

Kantowski–Sachs cosmological model with wet dark fluid in the general theory of relativity

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Abstract: The purpose of this study was to investigate the role of wet dark fluid (WDF) in Kantowski–Sachs space-time in the general theory of relativity. In this theory, we solved the field equations for the case where $\rho_{WDF} = -2p_{WDF}$. Various physical and geometrical properties of the model are also discussed.

Key words: Kantowski–Sachs, wet dark fluid, perfect fluid

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1. Introduction

The most successful theory of gravitation in terms of geometry is Einstein's general theory of relativity. Several alternative theories of gravitation are proposed from time to time to the general theory of relativity. Kantowski–Sachs space-time has a major role in studies of the cosmological model of the universe. As the models are considered to be possible candidates for an early era in cosmology, their astrophysical importance cannot be ignored. Reddy [1] discussed a Bianchi type V inflationary universe in general relativity and investigated the Kantowski–Sachs cosmological model in the presence of a massless scalar field with a flat potential relativity. Mishra [2] used the gauge function approach to construct the cosmological model of the universe in the space-time described by Kantowski–Sachs with perfect fluid as the matter field.

Here, we constructed the cosmological model of Kantowski–Sachs space-time with wet dark fluid (WDF) in the general theory of relativity. In section 2, WDF is discussed in detail. In section 3, with Kantowski–Sachs metric the field equations and its solutions are obtained. Some physical and geometrical properties of the model are discussed in section 4 and concluding remarks are given in section 5. Finally a list of references is given.

Many astronomical observations such as type I supernovae [3–10] give us the result that the universe is flat. Moreover, it is full of the undamped form of energy density. The undamped energy is called dark energy (DE). While 74% of the total energy is attributed to this DE, the other 26% of the energy density consists of matter: 22% dark matter density and 4% baryon matter density. At present, the most interesting problem in modern astrophysics and cosmology is to understand the behavior of DE. Jain et al. [11] studied the axially symmetric cosmological model with WDF in the bimetric theory of gravitation. It is to be noted that our universe experiences accelerated expansion according to the recent cosmological experiments. Hence, we were motivated to use WDF as a model for DE in Kantowski–Sachs space-time. We used an empirical equation of

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state proposed by Tait [12] and Hayward [13] to treat water and aqueous solution.

The equation for WDF is

$$p_{WDF} = \alpha (\rho_{WDF} - \rho^*), \quad (1)$$

where the parameters α and ρ^* are taken to be positive and $0 \leq \alpha \leq 1$. p_{WDF} and ρ_{WDF} represent the pressure and energy density of WDF, respectively.

Cosmologists have proposed many candidates for dark energy to fit the current observations such as cosmological constant, tachyon, quintessence, phantom, and so on. Current studies to extract the properties of the DE component of the universe from observational data focus on the determination of its equation of state $w(t)$, which is the ratio of the DE's pressure to its energy density $w(t) = \frac{p}{\rho}$, which is not necessarily a constant. Methods of restoration of the quantity $w(t)$ from experimental data have been developed and an analysis of the experimental data has been conducted to determine this parameter as a function of cosmic time. The importance of WDF is derived from the fact that it is a good approximation for many fluids, including water, in which the internal attraction of the molecules makes negative pressure possible. One of the virtues of this model is that the square of the sound speed, c_s^2 , which depends on $\frac{\partial p}{\partial \rho}$, can be positive (as opposed to the case of phantom energy, say), which still gives rise to the cosmic acceleration in the current epoch.

The energy conservation equation is

$$\dot{\rho}_{WDF} + 3H (p_{WDF} + \rho_{WDF}) = 0, \quad (2)$$

where H is the Hubble parameter.

Applying the equation of state $3H = \frac{\dot{v}}{v}$, Eq. (2) reduces to

$$\rho_{WDF} = \left(\frac{\alpha}{1 + \alpha} \right) \rho^* + \frac{c}{v(1 + \alpha)} \quad (3)$$

where c is the velocity of light and v is the volume expansion.

WDF has 2 components: one component behaves as a cosmological constant and the other component acts as the standard fluid. The required equation of state is $p = \alpha\rho$. If we take $c > 0$ in Eq. (3), the strong energy condition will not be violated:

$$p_{WDF} + \rho_{WDF} = (1 + \alpha) \rho_{WDF} - \alpha\rho^* = (1 + \alpha) \frac{c}{v(1 + \alpha)} \geq 0 \quad (4)$$

Holman and Naidu [14] used WDF as DE in the homogeneous, isotropic FRW case. Singh and Chaubey [15] studied the Bianchi type I universe with WDF.

2. Field equations and solutions

The Kantowski–Sachs space-time is considered in the form

$$ds^2 = dt^2 - \lambda^2 dr^2 - k^2 (d\theta^2 + \sin^2\theta d\varphi^2), \quad (5)$$

where $\lambda = \lambda(t)$, $k = k(t)$.

Space-time (5) represents homogeneous but anisotropically expanding (contracting) cosmologies. It also provides models, where the effects of anisotropy can be estimated and compared with the Friedman–Robertson–Walker class of cosmologies [16].

The relativistic field equations are

$$G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij} = -\kappa T_{ij} \quad (6)$$

where G_{ij} is the conventional Einstein tensor, R_{ij} is the Ricci tensor, $R = g^{ij}R_{ij}$ is the Ricci scalar, κ is a scalar quantity, and T_{ij} is the energy momentum tensor given as

$$T_{ij} = (\rho_{WDF} + p_{WDF})v_iv_j - p_{WDF}g_{ij} \quad (7)$$

with

$$g^{ij}v_iv_j = 1 \quad (8)$$

where v_i is the 4 velocity or the flow of the fluid. p_{WDF} and ρ_{WDF} are respectively the pressure and energy density of WDF.

Using the co-moving coordinates system $(0, 0, 0, 1)$, the field equation (6) for the metric (5) can be written as

$$2\frac{k_{44}}{k} + \frac{k_4^2}{k^2} + \frac{1}{k^2} = -8\pi\kappa p_{WDF} \quad (9)$$

$$\frac{k_{44}}{k} + \frac{\lambda_4 k_4}{\lambda k} + \frac{\lambda_{44}}{\lambda} = -8\pi\kappa p_{WDF} \quad (10)$$

$$\frac{k_4^2}{k^2} + 2\frac{\lambda_4 k_4}{\lambda k} + \frac{1}{k^2} = 8\pi\kappa \rho_{WDF} \quad (11)$$

The suffix 4 after the field variable λ and k denotes ordinary differentiation with respect to the cosmic time t only.

The energy conservation equation $T_{;b}^{ab} = 0$ yields

$$[p_{WDF} + \rho_{WDF}] \left(\frac{\lambda_4}{\lambda} + 2\frac{k_4}{k} \right) = 0 \quad (12)$$

Eq. (12) gives 2 cases:

Case 1. $p_{WDF} + \rho_{WDF} = 0$

In this case, because of the reality condition $p_{WDF} = \rho_{WDF} = 0$ and hence the model reduces to the vacuum case. The physical parameters pressure (p_{WDF}) and energy density (ρ_{WDF}) of the WDF are identically zero. Hence the metric for the vacuum case can be written as

$$ds^2 = dt^2 - (d_1 \sin\sqrt{c_1}t + d_2 \cos\sqrt{c_2}t)^2 dr^2 - \left(b_1 e^{\sqrt{\frac{c_3}{2}}t} \right)^2 (d\theta^2 + \sin^2\theta d\varphi^2) \quad (13)$$

where b_1, c_1, d_1, d_2 are constants of integration.

Case 2.

$$\frac{\lambda_4}{\lambda} + 2\frac{k_4}{k} = 0 \quad (14)$$

Using Eq. (14) in Eqs. (9)–(11), we obtain

$$2\frac{k_{44}}{k} + \frac{k_4^2}{k^2} + \frac{1}{k^2} = -8\pi\kappa p_{WDF} \quad (15)$$

$$-\frac{k_{44}}{k} + 4\frac{k_4^2}{k^2} = -8\pi\kappa p_{WDF} \quad (16)$$

$$-3\frac{k_4^2}{k^2} + \frac{1}{k^2} = -16\pi\kappa\rho_{WDF} \quad (17)$$

Due to the nonlinear nature of the field equations, we take the following equation of state in order to obtain a solution to the model as

$$\rho_{WDF} + 2p_{WDF} = 0 \quad (18)$$

Now, using Eq. (18), field equations (15)–(17) reduce to

$$2\frac{k_{44}}{k} + \frac{k_4^2}{k^2} + \frac{1}{k^2} = -8\pi\kappa p_{WDF} \quad (19)$$

$$-\frac{k_{44}}{k} + 4\frac{k_4^2}{k^2} = -8\pi\kappa p_{WDF} \quad (20)$$

$$-3\frac{k_4^2}{k^2} + \frac{1}{k^2} = -16\pi\kappa p_{WDF} \quad (21)$$

Eqs. (19)–(21) give

$$\frac{k_{44}}{k} + 8\frac{k_4^2}{k^2} = 0 \quad (22)$$

On solving Eq. (22), we obtain the result of the metric potential

$$k = (At + B)^{\frac{1}{9}} \quad (23)$$

and subsequently

$$\lambda = C(AT + B)^{-\frac{2}{9}}, \quad (24)$$

where A, B, and C are constants of integration.

Hence the metric (5) of the space is

$$ds^2 = dt^2 - C^2(Q(t))^{-2}dr^2 - Q(t)(d\theta^2 + \sin^2\theta d\varphi^2) \quad (25)$$

where $Q(t) = (At + B)^{\frac{2}{9}}$

3. Physical and geometrical properties of the model

In this section, various physical and geometrical properties of the model are discussed.

The pressure and energy density for the WDF are found to be

$$\rho_{WDF} = -2p_{WDF} = -\frac{1}{4\pi\kappa} \left[\frac{A^2}{27} (At + B)^{-2} - (At + B)^{-\frac{2}{9}} \right] \quad (26)$$

The scalar expansion $\Theta = u^i_{;i}$ is calculated as

$$\Theta = u^i_{;i} = 3A(At + B)^{-1}, \quad (27)$$

from which it is evident that $\Theta \rightarrow 0$ as $t \rightarrow \infty$, i.e. the universe is expanding with increase in time and the rate of expansion is slow with increase in time.

Moreover, the rotation ω and acceleration \dot{u}_i turn out to be zero. The streamlines of the fluid are geodesics.

The shear scalar of the model is

$$\sigma^2 = \frac{3A^2}{(At + B)^2} \quad (28)$$

since $\sigma^2 \rightarrow 0$ as $t \rightarrow \infty$ and $\sigma^2 \rightarrow 0$ constant as $t \rightarrow 0$. The shape of the universe changes uniformly in the x and y directions only and the rate of change of the universe becomes slow with increase in time.

The ratio to anisotropy to expansion is

$$\frac{\sigma^2}{\Theta^2} = \frac{1}{3A} \neq 0 \text{ for } t = 0 \quad (29)$$

Thus, there is a singularity at $t = 0$ for A is not very large.

The spatial volume is found to be

$$V = C \sin \theta \quad (30)$$

Hence $V = 0$, when $C = 0$, for $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \pm 3, \dots$

Hence the universe starts evolution with constant volume until infinite future.

4. Concluding remarks

In understating the early stage of evolution of the universe, the Kantowski–Sachs cosmological model plays an important role. Here, we found the Kantowski–Sachs cosmological model in WDF. The model obtained here is expanding and starts evolution with constant volume until infinite future.

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