2.1.

Integer n can be determined by considering the homogeneous balance between the nonlinear term and the highest order derivative of u in Eq. (2.1). Substituting (2.2) and (2.3) into (2.1) with Maple and equating

to zero the coefficients of all powers of $z(\xi)$ yields a set of algebraic equations for $a_i (i = 1, 2, \dots, n)$, A , B , C , and ω . The exact traveling wave solutions are then obtained by substituting each solution of this set of algebraic equations with $u(x, t) = u(\xi)$, $\xi = x - \omega t$ into (2.2).

Remark 2.1. As shown in this Remark, we present the exact traveling wave solutions of the AODE (2.3) as follows [40].

Solutions of AODE (2.3) with $\Delta = B^2 - 4AC$, $\varepsilon = \pm 1$.

Family 1. If $a > 0$, then $z(\xi) = \frac{-AB\text{sech}^2(\frac{\sqrt{A}}{2}\xi)}{B^2 - aAC(1 + \varepsilon \tanh(\frac{\sqrt{A}}{2}\xi))^2}$.

Family 2. If $A > 0$, then $z(\xi) = \frac{AB\text{csch}^2(\frac{\sqrt{A}}{2}\xi)}{B^2 - AC(1 + \varepsilon \coth(\frac{\sqrt{A}}{2}\xi))^2}$.

Family 3. If $A > 0, \Delta > 0$, then $z(\xi) = \frac{2A\text{sech}(\sqrt{A}\xi)}{\varepsilon\sqrt{\Delta} - B\text{sech}(\sqrt{A}\xi)}$.

Family 4. If $A < 0, \Delta > 0$, then $z(\xi) = \frac{2A\text{sech}(\sqrt{-A}\xi)}{\varepsilon\sqrt{\Delta} - B\text{sech}(\sqrt{-A}\xi)}$.

Family 5. If $A > 0, \Delta < 0$, then $z(\xi) = \frac{2a\text{csch}(\sqrt{a}\xi)}{\varepsilon\sqrt{-\Delta} - b\text{csch}(\sqrt{a}\xi)}$.

Family 6. If $a < 0, \Delta > 0$, then $z(\xi) = \frac{2A\text{csc}(\sqrt{-A}\xi)}{\varepsilon\sqrt{\Delta} - B\text{csc}(\sqrt{-A}\xi)}$.

Family 7. If $A > 0, C > 0$, then $z(\xi) = \frac{-A\text{sech}^2(\frac{\sqrt{A}}{2}\xi)}{B + 2\varepsilon\sqrt{AC}\tanh(\frac{\sqrt{A}}{2}\xi)}$.

Family 8. If $A < 0, C > 0$, then $z(\xi) = \frac{-A\text{sech}^2(\frac{\sqrt{-A}}{2}\xi)}{B + 2\varepsilon\sqrt{-AC}\tanh(\frac{\sqrt{-A}}{2}\xi)}$.

Family 9. If $A > 0, C > 0$, then $z(\xi) = \frac{A\text{csch}^2(\frac{\sqrt{A}}{2}\xi)}{B + 2\varepsilon\sqrt{AC}\coth(\frac{\sqrt{A}}{2}\xi)}$.

Family 10. If $A > 0, C > 0$, then $z(\xi) = \frac{-A\text{csc}^2(\frac{\sqrt{-A}}{2}\xi)}{B + 2\varepsilon\sqrt{-AC}\cot(\frac{\sqrt{-A}}{2}\xi)}$.

Family 11. If $A > 0, \Delta = 0$, then $z(\xi) = -\frac{A}{B}[1 + \varepsilon \tanh(\frac{\sqrt{A}}{2}\xi)]$.

Family 12. If $A > 0, \Delta = 0$, then $z(\xi) = -\frac{A}{B}[1 + \varepsilon \coth(\frac{\sqrt{A}}{2}\xi)]$.

Family 13. If $A > 0$, then $z(\xi) = \frac{4Ae^{\varepsilon\sqrt{A}\xi}}{(e^{\varepsilon\sqrt{A}\xi} - B)^2 - 4AC}$.

Family 14. If $A > 0, B = 0$, then $z(\xi) = \frac{\pm 4Ae^{\varepsilon\sqrt{A}\xi}}{1 - 4ACe^{2\varepsilon\sqrt{A}\xi}}$.

Remark 2.2. It is worth mentioning that the idea of transforming nonlinear PDEs into a solvable ODE (2.1) was presented in [14]. Moreover, the transformed rational function method [17] provides a more general process of constructing exact solutions this way, since a polynomial is just a special case of a rational function. More precisely, it was shown that the transformed rational function method provides a more systematical and convenient handling of the solution process of nonlinear equations, unifying the tanh-function type methods, the homogeneous balance method, the exp-function method, the mapping method, and the F-expansion type methods. Its key point is to search for rational solutions to variable-coefficient ordinary differential equations transformed from given partial differential equations.

3. Exact traveling wave solutions for Eq. (1.1)

In this section, we apply the AODE method presented in Section 2 and obtain the exact traveling wave solutions for Eq. (1.1).

By the basic ideas of the AODE method on the PKGE (1.1), we obtain

$$(\omega^2 - 1 + \varepsilon\beta\omega - \varepsilon\gamma\omega^2)u'' + (a - \varepsilon\alpha)u - bu^2 = 0,$$

where $u'' = u_{\xi\xi}$. Assume that $D = \omega^2 - 1 + \varepsilon\beta\omega - \varepsilon\gamma\omega^2$, $E = a - \varepsilon\alpha$, $F = -b$, such that the above equation is transformed into the following form:

$$Du''(\xi) + Eu(\xi) + Fu^2(\xi) = 0, \tag{3.1}$$

where D, E, F are constants.

Investigating the homogeneous balance between $u''(\xi)$ and $u^2(\xi)$ in Eq. (3.1), we obtain $n = 2$, and so the solution of (3.1) is in the form

$$\phi(\xi) = a_0 + a_1z(\xi) + a_2z(\xi)^2, \tag{3.2}$$

where a_0, a_1, a_2, ω are constants to be determined. $z(\xi)$ satisfies Eq. (2.3). After substituting (2.3) and (3.2) into (3.1) with the help of Maple and collecting coefficients of $z^j(\xi)$ ($j = 0, 1, 2, 3, 4$), we have the following algebraic equations:

$$\begin{cases} 6Da_2 + Fa_2^2 = 0, \\ Ea_0 + Fa_0^2 = 0, \\ Ea_1 + 2Fa_0a_1 + DAA_1 = 0, \\ \frac{3}{2}DBa_1 + 2Fa_0a_2 + Fa_1^2 + Ea_2 + 4DAa_2 = 0, \\ 2DCA_1 - 5DBa_2 + 2Fa_1a_2 = 0. \end{cases} \tag{3.3}$$

Solving this algebraic equation with the aid of Maple, we obtain

$$a_0 = 0, a_1 = \frac{3BD}{2F}, a_2 = 0, A = -\frac{E}{D}, C = 0, \tag{3.4}$$

$$a_0 = -\frac{E}{F}, a_1 = \frac{3BD}{2F}, a_2 = 0, A = \frac{E}{D}, C = 0, \tag{3.5}$$

$$a_0 = 0, a_1 = 0, a_2 = -\frac{6CD}{F}, A = -\frac{E}{4D}, B = 0, \tag{3.6}$$

$$a_0 = -\frac{E}{F}, a_1 = 0, a_2 = -\frac{6CD}{F}, A = \frac{E}{4D}, B = 0, \tag{3.7}$$

$$a_0 = 0, a_1 = -\frac{3BD}{F}, a_2 = \frac{3B^2D^2}{2EF}, A = -\frac{E}{D}, C = -\frac{B^2D}{4E}, \tag{3.8}$$

$$a_0 = -\frac{E}{F}, a_1 = -\frac{3BD}{F}, a_2 = \frac{3B^2D^2}{2EF}, A = \frac{E}{D}, C = \frac{B^2D}{4E}. \tag{3.9}$$

Substituting (3.4) and $u(x, t) = u(\xi)$, $\xi = x - \omega t$ with $z(\xi)$ in Remark 2.1 into (3.2) gives rise to the exact traveling wave solutions of Eq. (1.1) as follows:

$$u_1(x, t) = -\frac{3E}{2F} \operatorname{sech}^2\left(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t)\right), \quad ED < 0,$$

$$u_2(x, t) = \frac{3E}{2F} \operatorname{csch}^2\left(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t)\right), \quad ED < 0,$$

$$u_3(x, t) = \frac{3E \operatorname{sech}(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))}{-F[\varepsilon + \operatorname{sech}(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))]}, \quad ED < 0,$$

$$u_4(x, t) = \frac{3E \operatorname{sec}(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))}{-F[\varepsilon + \operatorname{sec}(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))]}, \quad ED > 0,$$

$$u_5(x, t) = \frac{3E \operatorname{csch}(\sqrt{-\frac{E}{D}}(x - \omega t))}{-F[\varepsilon + \operatorname{csch}(\sqrt{-\frac{E}{D}}(x - \omega t))]}, \quad ED < 0,$$

$$u_6(x, t) = \frac{3E \operatorname{csc}(\sqrt{\frac{E}{D}}(x - \omega t))}{-F[\varepsilon + \operatorname{csc}(\sqrt{\frac{E}{D}}(x - \omega t))]}, \quad ED > 0,$$

$$u_7(x, t) = -\frac{3E}{2F} \operatorname{sec}^2(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t)), \quad ED > 0,$$

$$u_8(x, t) = -\frac{3E}{2F} \operatorname{csc}^2(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t)), \quad ED > 0,$$

$$u_9(x, t) = \frac{6BEe^{\varepsilon\sqrt{-\frac{E}{D}}(x-\omega t)}}{F(e^{\varepsilon\sqrt{-\frac{E}{D}}(x-\omega t)} - B)^2}, \quad ED < 0,$$

where ω and B are arbitrary constants.

Taking (3.5) and $u(x, t) = u(\xi)$, $\xi = x - \omega t$ with $z(\xi)$ in Remark 2.1 into (3.2), using Maple, we obtain the following exact traveling wave solutions of Eq. (1.1):

$$u_{10}(x, t) = -\frac{E}{2F}[2 - 3\operatorname{sech}^2(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))], \quad ED > 0,$$

$$u_{11}(x, t) = -\frac{E}{2F}[2 + 3\operatorname{csch}^2(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))], \quad ED > 0,$$

$$u_{12}(x, t) = \frac{E[\varepsilon + 2\operatorname{sech}(\sqrt{\frac{E}{D}}(x - \omega t))]}{-F[\varepsilon - \operatorname{sech}(\sqrt{\frac{E}{D}}(x - \omega t))]}, \quad ED > 0,$$

$$u_{13}(x, t) = \frac{E[\varepsilon + 2\operatorname{sec}(\sqrt{-\frac{E}{D}}(x - \omega t))]}{-F[\varepsilon - \operatorname{sec}(\sqrt{-\frac{E}{D}}(x - \omega t))]}, \quad ED < 0,$$

$$u_{14}(x, t) = \frac{E[\varepsilon + 2\operatorname{csc}(\sqrt{-\frac{E}{D}}(x - \omega t))]}{-F[\varepsilon - \operatorname{csc}(\sqrt{-\frac{E}{D}}(x - \omega t))]}, \quad ED < 0,$$

$$u_{15}(x, t) = -\frac{E}{2F}[2 - 3\operatorname{sec}^2(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))], \quad ED < 0,$$

$$u_{16}(x, t) = -\frac{E}{2F} [2 - 3csc^2(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))], \quad ED < 0,$$

$$u_{17}(x, t) = -\frac{E}{F} [1 - \frac{6Be^\varepsilon\sqrt{\frac{E}{B}}(x-\omega t)}{F(e^\varepsilon\sqrt{\frac{E}{B}}(x-\omega t) - B)^2}], \quad ED > 0,$$

where ω and B are arbitrary constants.

The exact traveling wave solutions of Eq. (1.1) obtained from (3.6) and (3.7) are given by:

$$u_{18}(x, t) = \pm \frac{6CDE^2 e^{\varepsilon\sqrt{-\frac{E}{B}}(x-\omega t)}}{F(De^{\varepsilon\sqrt{-\frac{E}{B}}(x-\omega t)} + EC)^2}, \quad ED < 0,$$

$$u_{19}(x, t) = -\frac{E}{F} [1 - \frac{6CDE^2 e^{\varepsilon\sqrt{\frac{E}{B}}(x-\omega t)}}{(EC - De^{\varepsilon\sqrt{\frac{E}{B}}(x-\omega t)})^2}], \quad ED > 0,$$

respectively, in which ω and C are arbitrary constants.

Substituting (3.8) and $u(x, t) = u(\xi)$, $\xi = x - \omega t$ with $z(\xi)$ in Remark 2.1 into (3.2) gives rise to the exact traveling wave solutions of Eq. (1.1) as follows:

$$u_{20}(x, t) = -\frac{3E}{2F} \left[\frac{2sech^2(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))}{1 - \frac{1}{4}(1 + \varepsilon tanh(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t)))^2} - \frac{sech^4(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))}{[1 - \frac{1}{4}(1 + \varepsilon tanh(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t)))^2]^2} \right],$$

$$u_{21}(x, t) = \frac{3E}{2F} \left[\frac{2csch^2(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))}{1 - \frac{1}{4}(1 + \varepsilon coth(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t)))^2} + \frac{csch^4(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))}{[1 - \frac{1}{4}(1 + \varepsilon coth(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t)))^2]^2} \right],$$

$$u_{22}(x, t) = -\frac{3E}{2F} \left[\frac{2sech^2(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))}{1 + \varepsilon tanh(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))} - \frac{sech^4(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))}{(1 + \varepsilon tanh(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t)))^2} \right],$$

$$u_{23}(x, t) = \frac{3E}{2F} \left[\frac{2csch^2(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))}{1 + \varepsilon coth(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))} + \frac{csch^4(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))}{(1 + \varepsilon coth(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t)))^2} \right],$$

$$u_{24}(x, t) = -\frac{3E}{2F} [2(1 + \varepsilon tanh(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))) - (1 + \varepsilon tanh(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t)))^2],$$

$$u_{25}(x, t) = -\frac{3E}{2F} [2(1 + \varepsilon coth(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t))) - (1 + \varepsilon coth(\frac{1}{2}\sqrt{-\frac{E}{D}}(x - \omega t)))^2],$$

$$u_{26}(x, t) = \frac{12BEe^\varepsilon\sqrt{-\frac{E}{B}}(x-\omega t)}{F[e^\varepsilon\sqrt{-\frac{E}{B}}(x-\omega t) - 2B]^2},$$

with $ED < 0$, and ω and C are arbitrary constants.

Substituting (3.9) and $u(x, t) = u(\xi)$, $\xi = x - \omega t$ with $z(\xi)$ in Remark 2.1 into (3.2) gives rise to the exact traveling wave solutions of Eq. (1.1) as follows:

$$\begin{aligned}
 u_{27}(x, t) &= -\frac{E}{F} + \frac{3E}{2F} \left[\frac{2\operatorname{sech}^2(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))}{1 - \frac{1}{4}(1 + \varepsilon \tanh(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t)))^2} - \frac{2\operatorname{sech}^4(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))}{[1 - \frac{1}{4}(1 + \varepsilon \tanh(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t)))^2]^2} \right], \\
 u_{28}(x, t) &= -\frac{E}{F} - \frac{3E}{2F} \left[\frac{2\operatorname{csc}^2(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))}{1 - \frac{1}{4}(1 + \varepsilon \coth(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t)))^2} + \frac{\operatorname{csc}^4(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))}{[1 - \frac{1}{4}(1 + \varepsilon \coth(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t)))^2]^2} \right], \\
 u_{29}(x, t) &= -\frac{E}{F} + \frac{3E}{2F} \left[\frac{2\operatorname{sech}^2(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))}{1 - \frac{1}{4}(1 + \varepsilon \tanh(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t)))^2} - \frac{\operatorname{sech}^4(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))}{(1 + \varepsilon \tanh(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t)))^2} \right], \\
 u_{30}(x, t) &= -\frac{E}{F} - \frac{3E}{2F} \left[\frac{2\operatorname{csch}^2(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))}{1 + \varepsilon \coth(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))} + \frac{\operatorname{csch}^4(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))}{(1 + \varepsilon \coth(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t)))^2} \right], \\
 u_{31}(x, t) &= -\frac{E}{F} + \frac{3E}{2F} [2(1 + \varepsilon \tanh(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))) - (1 + \varepsilon \tanh(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t)))^2], \\
 u_{32}(x, t) &= -\frac{E}{F} + \frac{3E}{2F} [2(1 + \varepsilon \coth(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t))) - (1 + \varepsilon \coth(\frac{1}{2}\sqrt{\frac{E}{D}}(x - \omega t)))^2], \\
 u_{33}(x, t) &= -\frac{E}{F} \left[1 - \frac{12Be^{\varepsilon\sqrt{\frac{E}{D}}(x-\omega t)}}{(e^{\varepsilon\sqrt{\frac{E}{D}}(x-\omega t)} - 2B)^2} \right]
 \end{aligned}$$

with $ED > 0$, and ω and C are arbitrary constants.

4. Conclusion

In our contribution, we have employed the AODE method to investigate the traveling solutions of the perturbed KGEs in the (1+1)-dimension with quadratic nonlinearity without local inductance and dissipation effect. By using this method, we have successfully found many new types of exact traveling wave solutions. More importantly, different types of exact solutions of the auxiliary equation, Eq. (2.3), may lead to different types of exact traveling wave solutions of NPDEs. It is also clear that one can find new solutions of NPDEs by choosing other types of auxiliary ODEs. The problem of how to give new auxiliary equations and how to find their traveling wave solutions is thus worthy of further research.

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References

[1] Z. Y. Zhang, Z. H. Liu, X. J. Miao and Y. Z. Chen, *Appl. Math. Comput.*, **216**, (2010), 3064.
 [2] Z. Y. Zhang, *Turk. J. Phys.*, **32**, (2008), 235.

- [3] Z. Y. Zhang, Y. X. Li, Z. H. Liu and X. J. Miao, *Commun. Nonlinear Sci. Numer. Simulat.*, **16**, (2011), 3097.
- [4] Z. Y. Zhang, Z. H. Liu, X. J. Miao and Y. Z. Chen, *Phys. Lett. A*, **375**, (2011), 1275.
- [5] X. J. Miao and Z. Y. Zhang, *Commun. Nonlinear Sci. Numer. Simulat.*, **16**, (2011), 4259.
- [6] Z. Y. Zhang, X. Y. Gan and D. M. Yu, *Z. Naturforsch.*, **66a**, (2011), 721.
- [7] A. Yildırım and Z. Pınar, *Comput. Math. Appl.*, **60**, (2010), 1873.
- [8] T. Özis and A. Yildırım, *Chaos, Solitons and Fractals*, **38**, (2008), 209.
- [9] S. T. Mohyud-Din and A. Yildırım, *Z. Naturforsch.*, **65a**, (2010), 1077.
- [10] A. Yildırım, *Z. Naturforsch.*, **63a**, (2008), 621.
- [11] A. Yildırım, S. T. Mohyud-Din and D. H. Zhang, *Comput. Math. Appl.*, **59**, (2010), 2473.
- [12] S. T. Mohyud-Din and A. Yildırım, *Z. Naturforsch.*, **65a**, (2010), 1045.
- [13] A. Yildırım, *Z. Naturforsch.*, **65a**, (2010), 418.
- [14] W. X. Ma and B. Fuchssteiner, *Int. J. Nonlinear Mechanics*, **31**, (1996), 329.
- [15] W. X. Ma, T. W. Huang and Y. Zhang, *Phys. Scr.*, **82**, (2010), 065003.
- [16] W. X. Ma and Z. N. Zhu, *Appl. Math. Computat.*, **218**, (2012), 11871.
- [17] W. X. Ma and J. H. Lee, *Chaos, Solitons and Fractals*, **42**, (2009), 1356.
- [18] W. X. Ma and M. Chen, *Appl. Math. Comput.*, **215**, (2009), 2835.
- [19] W. X. Ma and Y. You, *Trans. Amer. Math. Soc.*, **357**, (2005), 1753.
- [20] W. X. Ma, C. X. Li and J. S. He, *Nonlinear Anal. TMA*, **70**, (2009), 4245.
- [21] W. X. Ma and E. G. Fan, *Comput. Math. Appl.*, **61**, (2011), 950.
- [22] W. X. Ma, Y. Zhang, Y. N. Tang and J. Y. Tu, *Appl. Math. Computat.*, **218**, (2012), 7714.
- [23] A. Biswas, C. Zony and E. Zerrad, *Appl. Math. Comput.*, **203**, (2008), 153.
- [24] R. Sassaman and A. Biswas, *Physics Express*, **1**, (2011), 9.
- [25] Z. Y. Zhang and X. J. Miao, *Comput. Math. Appl.*, **59**, (2010), 1003.
- [26] Z. Y. Zhang, Z. H. Liu, X. J. Miao and Y. Z. Chen, *J. Math. Phys.*, **52**, (2011), 023502.
- [27] C. E. Xiao and Z. Y. Zhang, *Far East J. Appl. Math.*, **60**, (2011), 131.
- [28] M. A. Aguero, M. L. Najera, J. A. Aguilar and J. Sanchez, *Int. J. Theor. Phys.*, **48**, (2009), 2098.
- [29] K. C. Basak, P. C. Ray and R. K. Bera, *Commun. Nonlinear Sci. Numer. Simulat.*, **14**, (2009), 718.
- [30] R. Sassaman and A. Biswas, *Appl. Math. Computat.*, **215**, (2009), 212.
- [31] R. Sassaman and A. Biswas, *Nonlinear Dyn.*, **61**, (2010), 23.
- [32] R. Sassaman and A. Biswas, *Studies in Math. Sci.*, **1**, (2010), 30.
- [33] R. Sassaman, A. Heidari, F. Majid and A. Biswas, *Dynam. Cont. Discr. Impul. Sys. Series A*, **17**, (2010), 275.
- [34] R. Sassaman and A. Biswas, *MESA*, **2**, (2011), 99.
- [35] R. Sassaman, A. Heidari and A. Biswas, *J. Franklin Institute*, **347**, (2010), 1148.
- [36] R. Sassaman and A. Biswas, *Commun. Nonlinear Sci. Numer. Simulat.*, **14**, (2009), 3239.
- [37] A. Esfahani, *Appl. Math. Lett.*, **24**, (2011), 204.
- [38] E. J. Parkes, B. R. Duffy and P. C. Abbott, *Phys. Lett. A*, **295**, (2002), 280.
- [39] Sirendaoreji, *Chaos, Solitons and Fractals*, **19**, (2004), 147.
- [40] Sirendaoreji and J. Sun, *Phys. Lett. A*, **309**, (2003), 387.
- [41] Y. Chen and Z. Y. Yan, *Chaos, Solitons and Fractals*, **26**, (2005), 393.
- [42] Y. Chen, Q. Wang and B. Li, *Chaos, Solitons and Fractals*, **26**, (2005), 231.